Chapter 1

CM Newton's Laws + $\vec{F} = m\vec{a}$ or for a conservative potential $m\ddot{\vec{r}} = -\vec{\nabla}V$

QM → <u>Not deterministic</u>

Want $\Psi(\vec{r}, t)$, for now $\Psi(x, t)$

Ψ is "Wavefunction"

SE $i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V\Psi$ → Solving for wavefunction of particle

V ← potential

TIME Evolution

h = planck's constant $\hbar = \frac{h}{2\pi} = 1.054572 \cdot 10^{-34}$ J·s

---

QM is <u>Statistical</u> in nature

Steps
1: Solve SE for given V
2: Get Ψ and Normalize it.
3: To find probability that particle is in region $[a, b]$ $\int_a^b |\Psi|^2 dx$
4: To measure physical variable apply corresponding operator $\hat{Q}\Psi = q\Psi$ → later

Two Examples

$V=0 \qquad \psi = e^{i(kx-\omega t)}$

$\psi = \rightarrow \underline{\underline{e^{ikx}e^{-i\omega t}}}$

$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}$

$i\hbar(-i\omega)e^{i(kx-\omega t)} = -\frac{\hbar^2}{2m}(ik)^2 e^{i(kx-\omega t)}$

$-i^2\hbar\omega = -i^2\frac{\hbar^2k^2}{2m}$

$\hbar\omega = \frac{\hbar^2k^2}{2m} \checkmark \qquad \hbar k = p$

What potential is required for $\psi = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega}{2\hbar}x^2} e^{-i\frac{\omega t}{2}}$

call this α This β

$\psi = \alpha e^{-\beta x^2} e^{-i\frac{\omega t}{2}}$

$i\hbar\frac{\partial\psi}{\partial t} = \alpha e^{-\beta x^2}\left(\frac{-i\omega}{2}\right)e^{-i\frac{\omega t}{2}}(i\hbar)$

$= \underline{-\frac{i\omega}{2}\psi(i\hbar)}$

$= \frac{\hbar\omega}{2}\psi$

$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} = -\frac{\hbar^2}{2m}\frac{\partial}{\partial x}\left[\frac{\partial}{\partial x}\psi\right]$

$= -\frac{\hbar^2}{2m}\frac{\partial}{\partial x}\left[\alpha e^{-\beta x^2}e^{\frac{i\omega t}{2}}(-2\beta x)\right]$

$= \frac{\hbar^2}{2m}\cdot 2\beta\alpha e^{\frac{i\omega t}{2}}\left[\underbrace{\frac{\partial}{\partial x}\, x e^{-\beta x^2}}_{\left[e^{-\beta x^2} - 2\beta x^2 e^{-\beta x^2}\right]}\right]$

$= \frac{\hbar^2\beta}{m}\left[\psi - 2\beta x^2\psi\right]$

You will NOT be tested on various philosophies

Probabilities

Discrete and continuous

Discrete → one and only one value per variable → ex → age, Energy

Continuous → range of values → location, momentum, etc

Ex. Age

$N(j)$	(j) ← value measured
1 − 14	$N(14) = 1$
1 − 15	$N(15) = 1$
3 − 16	$N(16) = 3$
2 − 22	$N(22) = 2$
2 − 24	$N(24) = 2$
5 − 25	$N(25) = 5$

$$N = \sum_{j=0}^{\infty \text{ or cutoff}} N(j) \qquad N = 14$$

Probabilities of Measurement of j $\quad P(j) = \dfrac{N(j)}{N}$

Need $\sum P(j) = 1$ All possible outcomes $= 1$

→ Most probable j is one with highest $N(j)$

→ 25

→ Median j (may not exist) one in the middle

→ Mean or Average (Expectation value → many measurements)

$$\langle j \rangle = \frac{\sum j N(j)}{N} = \sum j P(j)$$

$$\frac{14 + 15 + 3\cdot 16 + 2\cdot 22 + 2\cdot 24 + 5\cdot 25}{14} = \frac{294}{14} = 21$$

↖ Does not exist

Expectation Value → bad name but it's here to stay

$\langle j^2 \rangle = \sum j^2 P(j)$ in general $\langle f(j) \rangle \; (j^2) = \sum f(j) P(j)$

Be Careful $\langle j^2 \rangle \neq \langle j \rangle^2$

Two babies $j_1 = 1, \; j_2 = 3$

$$\langle j \rangle^2 = \left(\sum j P(j)\right)^2 = \left(1\cdot\tfrac{1}{2} + 3\cdot\tfrac{1}{2}\right)^2 = \left(\tfrac{1}{2} + \tfrac{3}{2}\right)^2 = 2^2 = 4$$

$$\langle j^2 \rangle = \sum j^2 P(j) = \left(1^2\cdot\tfrac{1}{2} + 3^2\cdot\tfrac{1}{2}\right) = \tfrac{1}{2} + \tfrac{9}{2} = 5$$

Refer To Figure 1.5 Same Median, Same Average, Same most Probable

Not Same Spread

How To calculate Spread from $\langle j \rangle$?

$\Delta j = j - \langle j \rangle$? difference from Mean?

