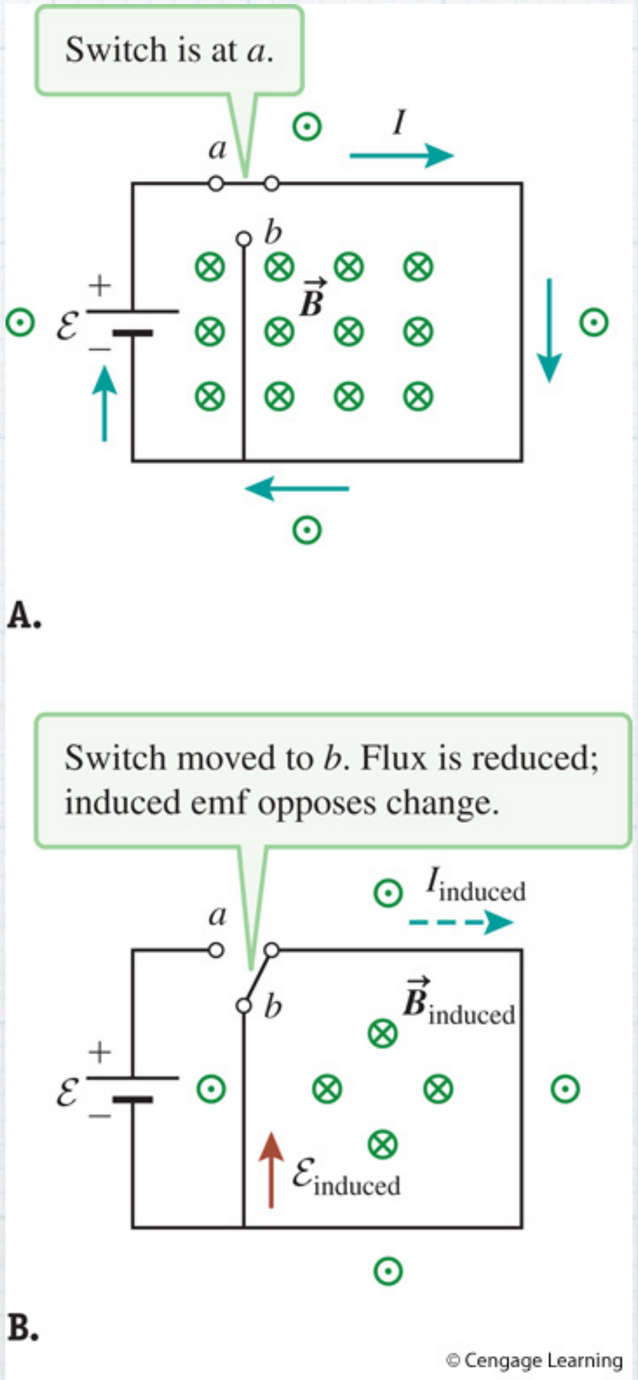
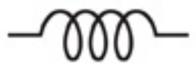
AC Circuits and Inductors

Inductors

* Circuit element designed to store more magnetic flux and hence more magnetic energy



Capacitor	Inductor
	
$E = \frac{V_C}{d}$ (parallel-plate capacitor) (27.21)	$B = \mu_0 n I$ (solenoid) (31.6)
Electric field E between plates	Magnetic field B in coils
Voltage V_C across plates	Current I in wire
Charge Q stored by capacitor	Magnetic flux Φ_B through inductor
Capacitance C	Inductance L
$Q = CV_C$ (27.1)	$\Phi_B = LI$ (33.1)
$C = \frac{\varepsilon_0}{d} A$ (parallel-plate capacitor) (27.10)	$L = \mu_0 n^2 \ell A = \frac{\mu_0 N^2}{\ell} A$ (solenoid) (33.5)
$U_E = \frac{1}{2} C V_C^2$ (27.3)	$U_B = \frac{1}{2} L I^2$ (33.3)

