The Ergodic Theory of History & Zeno Contours

John Gill 6 January 2013

Abstract: A simple and non-predictive analogue of the paths of history, using complex variable theory. Not meant as a serious undertaking!

Are there periods in history in which processes are so powerful that if minor incidents were changed there would be little or no change in the outcome? Consider, for example, the rise of the Nazi cult in Weimarer Germany during the first half of the 20th century. What if Hitler had been assassinated early on . . . would the cruel atrocities and attempt at world domination simply not have occurred? Or would the process in play have been so compelling and strong that the horrors would appear nevertheless?

The science fiction writer, Stanislaw Lem, described an *ergodic theory of history* that assumes such phenomena [1]. The counter-argument – "the butterfly effect" – cites chaos theory that includes as its principal tenant the mathematical concept of "sensitive dependence on initial conditions" (SDIC) wherein very minor alterations of events can have hugely divergent consequences.

Here is some of what Lem had to say (in the context of time travel stories):

"Even though a circular causal structure may signalize a frivolous type of content, this does not mean that it is necessarily reduced to the construction of comic antinomies for the sake of pure entertainment. The causal circle may be employed not as the goal of the story, but as a means of visualizing certain theses, e.g. from the philosophy of history. Slonimaki's story of the Time Torpedo[3] belongs here. It is a [belletristic] assertion of the "ergoness" or ergodicity of history: monkeying with events which have had sad consequences does not bring about any improvement of history; instead of one group of disasters and wars there simply comes about another, in no way better set.

A diametrically opposed hypothesis, on the other hand, is incorporated into Ray Bradbury's "A Sound of Thunder" (1952). In an excellently written short episode, a participant in a "safari for tyrannosaurs" tramples a butterfly and a couple of flowers, and by that microscopic act causes such perturbances of causal chains involving millions of years, that upon his return the English language has a different orthography and a different candidate not-- liberal but rather a kind of dictator-- has won in the presidential election. It is only a pity that Bradbury feels obliged to set in motion complicated and unconvincing explanations to account for the fact that hunting for reptiles, which indeed fall from shots, disturbs nothing in the causal chains, whereas the trampling of a tiny flower does (when a tyrannosaur drops to the ground, the quantity of ruined flowers must be greater than when the safari participant descends from a safety zone to the ground). "A Sound of Thunder" exemplifies an "anti-ergodic" hypothesis of history, as opposed to Slonimski's story. In a way, however, the two are reconcilable: History can as a whole be "ergodic" if not very responsive to local disturbances, and at the same time such exceptional hypersensitive points in the causal chains can exist, the vehement disturbance of which produces more intensive results. In personal affairs such a "hyperallergic point" would be, for example, a situation in which a car attempts to pass a truck at the same time that a second car is approaching from the opposite direction."

The dynamical systems of mathematics (sets of points and functions to be iterated), particularly in the complex plane, show both sorts of narratives. There are instances of very stable regions in which all points under iteration of a particular complex function converge to a central point, an "attractor"; and there are at times very sensitive regions where starting the iteration process at two different but neighboring points leads to severe divergence of outcomes.

Before addressing these issues, let us turn briefly to the passage of time. In what fashion does history – or time itself – progress? What appears in everyday experience is a sort of continuum in the evolution of an event, but are there infinitesimal "particles" of time wherein motion is a sequence of tiny changes progressing in a discrete fashion from instant to instant? Some physicists speculate there is a lower bound on the length of a time interval that can be measured, or that even exists: an indivisible unit called a *chronon*, whose dimension depends upon a particle's charge and mass. Or perhaps there is no conflict between continuous and static . . . the former arising from the latter? The nature of elementary calculus supports this view. Here, an analogue between a certain dynamical system and motion or evolution in the normal world is developed, with an analysis of change during an interval of time inferring the continuum arises from the discrete. We begin with a point z in the complex plane and an interval of time I: $0 \le t \le 1$.

The point z might be thought of as an analogue to a *set of circumstances* in the normal world: an *event* for short. Starting at one such point yields a path over the time interval. Shifting from one initial point to another means a different set of initial circumstances, leading perhaps to a different outcome. The motion of the point over this period will be thought of as its *evolution*. The evolution might be strongly influenced by relatively stable and ongoing circumstances, or the element of chance might prevail, leading to a complex system difficult to analyze.

For example, consider alternate history accounts of WWII. Suppose Hitler had been assassinated in the early 1930s. It remains highly probable that the atrocities and war that followed would still have been severe, with some equally talented and psychotic leader emerging. However, had he been assassinated during the course of the war there very well might have been a coup by the Wehrmacht that would have let to a rapid peace process.

