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DEPARTMENT:

Statistics

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PROJECT TITLE:

Kaplan-Meier Survival Curves in Three Samples

Kaplan-Meier Survival Curves in Three-Samples

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Motivating Question

Can we estimate three survival curves and uphold stochastic ordering?

Introduction

The Kaplan-Meier (KM) Estimator is the most well-known survival estimator in survival analysis; however, using data from a Stanford University study in 1977 comparing degree 1 and degree 2 Carcinoma Cancer patients, there is a violation of an assumption called stochastic ordering. We expect a degree 1 Carcinoma Cancer patient to live longer than a degree 2 patient due to the severity of the disease. We will look at an estimator by Rojo (2004) to force stochastic ordering as we evaluate the Carcinoma data as well as a theoretical three-sample case.

Definitions

- Cumulative Distributive Function (CDF):

the probability that the variable takes a value less than or equal to t, time:

$$F(t) \leq P(X \leq t)$$

For a continuous distribution, this can be expressed mathematically as:

$$F(t) = \int_{-\infty}^{t} f(x)\,dx$$

- Survival Function and the Kaplan-Meier:

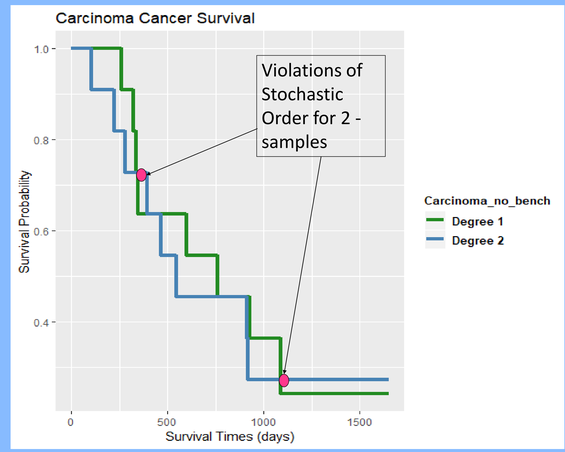
- The survival function, S(t) = 1 – F(t).
The survival function looks at the "Probability of surviving beyond time t".

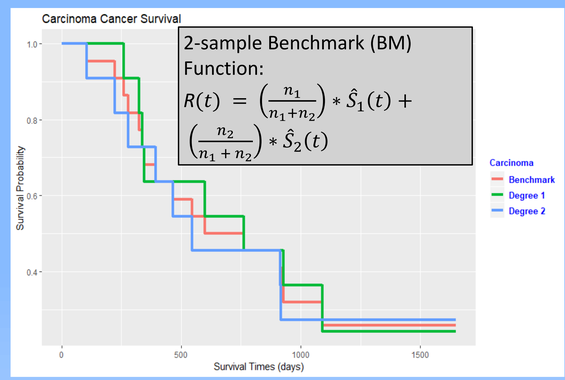
Let t_j = times of death; d_j = deaths at time t_j; n_j = number of participants remaining in the study at time t_j.
For $t \in [t_j, t_{j+1}), j = 1, 2, 3, \ldots,$ we have…

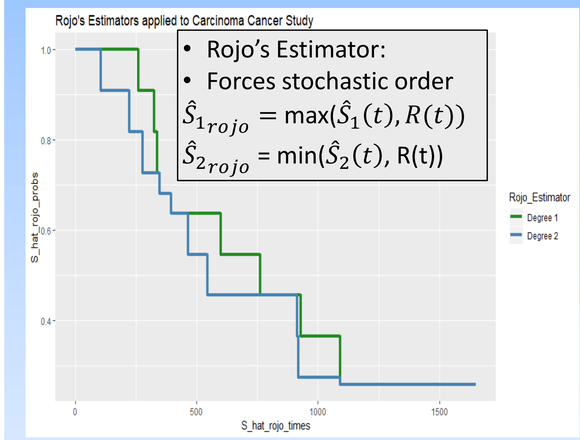
Kaplan-Meier Estimator: $\hat{S}(t) = \Pi_{i=1}^{j}\left(1 - \frac{d_i}{n_i}\right)$.