$$\Phi_t = LI$$

$$1H = 1\frac{Wb}{A}$$

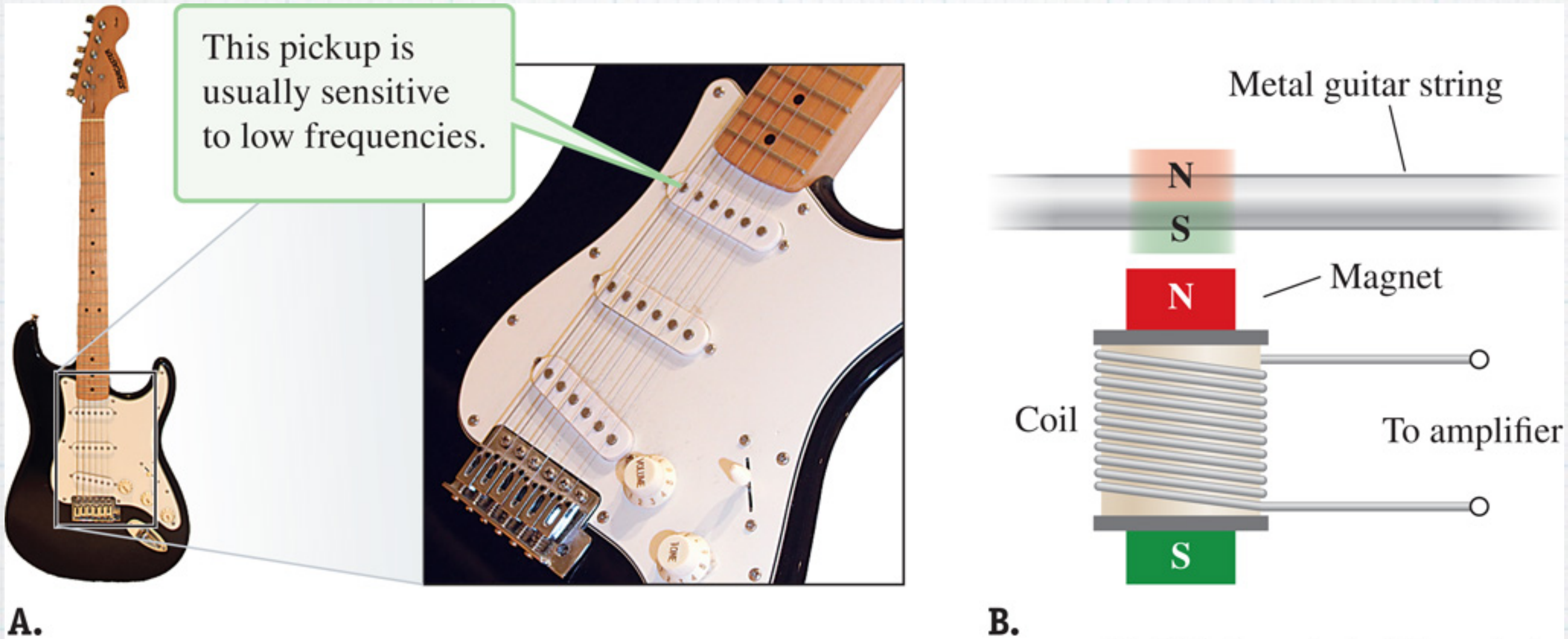
$$U_B = \frac{1}{2}LI^2$$

$$L \propto A$$

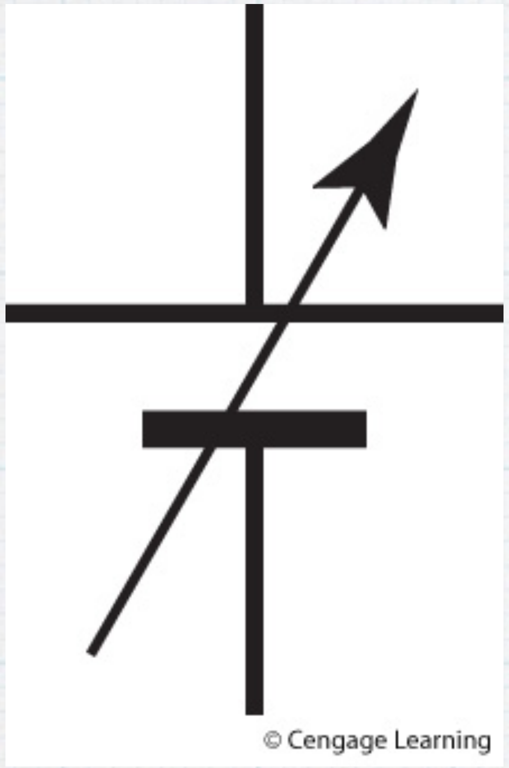
Inductance of a solenoid

$$L = \mu_0 n^2 l A$$

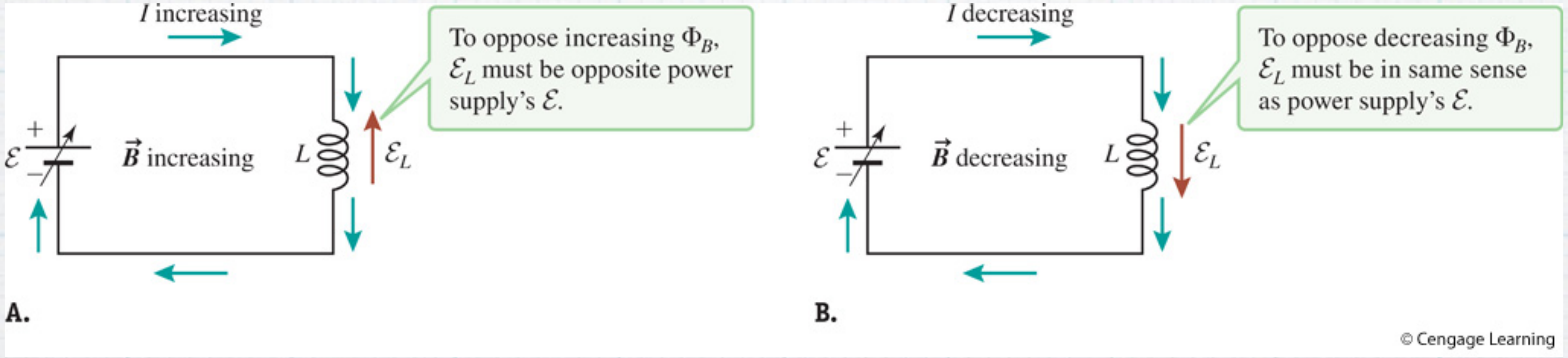
show



a) Peter McGahey/Cengage Learning; b: © Cengage Learning



Variable DC power supply



$$\xi_L = \frac{d\Phi}{dt} = L\frac{dI}{dt}$$

Back EMF $$\Delta V_L = -L\frac{dI}{dt}$$

Depends on what is happening to the current

Example

Transformers

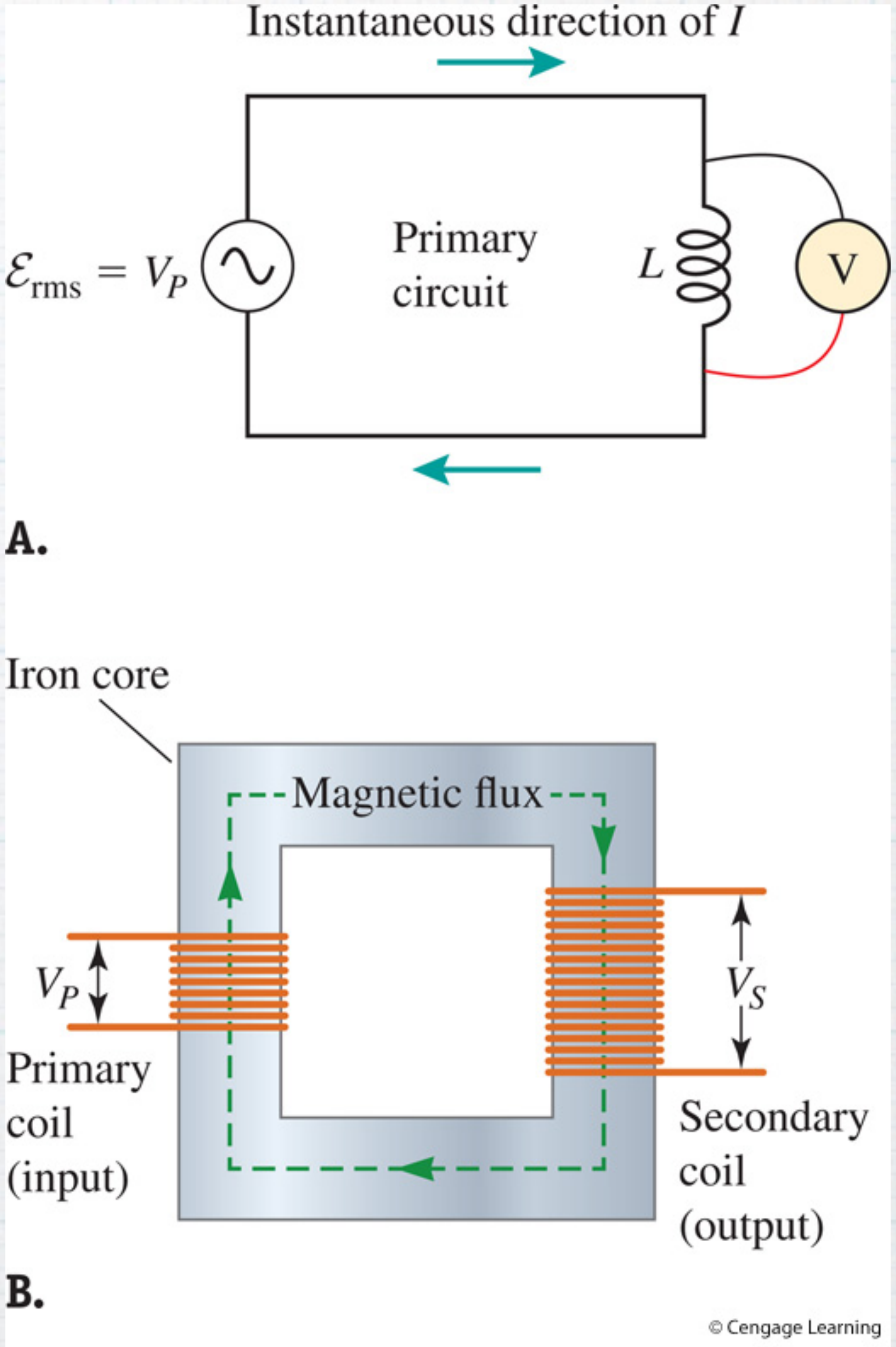
$$V_p = L\frac{dI}{dt} = N_p\frac{d\Phi_B}{dt}$$