In the first scenario there exists a powerful *attractor*, induced by the Treaty of Versailles and other movements, that might have made it virtually certain that the Nazis would ascend to power. The analogue of this kind of attractor is an *attractive fixed point* α (FP) in the complex plane $\mathbb{C}$. In the second scenario there might have been a problematic outcome, resulting from a more random environment.

Returning to mathematical technicalities, the time interval I will be subdivided into n equal subintervals, and the motion of the point over the kth subinterval will be the result of applying an (evolution) function $g_{k,n}$ to the value of the evolved point at the beginning of that subinterval. In the language of mathematics, the composition

$$G_{n,n}(z) = g_{n,n} \circ g_{n-1,n} \circ \cdots \circ g_{2,n} \circ g_{1,n}(z) = \mathop{\mathcal{L}}_{k=1}^{n} g_{k,n}(z)$$

describes the journey of the original point to its new position at the end of the time interval. Now, each application of an evolution function represents a static or discrete "jump" from one place to the next. But if the number of subdivisions of I is allowed to become infinite, the discrete is replaced by the continuous. I.e.,

$$\text{(i)} \qquad G(z) = \lim_{n\to\infty} \mathop{\mathcal{L}}_{k=1}^{n} g_{k,n}(z)$$

The nature of observed reality requires the size of the discrete jumps reduce to 0 as n becomes infinite:

$$\lim_{n\to\infty} g_{k,n}(z) = z \quad \forall k \le n .$$

Even the most dramatic and abrupt changes, when perceived over the tiniest of time intervals appears gradual. Think slow-motion photography of a bullet exiting a gun barrel.

(The analogue of $g_{k,n}$ in the real world is of course a mysterious and evasive concept, its existence a matter of metaphysical contemplation!)

But in our mathematical model we now see that the following conditions should be met:

1) $g_{k,n}(S) \subseteq S$, a suitable simply-connected domain,

2) $\lim_{n\to\infty} g_{k,n}(z) = z \quad \forall k \le n$, and

3) $G(z) = \lim_{n\to\infty} \mathop{\mathcal{L}}_{k=1}^{n} g_{k,n}(z)$ exists.

Toward this end a convenient set of functions is the following:

$$g_{k,n}(z) = z + \eta_{k,n}\varphi(z)$$

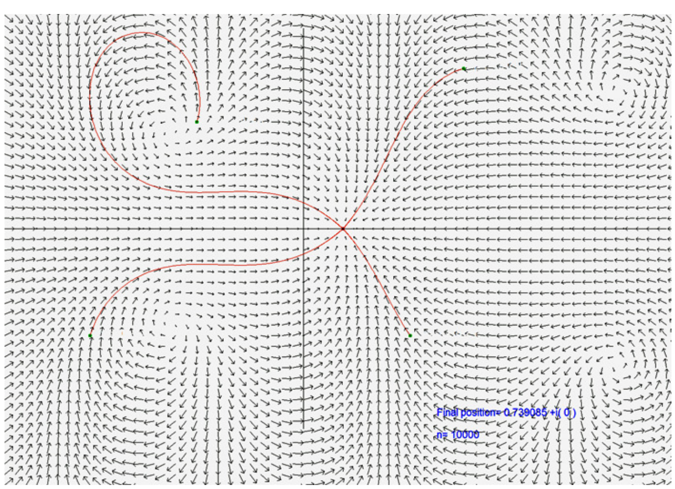
where $z \in S$ and $g_{k,n}(z) \in S$ for a convex set S in the complex plane. Require $\lim_{n\to\infty} \eta_{k,n} = 0$, where (usually) $k = 1, 2, \ldots, n$ (and $\lim_{n\to\infty} \eta_{k,n}\varphi(z) = 0$). Set $G_{1,n}(z) = g_{1,n}(z)$, $G_{k,n}(z) = g_{k,n}\left(G_{k-1,n}(z)\right)$ and $G_n(z) = G_{n,n}(z)$ with $G(z) = \lim_{n\to\infty} G_n(z)$, when that limit exists. I.e., (i). The *Zeno contour* is a graph of this iteration, i.e., *the path* followed by z . Normally, $\varphi(z) = f(z) - z$ for a vector field $\mathbb{F} = f$. [2]

Let us now return to Lem's ergodic theory of history. A complex analysis analogue of a situation in which minor changes in a narrative do not result in pronounced changes in outcome occurs when Zeno contours terminate at the same point – an attractive fixed point α of some function f : Suppose $\left|f(z) - \alpha\right| < \rho\left|z - \alpha\right|$, $0 \le \rho < 1$ in a convex region $z \in S$. Then [2]

$$\left|G_n(z) - \alpha\right| \le \left(1 - \eta_{k,n}(1-\rho)\right)^n \cdot \left|z - \alpha\right| \to 0 \text{ for appropriate } \eta_{k,n}$$

In the context of history or event evolution this strong force might be societal (the struggle for equal rights), political (the rise of the Nazi regime), geological (the eruption of Vesuvius) , astronomical (an asteroid impact), technological (the creation of nuclear weapons). Minor disruptions or alterations along the chronological path do not preclude major developments (or catastrophes).

