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DEPARTMENT:

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Dr. Ed Bonan-Hamada

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Amy Sullivan

PROJECT TITLE:

Cognitive Science and Mathematics Education

# Cognitive Science in Mathematics Education

**Amy Sullivan**
**Dr. Ed Bonan-Hamada**, *Faculty Advisor*

COLORADO MESA UNIVERSITY

## Abstract

Addition is regarded as a fundamental part of mathematical practice. Yet, we do not understand how students construct their understanding of this operation. This project will look at three methods of addition and then discuss the underlying logical systems that give rise to these methods. We will then consider the cognitive neuroscience and the human reasoning in mathematics behind one of these methods. Finally, we will consider some teaching methods and strategies that align with a supposed model of student learning.

## Motivating Questions

- Can we construct mathematics education in such a way that our teaching methods mirror how student's brains structure the mathematics they are learning?
- What is underlying the ability to do this?

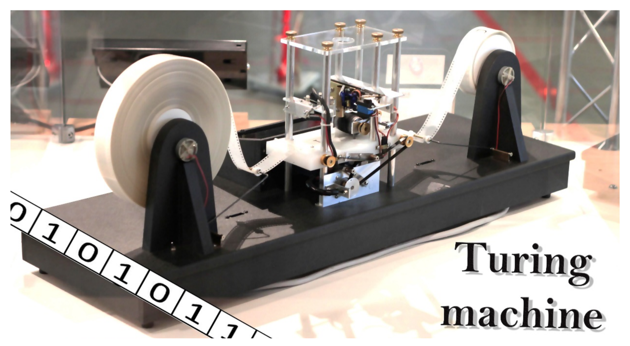
## Three Methods of Addition

**Zermelo-Fraenkel's Axiomatic Set Theory (ZFC) [3]**

- Addition is the cardinality of the union of disjoint sets.
- Theoretical and detailed in the descriptions
- Describes the abstract way of thinking about addition and the rules that apply, not how to calculate the values in a math problem

**Turing Machine (TM) [10]**

- Has an infinite tape with discrete cells which can hold a symbol and has a perfect memory
- Preforms a series of steps in a purely mechanical process – this requires no thought
- Does not need to have a fundamental understanding of the idea of addition




https://medium.com/free-code-camp/how-the-turing-machine-interactive-simulator-can-help-you-understand-algorithms-a0fa88d0e057

**Dr. Catherine Sophian [8]**

- Addition is a matter of finding the whole given the parts
- The whole of some material can be grouped into smaller sizes without changing the total size

## Neuroscience of Mathematics

**Three Systems of the brain which enable us to compute mathematics**

- Quantity System
- Visual System
- Verbal System

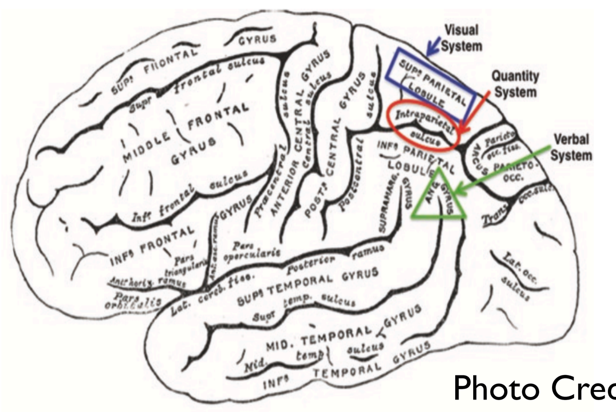



Photo Credit [2]

**Dr. Brown's Research [2]**

- There is not a locality in the brain that computes algebra problems
- Areas function together to give meaning, context, and interpretations to computational problems

Students use their existing knowledge to create new characteristics of a subject and to help students to develop their mathematical maturity, we cannot rush students through concepts [2].