$$V_s = N_s\frac{d\Phi_B}{dt}$$

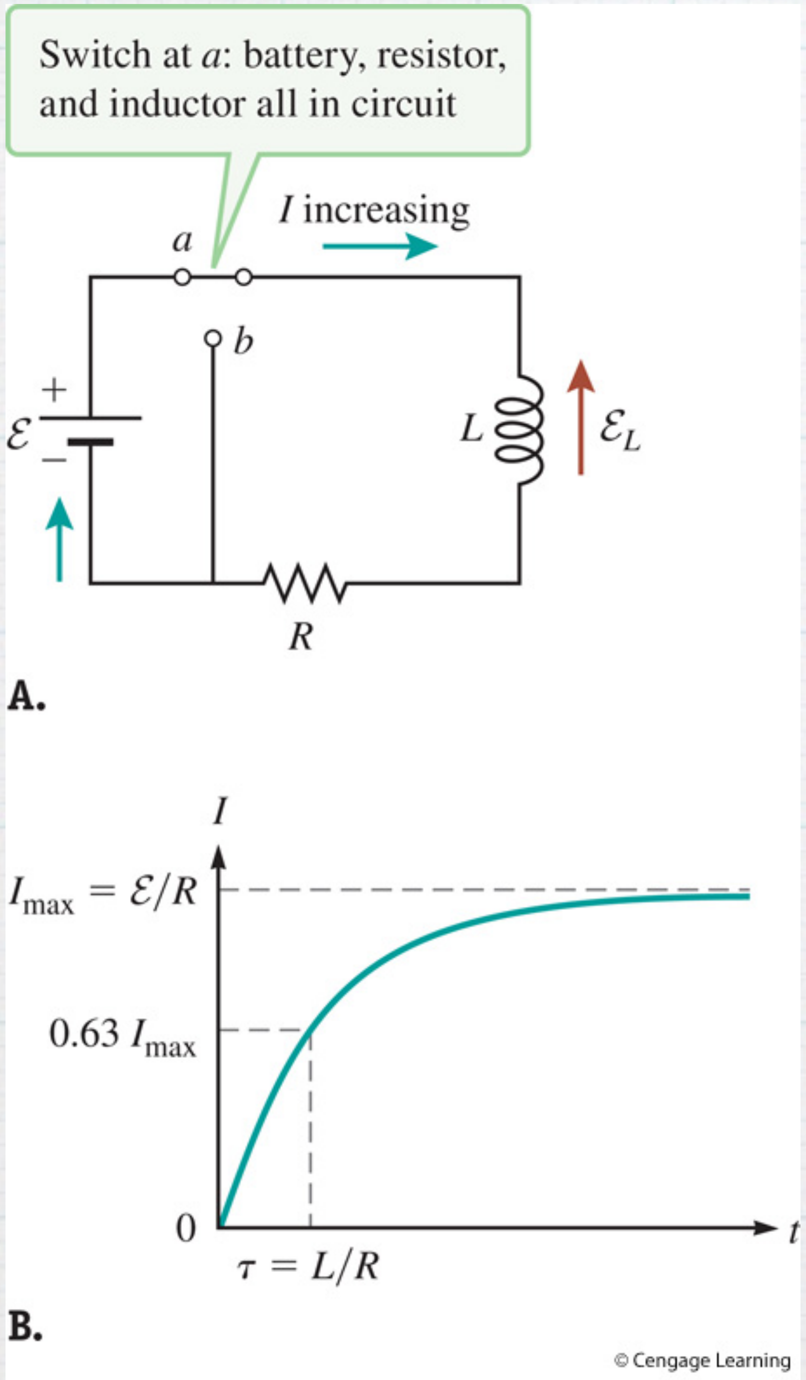
flux is the same

$$\frac{d\Phi_B}{dt} = \frac{V_s}{N_s} = \frac{V_p}{N_p}$$

Example



RL Circuits Power On



$$\xi - L\frac{dI}{dt} - IR = 0$$

$$\xi - L\frac{d^2q}{dt} - \frac{dq}{dt}R = 0$$

$$I(t) = \frac{\xi}{R}(1 - e^{-t/\tau})$$

$$\tau = \frac{L}{R}$$

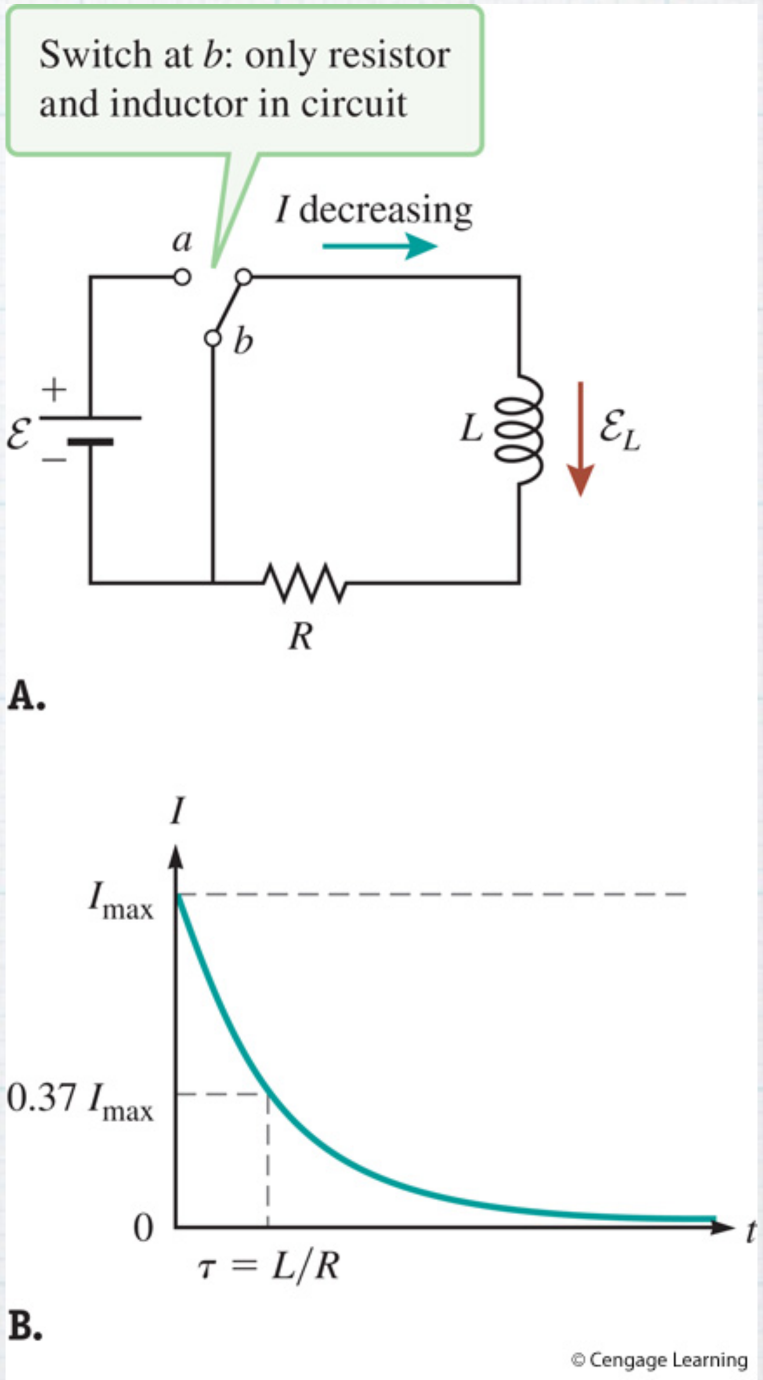
Example

RL circuits Power Off

$$L\frac{dI}{dt} + IR = 0$$

$$I(t) = I_{max}e^{-t/\tau}$$

Example



Energy Stored in the Magnetic Field

$$\xi - L\frac{dI}{dt} - IR = 0$$

Multiply by I, now its a differential equation for power

$$(I\xi)_1 - (LI\frac{dI}{dt})_2 - (I^2R)_3 = 0$$

1: power supply

2: Power stored in inductor

3: power dissipated in resistor

$$\frac{dU_B}{dt} = LI\frac{dI}{dt}$$

Continued

$$dU_B = LIdI$$

$$\int_0^{U_B} dU_B = L\int_0^I IdI$$

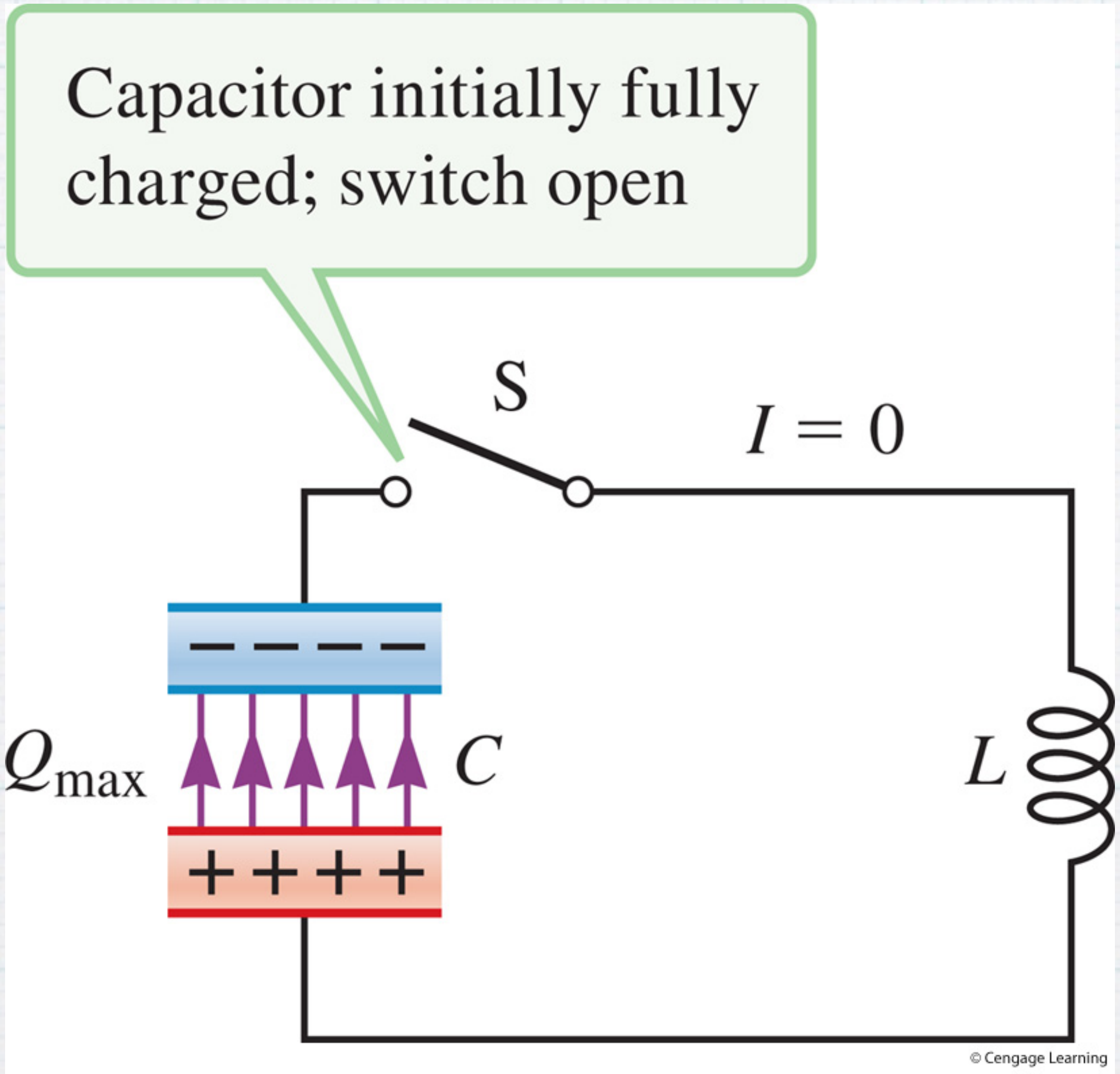
$$U_B = \frac{1}{2}LI^2$$

$$L_{solenoid} = \mu_0 n^2 lA$$

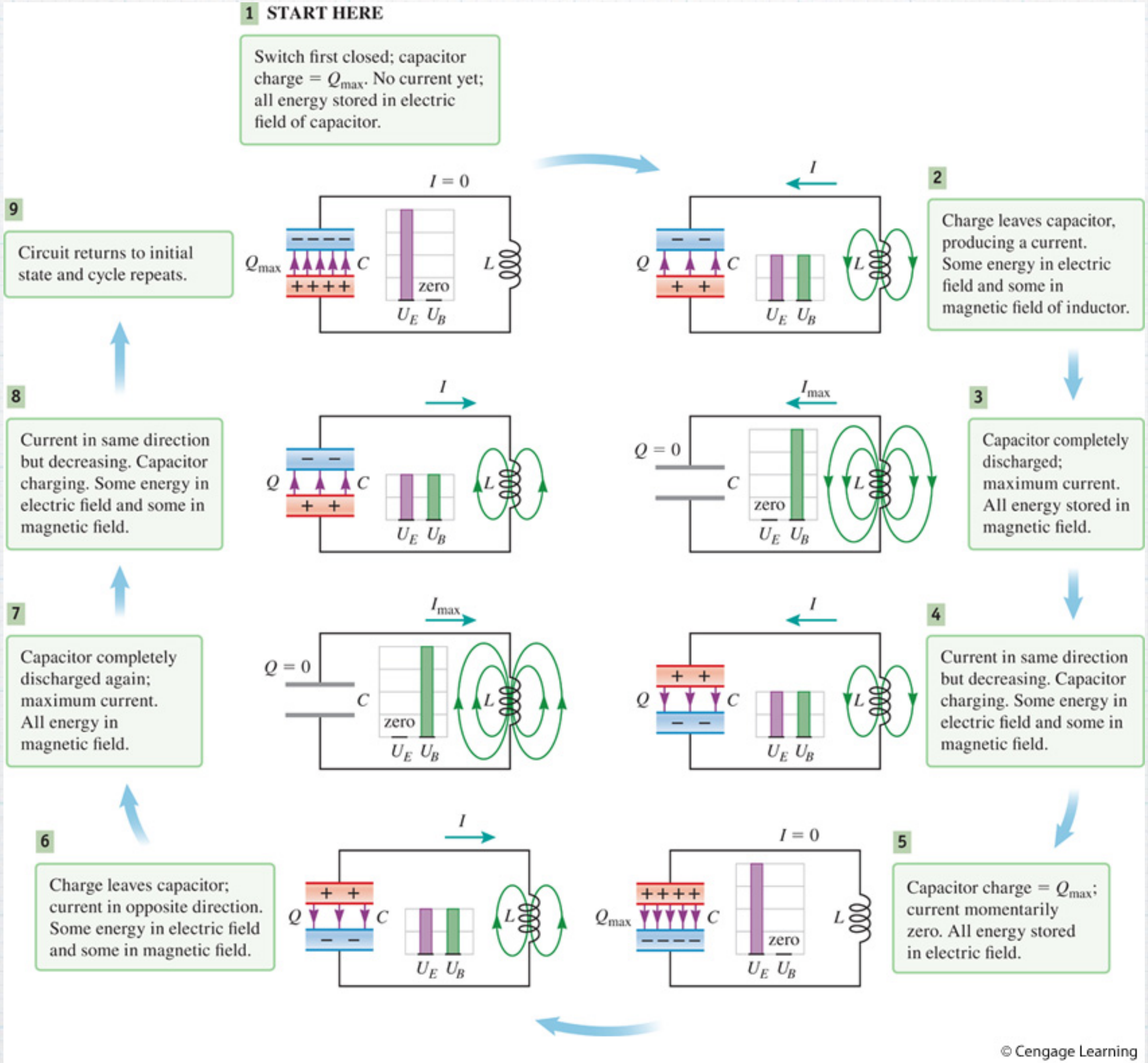
$$B_{solenoid} = \mu_0 nI$$

$$U_{Bsolenoid} = \frac{1}{2}\frac{B^2}{\mu_0}lA$$

LC circuit



$$E_{tot} = \frac{1}{2}\frac{Q^2}{C} + \frac{1}{2}LI^2 = \frac{1}{2}\frac{Q^2_{max}}{C} = \frac{1}{2}LI^2_{max} = U_{E,max} = U_{B,max}$$


