# Classroom Notes: Images of Fixed-Point Continued Fractions

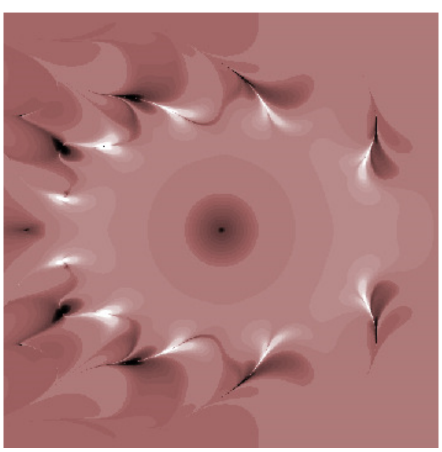
John Gill   December 2013

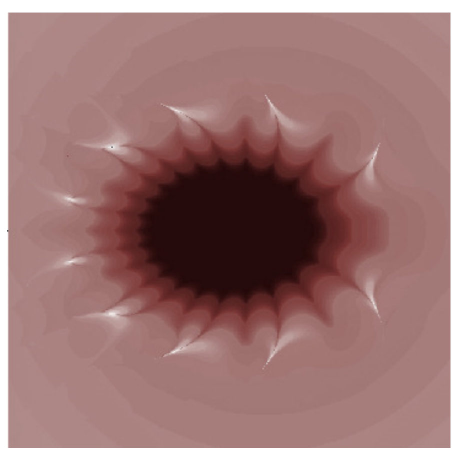
Abstract: Computer generated images in the complex plane of iterations of certain analytic continued fractions.

*Fixed-point continued fractions* have the form

(1) $\dfrac{\alpha_1\beta_1}{\alpha_1+\beta_1-}\;\dfrac{\alpha_2\beta_2}{\alpha_2+\beta_2-}\;\cdots$ where $\alpha_k,\beta_k$ are the fixed points of the function $f_k(\zeta)=\dfrac{\alpha_k\beta_k}{\alpha_k+\beta_k-\zeta}$. The nth approximant of the continued fraction then can be written $F_n(\zeta)=f_1\circ f_2\circ\cdots\circ f_n(\zeta)$. If $\alpha_k=\alpha_k(z)$ and $\beta_k=\beta_k(z)$, we have $F_n(z,\zeta)=f_1\circ f_2\circ\cdots\circ f_n(z,\zeta)$. The following images show magnitudes of simple complex flux (SCF), $\left|F_n(z,0)\right|$ , or fixed-point flux (FPF), $\left|F_n(z,0)-z\right|$. Dark=small, light=large. Several variations on FPCFs are shown, as well. [Liberty Basic V 4.04 (2013)]

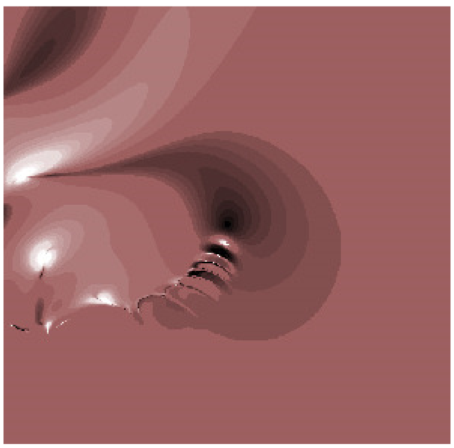
*Example 1*: $f_k(z,\zeta)=\dfrac{kz}{k+z-\zeta}$, $-6\le x,y\le 6$, $n=10$   SCF & FPF

