Physics 321, Final Exam Fall 2013

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RULES

You may use an equation sheet. This sheet may NOT have solutions to problems but may contain any formulae or integrals you choose to include. You would do well to read all of the problems first and tackle them in the order that works best. Please ask me to clarify any questions you may have. Some of these problems require zero clarification.

Problems

1: What does Schroedinger's equation solve for and what information does this solution contain? What may or may not be determined exactly from the solution? How is this solution's content fundamentally different from the solutions for Newton's laws? What are the limitations of applicability of Schroedinger's equation(3 pts)

2: Construct $\Psi_{3,2,1}$. Don't worry about normalizing $R_{n,l}$. (3 pts)

3: Ψ is determined to be

$$\frac{1}{4\pi\sqrt{2}}[1-\frac{1}{2}\frac{r}{a}]e^{-r/2a} \tag{1}$$

Find

A)The probability of finding the particle in the realm [0,2a] (4 pts)

B)The most likely location to find the particle in (3 pts)

4: The Klein-Gordon equation was an attempt towards finding an equation which was the relativistic generalization of the Schroedinger equation. Starting from the relativistic form for energy

$$E^2 = p^2c^2 + m^2c^4 \tag{2}$$

and using the full forms for the E and p operators write down the klein-Gordon differential equation in 3 dimensions. (3 pts)

5: Solutions to Schroedinger's equation are said to be complete and orthogonal. Assuming a solution set of $\phi's$ is found we may express a solution at $t = 0$ to be $F(x) = \sum c_n\phi_n$. What is the probability of measuring a particular ϕ? What is the expression for $< H >$? (2 pts)

6: A particle with energy E is incident upon a rectangular barrier of height V and width a. Set up the system of equations which would allow one to solve for the transmission coefficient T assuming E is less than V. Draw the geometry. Indicate which waves go away and what needs to be done to generate this system of equations. (4 pts)

7. Under what conditions are the solutions to Schroedinger's equations in terms of quantized

energies and what conditions are they in terms of a continuous spectrum of energy. Comment on the physicality of the eigenfunctions in the continuous case. (3 pts)

8. Imagine a system in which there are just two linear independent vectors given by $|1>= \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|2>= \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. The most generic solution can be written as $|\Psi>= a|1> + b|2>= \begin{pmatrix} a \\ b \end{pmatrix}$ where $a^2 + b^2 = 1$. The Hamiltonian for this system is given by by $\hat{H} = \begin{pmatrix} h & g \\ g & h \end{pmatrix}$.

A)Solve Schroedinger's equation $\hat{H}|\Psi>= i\hbar\frac{d}{dt}|\Psi>$. i.e. Determine the eigenvalues and normalized eigenvectors.

B) Write down the time dependent solution assuming the system starts out at $t = 0$ in state $|\Psi>=|1>= \begin{pmatrix} 1 \\ 0 \end{pmatrix}$. Don't forget that your solution is a linear combination of the eigenvectors.

C) What is the probability of measuring $|1>$ and of measuring $|2>$?

D) Calculate the expectation value of the Hamiltonian for this solution. (16 pts)

Relax, your toughest semester is ending

EQUATIONS

$$P_l(x) = \frac{1}{2^l l!}(\frac{d}{dx})^l (x^2 - 1)^l \tag{3}$$

$$P_l^m(x) = (1 - x^2)^{|m|/2}(\frac{d}{dx})^{|m|} P_l(x) \tag{4}$$

$$Y_l^m(\theta, \phi) = \epsilon \sqrt{\frac{(2l+1)}{4\pi}\frac{(l-|m|)!}{(l+|m|)!}} e^{im\phi} P_l^m(\cos\theta) \tag{5}$$

$$R_{n,l} = \frac{\rho^{l+1} e^{-\rho} v(\rho)}{r} \tag{6}$$

where $\rho = r/(an)$ and $v(\rho) = \sum c_j \rho^j$ with

$$c_{j+1} = c_j \frac{2(j+1+l-n)}{(j+1)(j+2l+1)}. \tag{7}$$