1 START HERE
Switch first closed; capacitor charge = Q_{max}. No current yet; all energy stored in electric field of capacitor.
2
Charge leaves capacitor, producing a current. Some energy in electric field and some in magnetic field of inductor.
3
Capacitor completely discharged; maximum current. All energy stored in magnetic field.
4
Current in same direction but decreasing. Capacitor charging. Some energy in electric field and some in magnetic field.
5
Capacitor charge = Q_{max}; current momentarily zero. All energy stored in electric field.
6
Charge leaves capacitor; current in opposite direction. Some energy in electric field and some in magnetic field.
7
Capacitor completely discharged again; maximum current. All energy in magnetic field.
8
Current in same direction but decreasing. Capacitor charging. Some energy in electric field and some in magnetic field.
9
Circuit returns to initial state and cycle repeats.
$I = 0$
I
I_{max}
Q_{max}
Q
$Q = 0$
C
L
zero
U_E U_B
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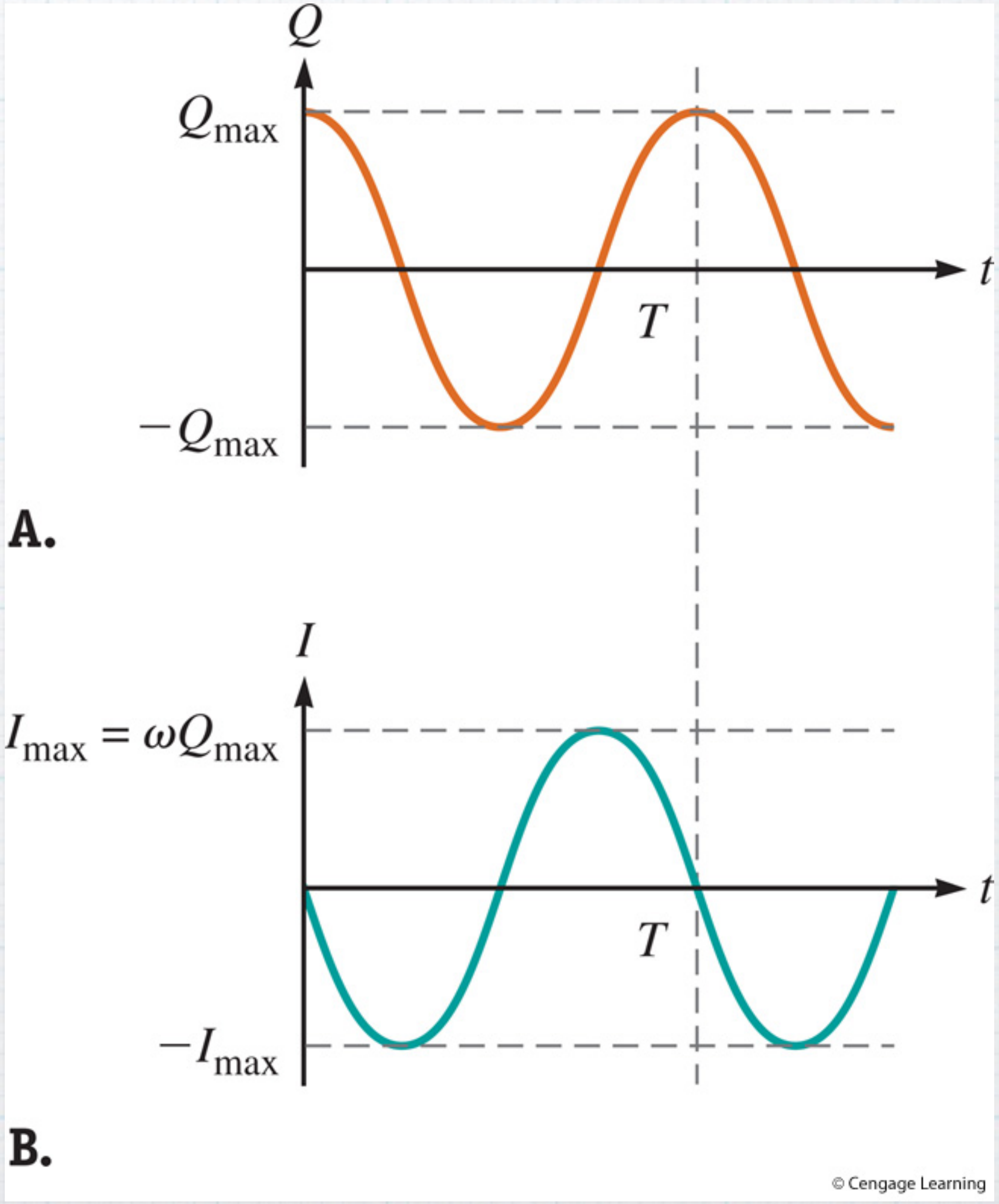
Current and Charge for the LC Circuit