$\langle \Delta j \rangle = \sum (j - \langle j \rangle) P(j) = \sum j P(j) - \sum \langle j \rangle P(j)$

average spread

$= \langle j \rangle - \langle j \rangle \underbrace{\sum P(j)}_{1} = \langle j \rangle - \langle j \rangle = 0$

just a #, Factorizable

<u>**No**</u>

How about $\sigma^2 = \langle (\Delta j)^2 \rangle$ inside

Variance, average of Square of Spread

Not AVG of Spread Squared!!

$\sigma^2 = \langle (\Delta j)^2 \rangle = \langle j^2 - 2j\langle j \rangle + \langle j \rangle^2 \rangle$

$= \sum j^2 P(j) - \sum 2j\langle j \rangle P(j) + \sum \langle j \rangle^2 P(j)$

# #

$= \langle j^2 \rangle - 2\langle j \rangle \sum j P(j) + \langle j \rangle^2 \sum P(j)$

$\langle j \rangle$ 1

$= \langle j^2 \rangle - 2\langle j \rangle^2 + \langle j \rangle^2$

$= \langle j^2 \rangle - \langle j \rangle^2$

$\sigma = \sqrt{\sigma^2}$ = Standard deviation $= \sqrt{\langle j^2 \rangle - \langle j \rangle^2}$

calculate $\langle j \rangle^2$, $\langle j^2 \rangle$

Let's do it for a dice $\langle j \rangle$, $\langle j^2 \rangle$

Write down j, $n(j)$, $P(j)$, N

j	$N(j)$	$P(j)$
1	1	$\frac{1}{6}$
2	1	$\frac{1}{6}$
3	1	$\frac{1}{6}$
4	1	$\frac{1}{6}$
5	1	$\frac{1}{6}$
6	1	$\frac{1}{6}$

$N = 6$

$\langle j \rangle = \Sigma\ 1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + 3 \cdot \frac{1}{6} + 4 \cdot \frac{1}{6} + 5 \cdot \frac{1}{6} + 6 \cdot \frac{1}{6}$

$= \dfrac{1+2+3+4+5+6}{6} = \dfrac{21}{6} = \dfrac{7}{2} \rightarrow \langle j \rangle^2 = \dfrac{49}{4}$

$\langle j^2 \rangle = \dfrac{1+4+9+16+25+36}{6} = \dfrac{91}{6}$

$\sigma = \sqrt{\dfrac{91}{6} - \dfrac{49}{4}}$

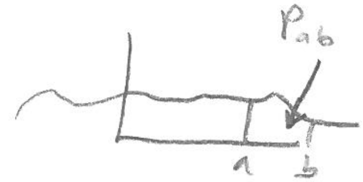
Continuous Variable

Probability density

$P(\in[x, x+dx])$ maybe Speed $= \rho(x)\,dx$

CAREFUL $\rho(x)$ for SE $= |\Psi|^2$ else use supplied $\rho(x)$ for HW.

$P_{ab} = \int_a^b \rho(x)\,dx \rightarrow$ for Ψ $P_{ab} = \int_a^b |\Psi|^2\,dx$



Must Be normalizable $\quad \sum P(j) = 1 \rightarrow \int_{-\infty}^{+\infty} \rho(x)\,dx = \int_{-\infty}^{+\infty} |\Psi|^2\,dx = 1$

If $\rho(x)$ or Ψ only non zero in realm $x_i \rightarrow x_f$

$\int_{x_i}^{x_f} \rho(x)\,dx$ or $\int_{x_i}^{x_f} |\Psi|^2\,dx = 1$ <u>Normalization</u>

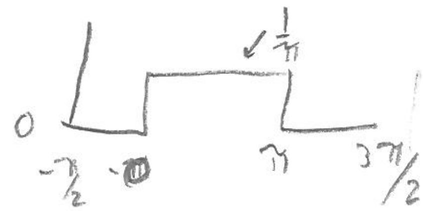
$\langle x \rangle = \int_{-\infty}^{+\infty} \rho(x)\,dx$, $\langle f(x) \rangle = \int_{-\infty}^{+\infty} f(x)\rho(x)\,dx$

Problem 1.11 all θ between $[0, \pi]$ same likelihood

constant probabilities

So $\int_0^{\pi} A \, d\theta = 1 \quad A\theta \Big|_0^{\pi} = 1 \quad A[\pi - 0] = 1 \quad A = \frac{1}{\pi}$

$\rho(\theta) = \frac{1}{\pi} \quad \theta \in [0, \pi]$



$$\langle \theta \rangle = \int_0^{\pi} \frac{\theta}{\pi} \, d\theta = \frac{1}{\pi}\left(\frac{\theta^2}{2}\right)\Bigg|_0^{\pi} = \frac{1}{\pi} \cdot \frac{\pi^2}{2} = \frac{\pi}{2}$$

$$\langle \theta^2 \rangle = \frac{1}{\pi} \int_0^{\pi} \theta^2 \, d\theta = \frac{1}{\pi} \frac{\theta^3}{3}\Bigg|_0^{\pi} = \frac{\pi^3}{3\pi} = \frac{\pi^2}{3}$$

$$\sigma^2 = \langle \theta^2 \rangle - \langle \theta \rangle^2 = \frac{\pi^2}{3} - \frac{\pi^2}{4} = \frac{4\pi^2 - 3\pi^2}{12} = \frac{\pi^2}{12}$$

$$\sigma = \sqrt{\frac{\pi^2}{3 \cdot 4}} = \frac{\pi}{2\sqrt{3}}$$

<u>HW</u> 1.1–1.4 1.9, 1.10 (c), 1.7