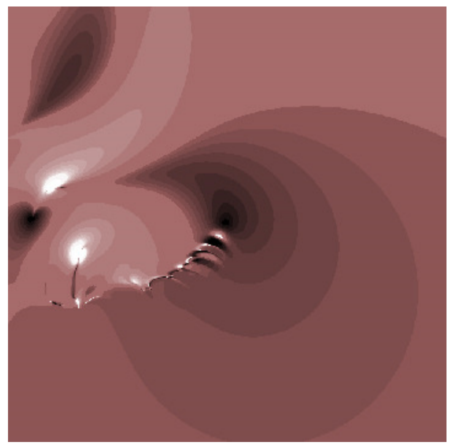




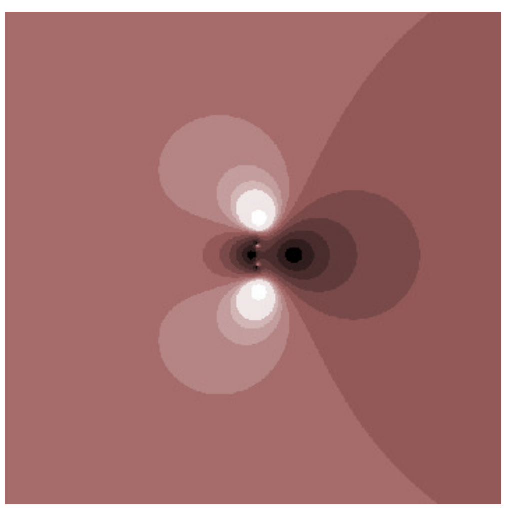
*Example 2*: $f_k(z,\zeta)=\dfrac{(1+ik)^2 z}{1+ik+z-\zeta}$ and $f_k(z,\zeta)=\dfrac{(.5+ik)^2 z}{.5+ik+z-\zeta}$, $-6\le x,y\le 6$, $n=10$ SCF

A Variation on fixed-point CFs. *Note*: a programming error produced these two images!

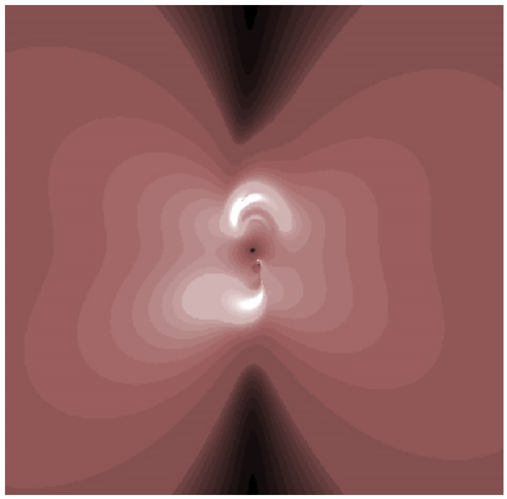




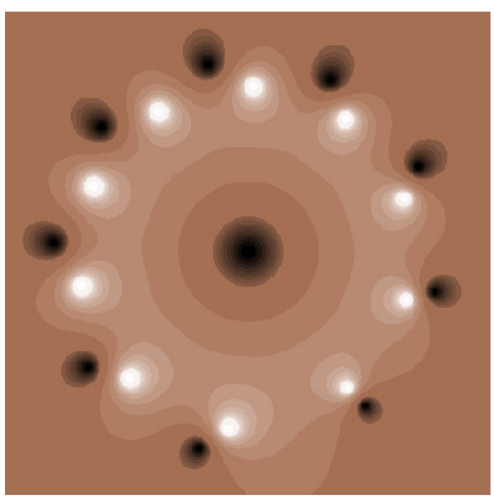
*Example 3*: $f_k(z,\zeta)=\dfrac{kz(1-kz)}{1-\zeta}$, $-6\le x,y\le 6$, $n=10$ SCF



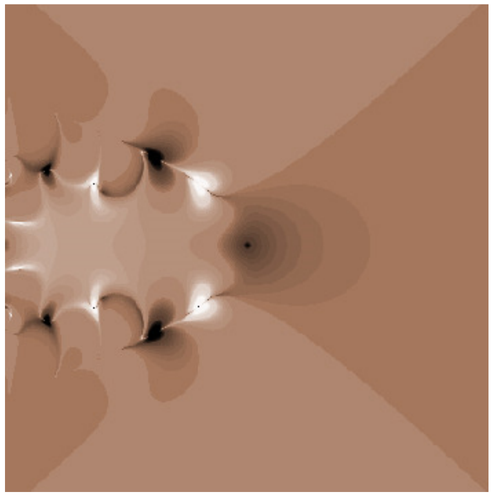
*Example 4*: $f_k(z,\zeta)=\dfrac{kz(1-kz)}{1-\dfrac{\alpha_k z}{\alpha_k+z-\zeta}}$ , $\alpha_k$ =1+ik , $-6\le x,y\le 6$, $n=10$ SCF