$$Q(t) = Q_{max}cos(\omega t + \psi)$$

$$\omega = \sqrt{\frac{1}{LC}}$$

$$I = \frac{dQ}{dt}$$

Example



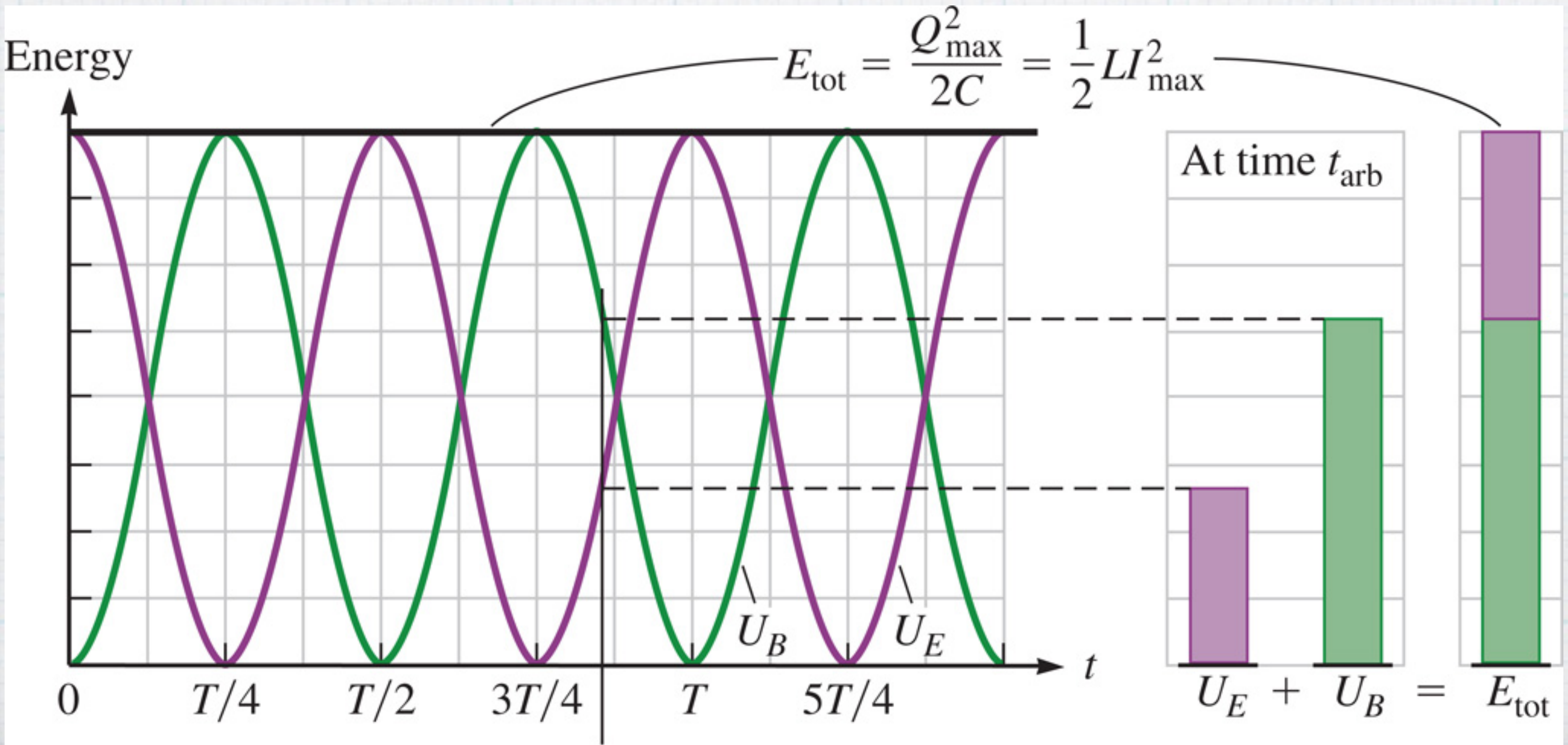
Energy in an LC Circuit

$$U_E(t) = \frac{Q_{max}^2}{2C} cos^2(wt + \psi)$$

$$U_B(t) = \frac{Q_{max}^2}{2C} sin^2(wt + \psi)$$

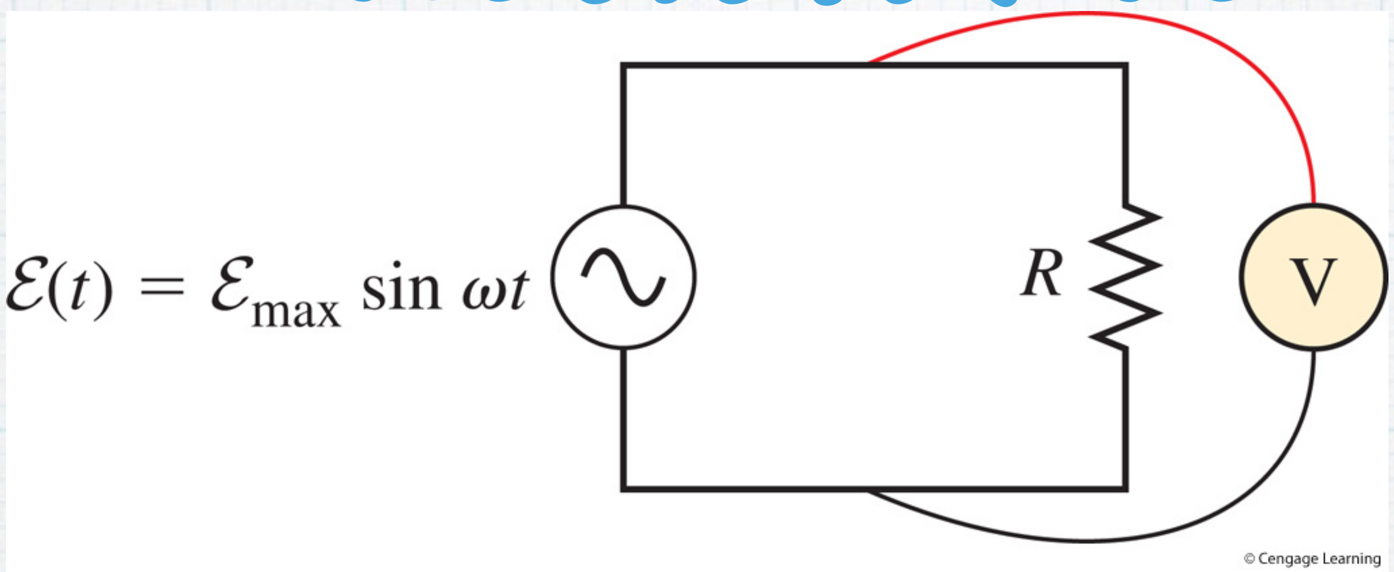
