ABSTRACT

Optimizing Gaussian Quadrature for Positive Definite Strong Moment Functionals
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Brian A. Hagler
University of Texas of the Permian Basin, Odessa, TX 79762 USA
hagler_b@utpb.edu

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Classical Gaussian quadrature is anchored in the space of real polynomials at polynomial degree 0 and is monotonically directed by successively increasing polynomial degree. The results of recent research, investigating weighing anchor and charting various courses in search of the best currents in the space of real Laurent polynomials, are presented. Standard error bound minimization anchoring and steering algorithms which utilize the moments of positive definite strong moment functionals to guide the construction of ordered orthogonal Laurent polynomial sequences are introduced.

Average Anchoring Algorithm.
INPUT (1) The maximum allowable magnitude *MaxAbsPower* of the initial power. (2) The function $r(x)$ being analyzed. (3) Moments μ_k.
OUTPUT An initial power $p(1)$ which, in a neighborhood of 0, minimizes the average of the two rank 2 Standard Error Bounds corresponding to the two possible directions $d(2) = \pm 1$.

Step 1 Set $p_1 = 0$ and $g = 1$, and define the measure
$$B_p = \max_{x\in[a,b]} \left| \frac{d^2}{dx^2}\left(r(x)/x^{2p}\right)\right| \times (\mu_{2p+2} - \mu_{2p+1}^2/\mu_{2p}). + \max_{x\in[a,b]} \left| \frac{d^2}{dx^2}\left(x^{2p}\, r(1/x)\right)\right| \times (\mu_{2p-2} - \mu_{2p-1}^2/\mu_{2p}).$$
Step 2 If $|p_1| \geq MaxAbsPower$,
OUTPUT(Message: Power range restriction reached.)
and go to Step 5.
Step 3 If $B_{p_1+g} < B_{p_1}$, set $p_1 = p_1 + g$ and go to Step 2.
Step 4 If $B_{p_1+g} \geq B_{p_1}$ and $p_1 = 0$ and $g \neq -1$, set $g = -1$ and go to Step 3.
Step 5 Set $p(1) = p_1$ and OUTPUT(p_1).
STOP.

Steering Algorithm. With the initial power $p(1)$ chosen, direction $d(n)$ is determined by comparing the standard error bounds corresponding to $r(x)$ for the two possible directions to expand the OLPS at rank n.

References

[1] B. A. Hagler, *Ordered OLPS and CF Nth Numerators*, Linear Algebra and Its Applications 422 (2007) 100-118.