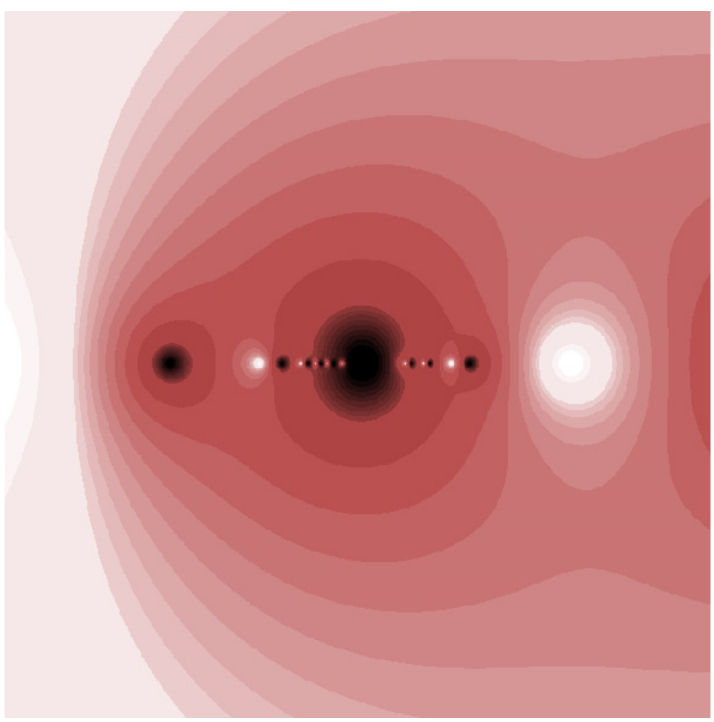
*Example 5*: $f_k(z,\zeta)=\dfrac{(1-ik)z}{1-ik+z-\zeta}$ , $-7\le x,y\le 7$, $n=10$ SCF



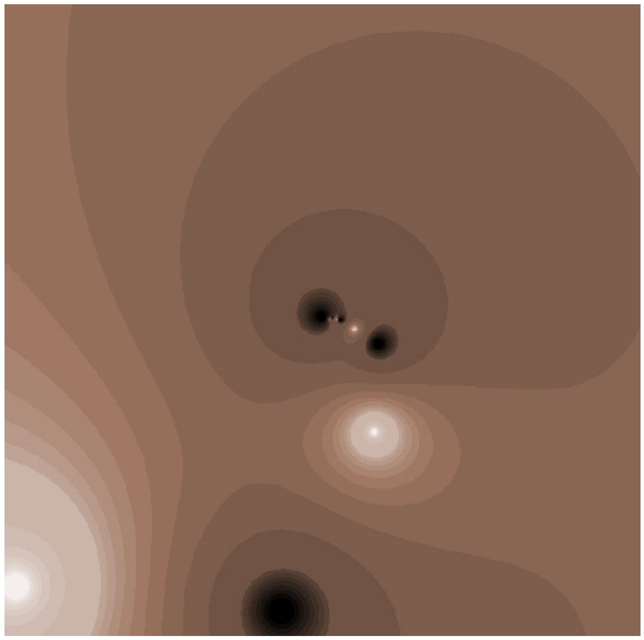
*Example 6*: $f(z,\zeta)=\dfrac{z}{k+z-\zeta}$ , $-6\le x,y\le 6$, $n=10$ SCF. Another variation on FPCFs.



*Example 7*: $f_k(z,\zeta)=\dfrac{kz^2}{k+z-\zeta}$, $-10\le x,y\le 10$, $n=10$ SCF. Another variation on FPCFs.



*Example 8*: $f(z,\zeta)=\dfrac{(1+2ki)^2 z}{1+2ki+z-\zeta}$ , $-10 \le x, y \le 10$ , $n=10$ SCF. Another variation on FPCFs.



*Example 9*: $f(z,\zeta)=\dfrac{i(1+ik)z}{1+ik+z-\zeta}$ , $-8 \le x, y \le 8$ , $n=10$ SCF. Variation on FPCFs.