Show

$$E(t) = U_E + U_B$$



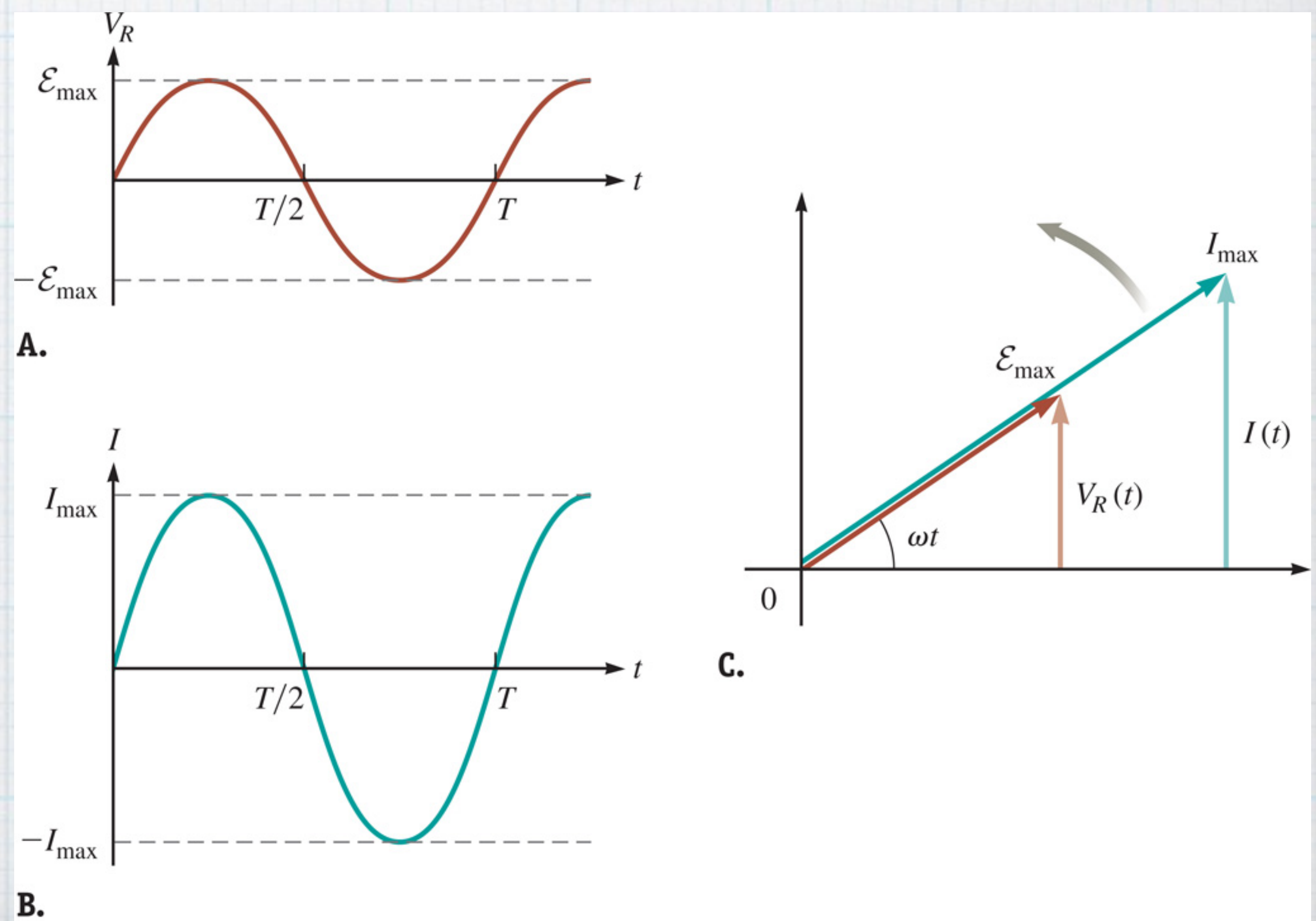
Example

AC Circuit with Resistance

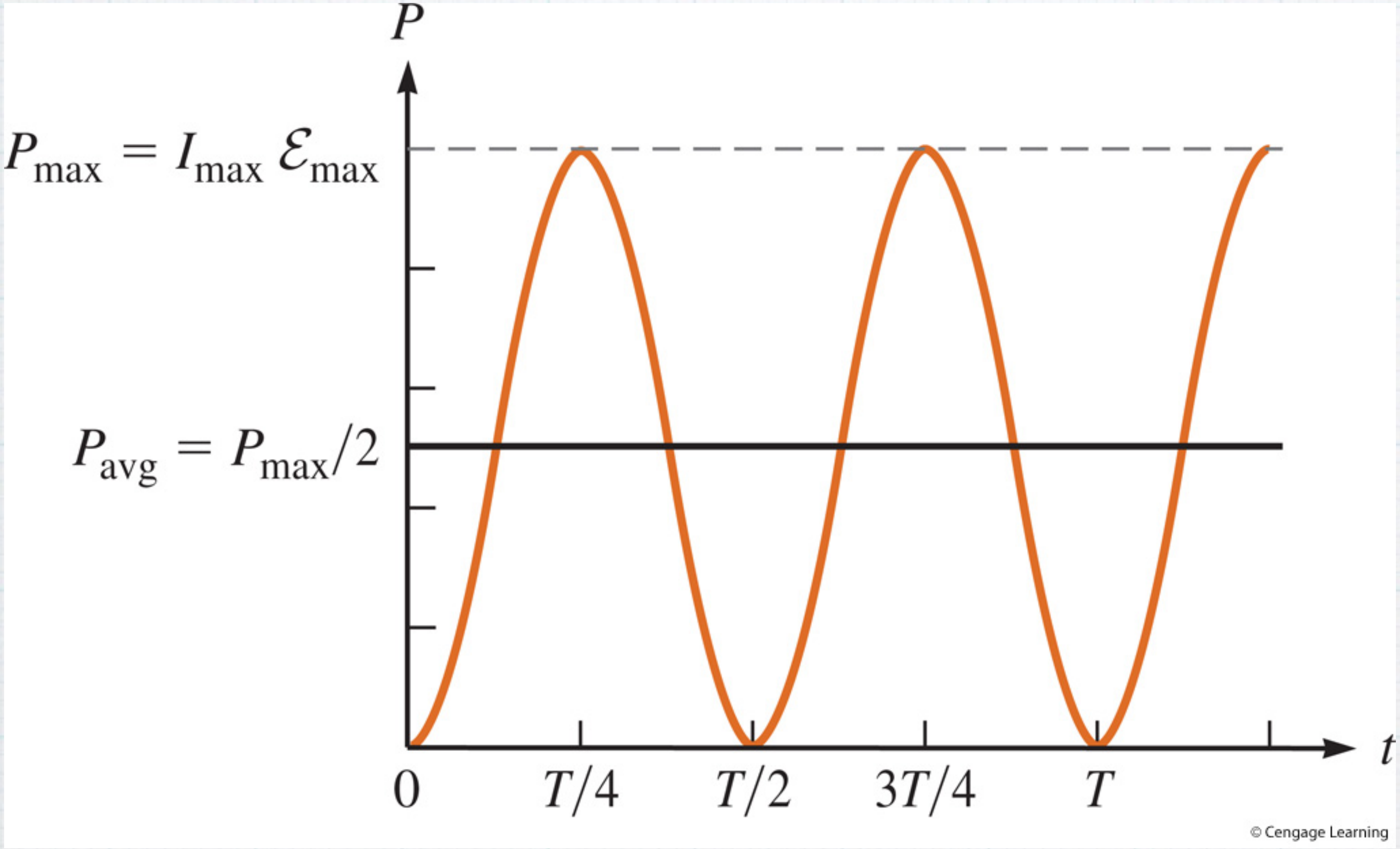


$$V_R(t) = \xi(t) = \xi_{max} sin(\omega t)$$

$$I(t) = \frac{\xi(t)}{R}$$

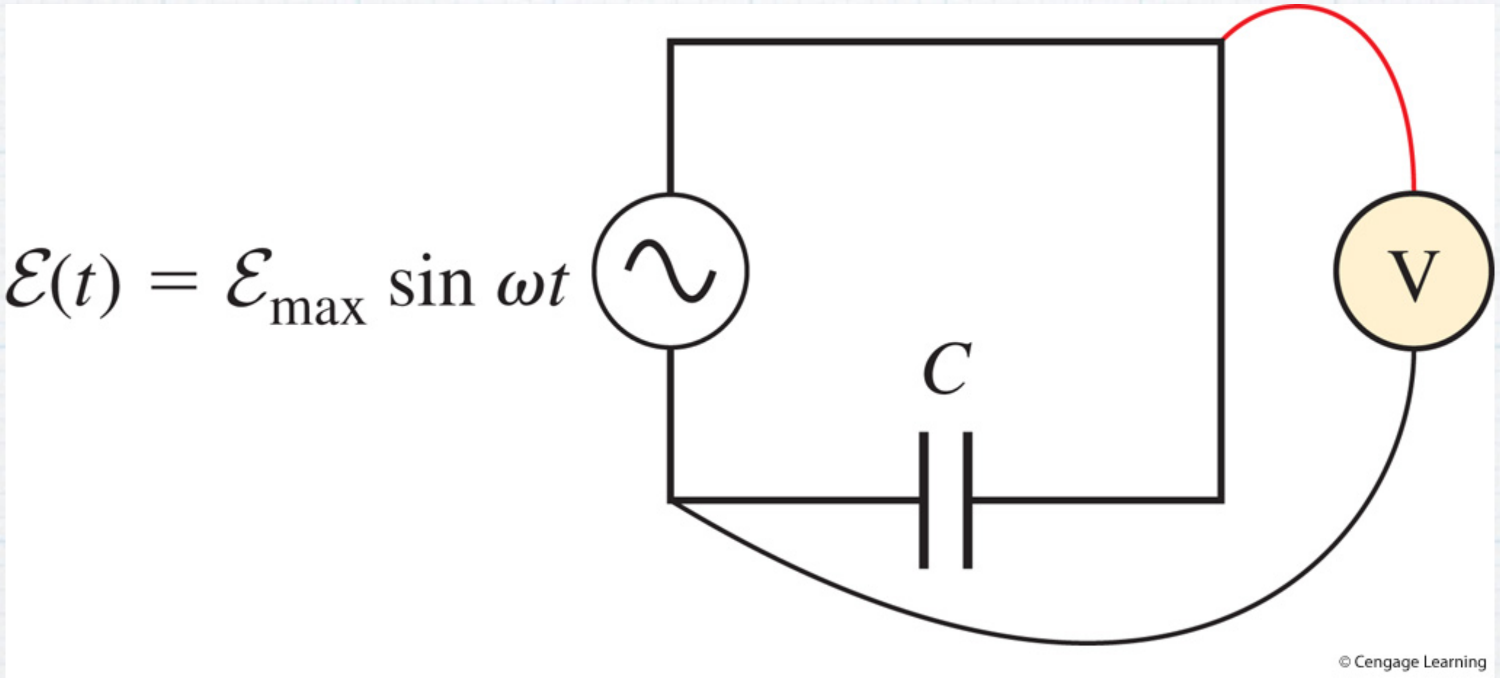


Example P(t)