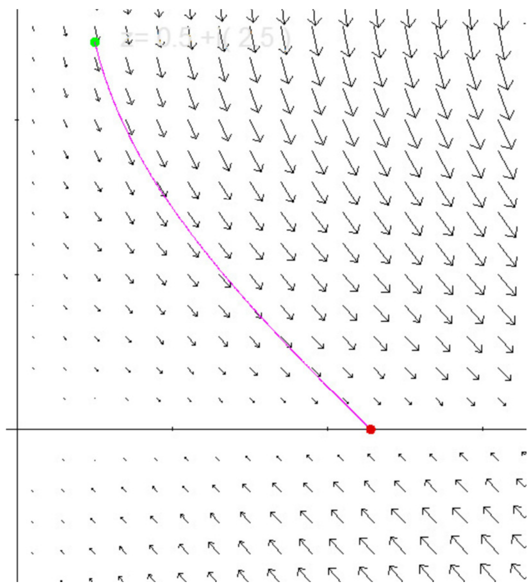
Figure 1: $\mathbb{F} = Cos(z)$. Four Zeno contours converge to the strongly-attractive fixed point. Observe that were a contour to begin in one of the whirls it might spin off, well away from the fixed point. It then might continue its wayward journey or ultimately return to the FP.



The system in figure 1 is extraordinarily stable. Virtually all initial values of z anywhere in the vicinity of the FP will travel to that point over the interval I.

Another scenario demonstrates the evolution of a point (or event) z to an attractor that is *predetermined* by the choice of z. Slight variations in starting positions lead to slight variations of terminal positions (or evolution outcome):

Figure 2: $\mathbb{F} = \dfrac{x\sin(y)}{3} + x + iy\left(1 - \dfrac{x}{3}\right)$

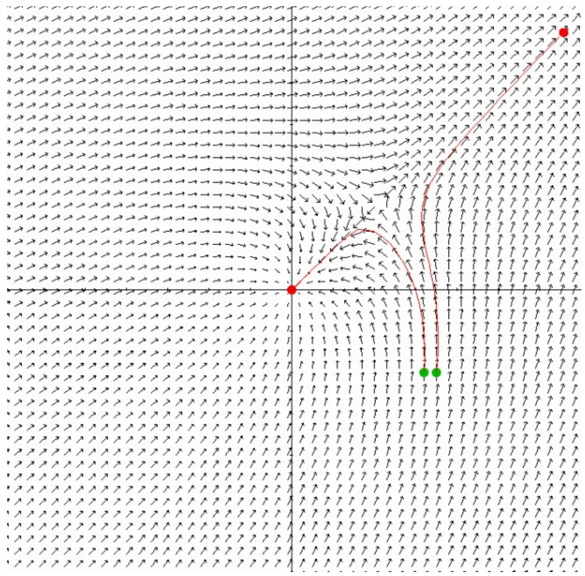


A real-world analogue might be any situation in which there is a certain inevitability associated with starting at a set of events that evolve to finales, and minor variations of initial choice lead to minor variations of outcome, in contrast to powerful attractive forces leading to a single outcome.

We now return to Lem's comment about "hyperallergic points" in the causation chain. Suppose a candidate is running for high political office, and is "bulletproof", so that only some highly improbable event would cause him to lose the election: such as passing a car into the path of an oncoming car. Then the strong attractor disintegrates and chaos results.

However, barring such an unfortunate event path, most paths – starting "near" the attractor - would lead to his election and perhaps a repudiation of the country's adventurous war efforts.

Figure 3: $\mathbb{F} = \dfrac{z^2}{z-\beta}$, $\beta = 2+2i$



A slight change in initial event results in a totally different outcome, with event evolution paths diverging dramatically.

[1] S. Lem, Science Fiction Studies, Vol. I, No. 3, Spring 1974

[2] J. Gill, *Informal Notes: Zeno Contours, Parametric Forms, & Integrals* , Comm. Anal. Th. Cont. Frac., Vol XX (2014)