ABSTRACT

Anchoring and Steering Gaussian Quadrature for Positive Definite Strong Moment Functionals, *Numerical Algorithms* 2012, DOI: 10.1007/s11075-012-9565-y.

Classical Gaussian quadrature is anchored in the space of real polynomials at polynomial degree 0 and is monotonically directed by successively increasing polynomial degree. The results of recent research, investigating weighing anchor and charting various courses in search of the best currents in the space of real Laurent polynomials, are presented in pursuit of a central question:

> $(*)$ Which choice of anchoring and steering gives the *optimal* rank n Gaussian quadrature for the numerical evaluation of $\mathcal{L}[r(x)] = \int_a^b r(x)\, w(x)\, dx$?

A plethora of considerations enter into the debate about how to assess quadrature optimality, and in practice it is often important to strike a balance between performance and expense. A *Standard Error Bound Minimization* (SEBM) algorithm was given in [1]. It searches for an initial power $p(1)$ in a neighborhood of 0 which minimizes the average of the two rank 2 standard error bounds $\mathbf{B}_2[r(x)]$ corresponding to the two possible directions ± 1. For purposes of this paper, this is referred to as **Average SEBM Anchoring**. Then, with $p(1)$ determined, the direction $d(n)$ is chosen recursively, rank by rank, by comparing the two rank n bounds $\mathbf{B}_n[r(x)]$ corresponding to the directions ± 1, herein referred to as **Anchored SEBM Steering**. The exact value of $\mathcal{L}[r(x)]$, hence $\mathcal{E}[r(x)]$, generally is not known *a priori*, and quadratures with smaller error bounds do not guarantee smaller actual error. Besides this criticism, *Anchored SEBM Steering* has relatively high associated computing costs.

The author introduces a selection of new algorithms for anchoring and steering strong Gaussian quadrature and provides a rough gauge for their costs in pursuit of an answer to the question $(*)$:

> $(**)$ An optimal rank n Gaussian quadrature for the numerical evaluation of $\mathcal{L}[r(x)] = \int_a^b r(x)\, w(x)\, dx$ is one with the lowest *cost product*, a term used here to mean the computing cost times the standard error bound.

Features and issues of the algorithms and comparisons of their cost products are illuminated with several tables of numerical examples.

References

[1] B. A. Hagler, Optimizing Gaussian Quadrature for Positive Definite Strong Moment Functionals, SIAM J. Numer. Anal. 49 (2011) 1111-1126.