Example

AC Circuit with Capacitance



$$V_C(t) = \xi(t) = \xi_{max} sin(\omega t)$$

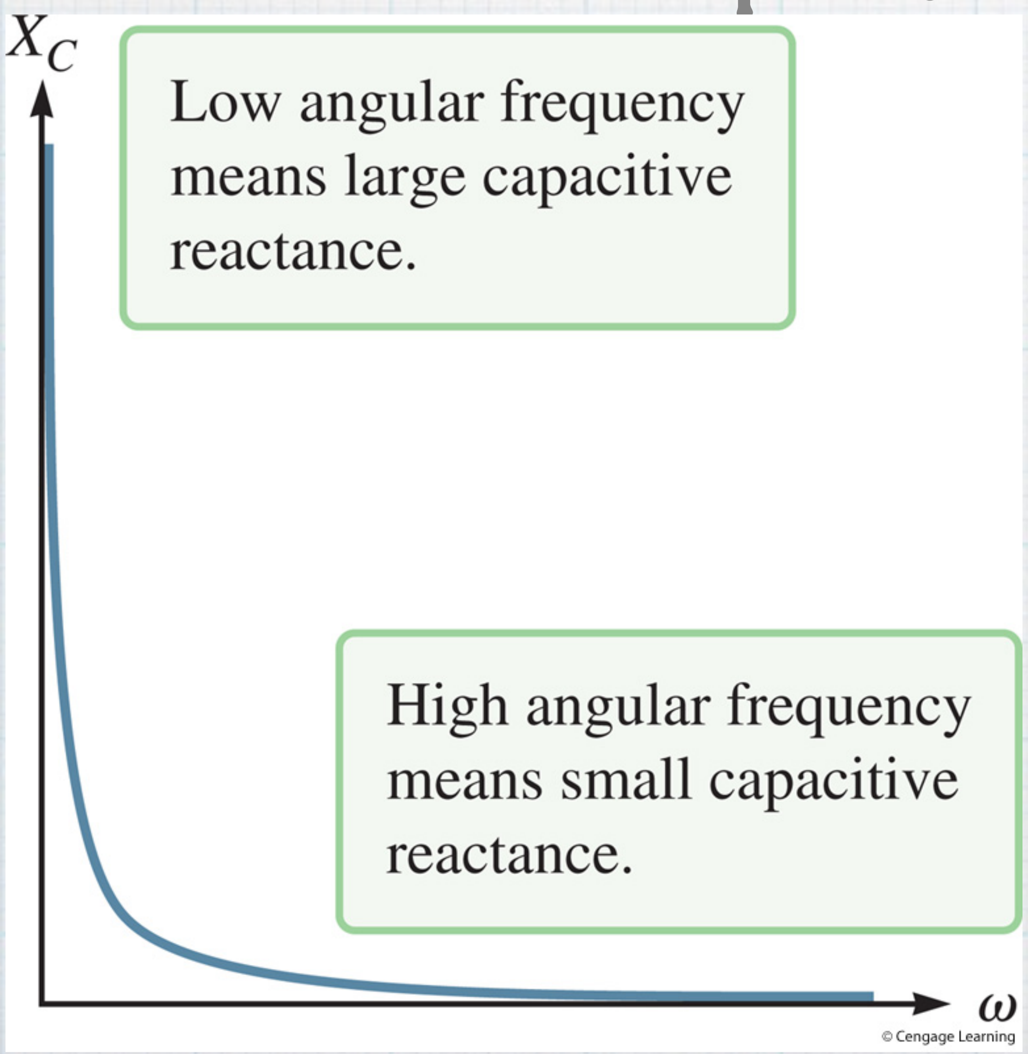
$$Q = CV_C \rightarrow Q(t) = C\xi_{max} sin(\omega t)$$

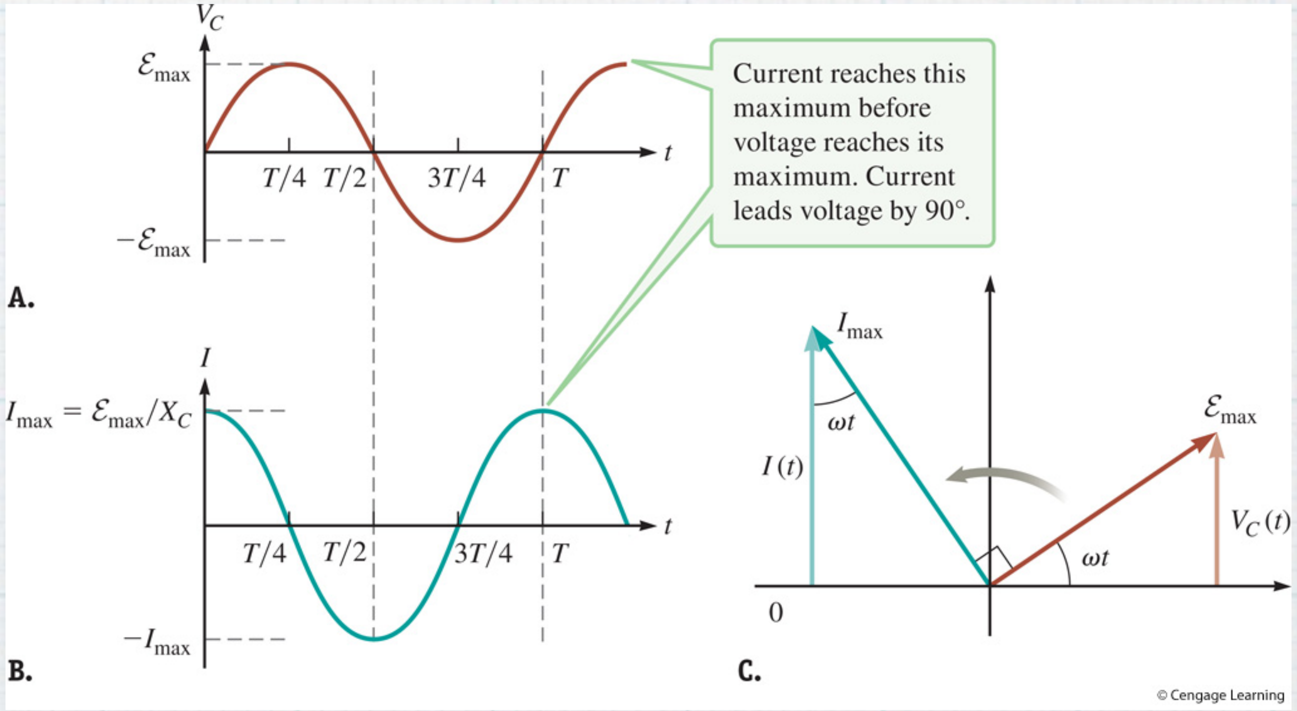
$$I(t) = C\omega\xi_{max} cos(\omega t) = C\omega\xi_{max} sin(\omega t + 90)$$