- Stochastic Ordering: Let $X_1(t) \sim F_1(t)$ and $X_2(t) \sim F_2(t)$ and $F_1(t) \leq F_2(t)$ for all t. In terms of respective survival functions :$S_2(t) \leq S_1(t)$ for all t.

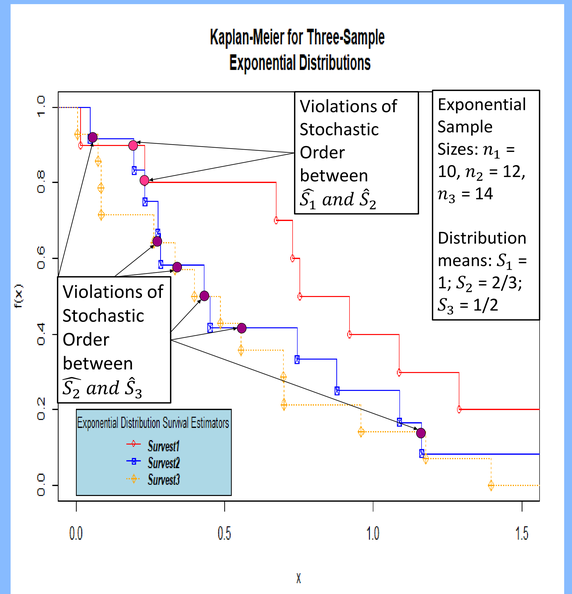
Carcinoma Cancer Patients – 2 sample Kaplan-Meier Survival Curves

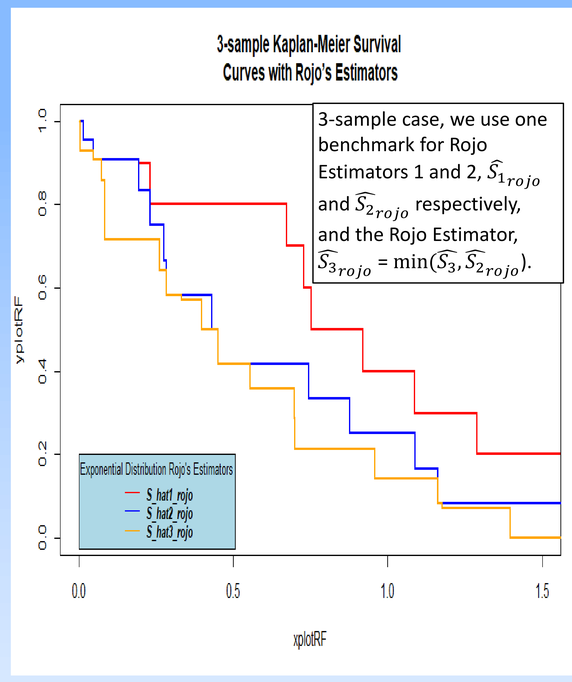






3-sample Exponential Kaplan-Meier Survival Curves





Future Statistical Research and Conclusion

Conclusions:

- We were successfully able to estimate three survival curves that uphold Stochastic order, where $\widehat{S_1}$ and $\widehat{S_2}$ by the benchmark to set Rojo's Estimators.
- We achieved getting $\widehat{S_{3_{rojo}}}$ by taking the $\min\left(\widehat{S_{2_{rojo}}}, \widehat{S_3}\right)$ where $\widehat{S_3}$ is the Kaplan – Meier survival curve.

Future Research:

- We can find the Benchmark (weighted average) between $\widehat{S_{2_{rojo}}}$ and $\widehat{S_{3_{rojo}}}$ instead and then find the $\widehat{S_1}$ $\max\left(\widehat{S_{1_{Rojo}}}, \widehat{S_1}\right)$ where $\widehat{S_1}$ is the Kaplan – Meier survival estimator.
- Finding the bias, mean squared error, and root mean squared errors for the three-sample case sampling from various distributions such as Weibull and Gamma. This can be done for the technique when the benchmark is between $\widehat{S_1}$ and $\widehat{S_2}$, as well as the case when the benchmark is found between $\widehat{S_2}$ and $\widehat{S_3}$.
- Apply similar model to up and coming data on CO-vid 19.

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