

PHYSICS SEMINAR

The Looking-Glass Transform: The Schrödinger Equation as a Centered-Difference Approximation of the Classical Liouville Equation

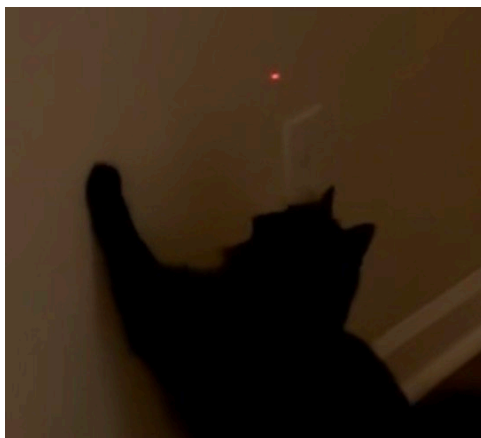
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12:30 – 1:30 p.m.

Wubben 264

The Schrödinger equation lies a single approximation step from the classical Liouville equation. Lifting classical Hamiltonian dynamics to a conservation law for the phase-space distribution, Fourier transforming in momentum, and applying a centered-difference approximation to the potential gradient yields an equation that separates into coupled Schrödinger equations; the phase-space distribution is the Wigner quasi-probability function, and the Born position density is its momentum marginal. The ingredients of this route are classical results of the phase-space formulation of quantum mechanics, and closely related Liouville-to-Schrödinger constructions exist. The contribution here is the framing and its consequences: the centered difference is kept finite rather than taken as a limit, so quantum dynamics appears as a controlled discretization of classical dynamics—exact for quadratic potentials, with explicit leading error $\frac{\hbar^2}{24} \frac{\partial^3 V}{\partial x^3} \frac{\partial^3 w}{\partial p^3}$, the Moyal correction read in reverse. Two observations follow from this viewpoint: Wigner negativity admits a classical reading as fluctuation below a uniform background, since $w \rightarrow w + c$ is invisible to the Liouville flux; and the transformed dynamics is well posed on a state space strictly larger than the quantum one, which contains the classical point preparations excluded by the uncertainty principle. We are explicit about what the framing does not do: where the approximation error accumulates—anharmonic potentials at long times—experiment follows the quantum dynamics, not the classical one. The value of the transform is structural and pedagogical: it locates exactly what must be added to probabilistic classical mechanics to obtain quantum mechanics, in the language of Fourier transforms and finite differences.



Refreshments!