

## PROB8: PERMS

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Count the number of permutations that have a specific number of inversions.

#### DESCRIPTION

Given a permutation  $a_1, a_2, a_3, \dots, a_n$  of the  $n$  integers  $1, 2, 3, \dots, n$ , an inversion is a pair  $(a_i, a_j)$  where  $i < j$  and  $a_i > a_j$ . The number of inversions in a permutation gives an indication on how "unsorted" a permutation is. If we wish to analyze the average running time of a sorting algorithm, it is often useful to know how many permutations of  $n$  objects will have a certain number of inversions.

In this problem you are asked to compute the number of permutations of  $n$  values that have exactly  $k$  inversions.

For example, if  $n = 3$ , there are 6 permutations with the indicated inversions as follows:

123	0 inversions
132	1 inversion ( $3 > 2$ )
213	1 inversion ( $2 > 1$ )
231	2 inversions ( $2 > 1, 3 > 1$ )
312	2 inversions ( $3 > 1, 3 > 2$ )
321	3 inversions ( $3 > 2, 3 > 1, 2 > 1$ )

Therefore, for the permutations of 3 things

- ◇ 1 of them has 0 inversions
- ◇ 2 of them have 1 inversion
- ◇ 2 of them have 2 inversions
- ◇ 1 of them has 3 inversions
- ◇ 0 of them have 4 inversions
- ◇ 0 of them have 5 inversions
- ◇ etc.

#### INPUT: `prob8.dat`

The input consists one or more problems. The input for each problem is specified on a single line, giving the integer  $n$  ( $1 \leq n \leq 15$ ) and a non-negative integer  $k$  ( $1 \leq k \leq 200$ ). The end of input is specified by a line with  $n = k = 0$ .

An example input file would be

```
column 1
1234567890
line 1:3 0[EOL]
2:3 1[EOL]
3:3 2[EOL]
4:3 3[EOL]
5:4 2[EOL]
```

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```
6:4 10[EOL]
7:13 23[EOL]
8:18 80[EOL]
9:0 0[EOL]
:[EOF]
```

### OUTPUT: prob8.out

For each problem, output the number of permutations of  $\{1, \dots, n\}$  with exactly  $k$  inversions.

The correct output corresponding to the example input file would be

```
column 11111111112222222223
123456789012345678901234567890
line 1:Program 8 by team 0[EOL]
2:1[EOL]
3:2[EOL]
4:2[EOL]
5:1[EOL]
6:5[EOL]
7:0[EOL]
8:46936280[EOL]
9:184348859235088[EOL]
10:End of program 8 by team 0[EOL]
:[EOF]
```

### NOTES

Even though only integer arithmetic is performed, use double precision values to represent quantities to avoid overflows.