## Reasoning by Construction of Models

- For children to properly use the concepts that they are learning, they must reason *from an interpretation to an interpretation* [9]
- This infrastructure is made up of prior knowledge that the student may have gained or experienced, and their conclusions are drawn from that structure
- For students to form a solid mental infrastructure, they need the time and opportunity to develop their interpretations on their own [1]

The goal of mathematics education should be to enable students to experiment or play with the mathematics so that they can renovate, create, and discover, which is how they construct their interpretation

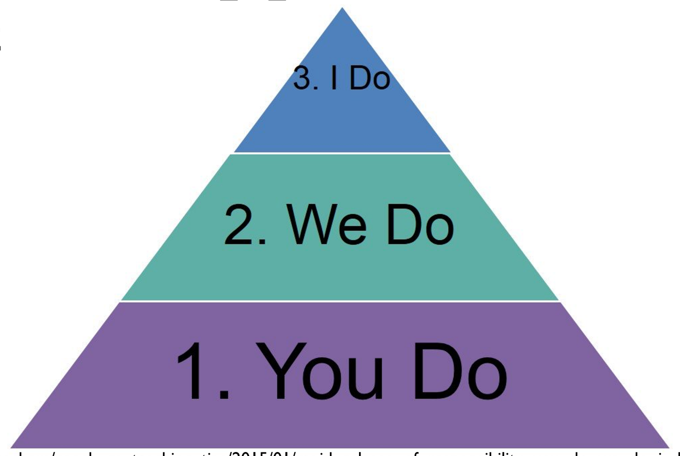
## Teaching Methods

**Over-imitation by Dr. Laurie Santos [7]**

- Children tend to try to over imitate the instructor's actions
- The direct instruction alters up the approach of the children

**Gradual Release of Responsibility Model Reversed [5]**

1. Students are given a problem that they might not be familiar with
2. Student work individually
3. Students pair up and collaborate
4. Students receive direct instruction




http://blogs.edweek.org/teachers/coach_gs_teaching_tips/2015/01/rapid_release_of_responsibility_you_do_we_do_i_do.html

## Conclusion

Reversing the Gradual Release of Responsibility Model encourages a deeper reasoning level in the teaching of mathematics. This disables student's natural tendency to over-imitate what the instructor does, which allows students to develop a stronger framework to reason from. Our brains do not have a mathematics portion, but rather collaborates the visual, mental, and vocal aspect of the brain to solve problems and compute arithmetic.

There is so much to learn and we have not truly discovered with how children learn mathematics. We need more research to find out how we can better revamp the curricula or change the timing or type of instruction so that it is more conducive to student learning and cognitive development.

## Future Directions

- Continue research in the psychology and cognition of mathematics
- Long-range goal: Integrate this information into curriculum design

## Acknowledgements

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## References

[1] Berch, D., Geary, D.C., Mann-Koepke, K. (2016). *Mathematical Cognition and Learning: Development of Mathematical Cognition*. n.p.: Academic Press

[2] Brown, R.D. (2018) *Neuroscience of Mathematical Cognition Development: From Infancy Through Emerging Adulthood*. Cincinnati, OH: Springer International Publishing

[3] Jech, T. (1978) *Set Theory*. New York, New York: Harcourt Brace Jovanovich Publishers

[4] McCarthy, J., (2013) *Addition on Turing Machines: Unary Addition.*

[5] NCTM. (2018) *Thinking about Instructional Routines in Mathematics Teaching and Learning.*

[6] Ping, R.M., Goldin-Meadow, S. (2008) *Hands in the Air: Using Ungrounded Iconic Gestures to Teach Children Conservation of Quantity*. University of Chicago: American Phycological Association

[7] Santos, L., (2019) *Hidden Brain: What Monkeys can Teach us About Being Human.*

[8] Sophian, C (2007) *The Origins of Mathematical Knowledge in Childhood.* New York, London: Lawrence Erlbaum Assoc,

[9] Stenning, K & Van-Lambalgen, M. (2008) *Human Reasoning and Cognitive Science.* Cambridge, MA, London, England: Massachusetts Institute of Technology Press

[10] Ullman J.D., Hopcroft, J.E. (1979). Introduction to Automata Theory, Languages, and Computation. Reading, MA: Addison-Wesley Publishing Company