$$I(t) = \frac{\xi_{max}}{X_C} sin(\omega t + 90)$$

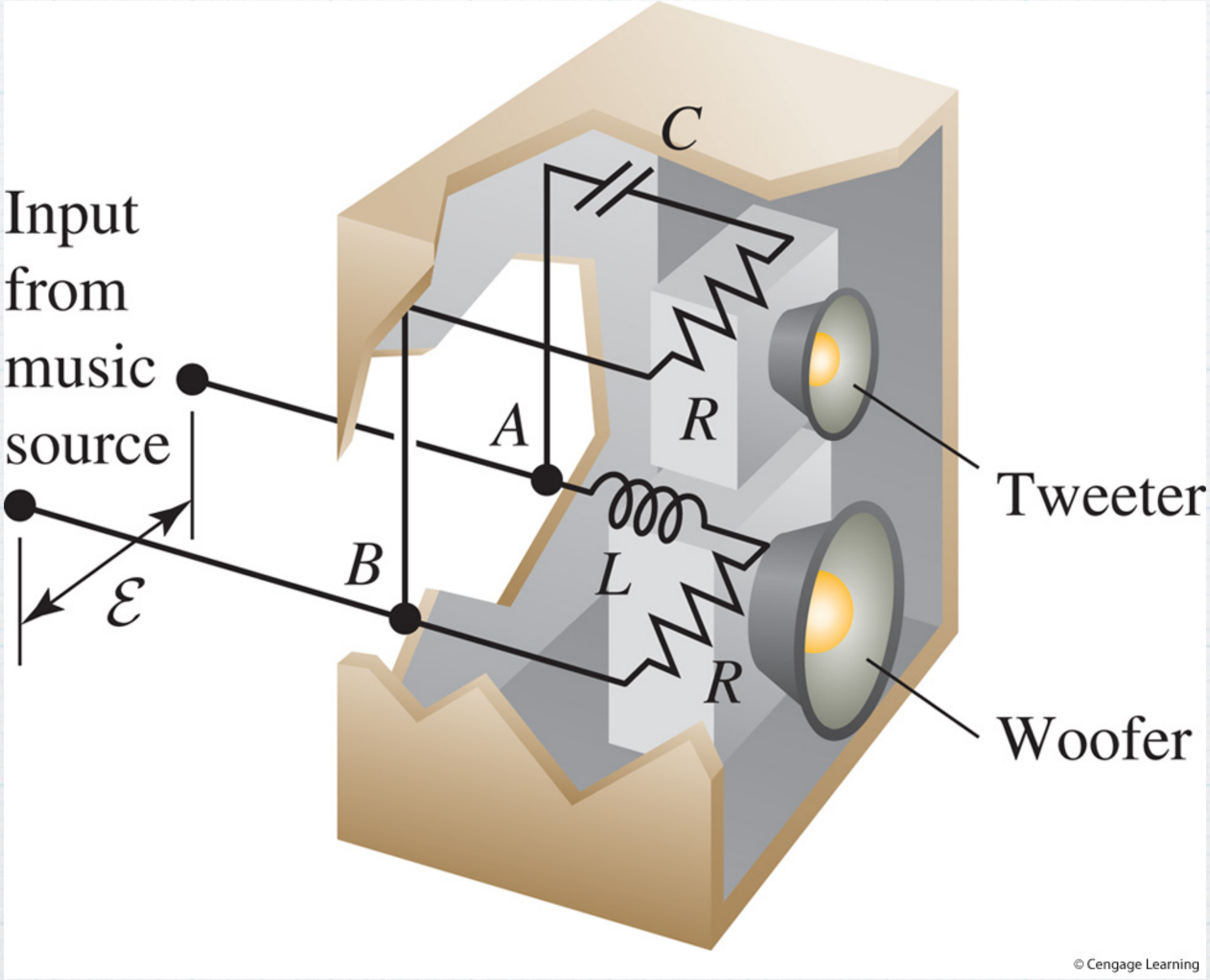
$$X_C = \frac{1}{\omega C}$$

capacitive resistance





Example, power in AC circuits



Input
from
music
source
C
A
R
Tweeter
B
L
$\mathcal{E}$
R
Woofer
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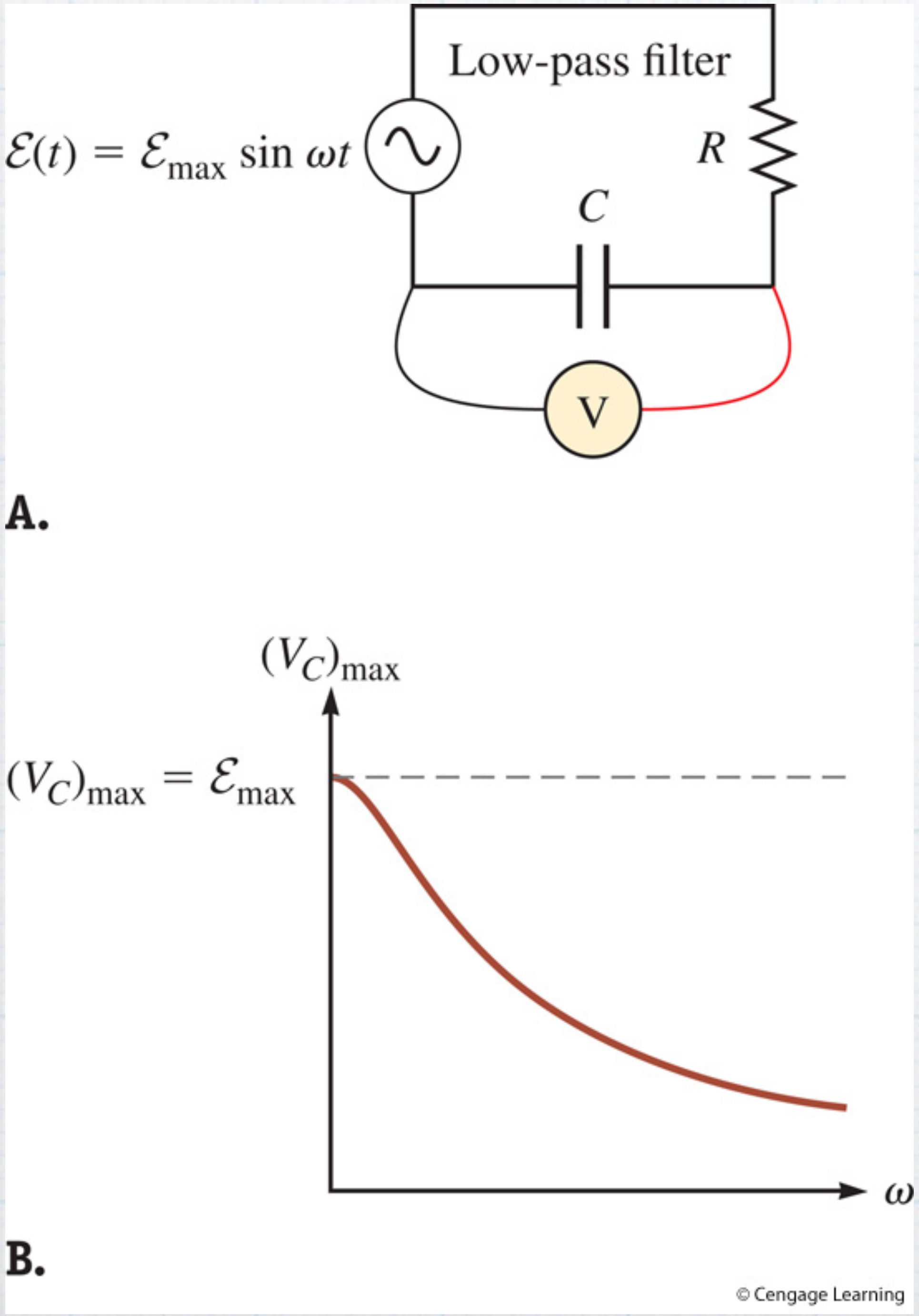
RC Filters

No derivations



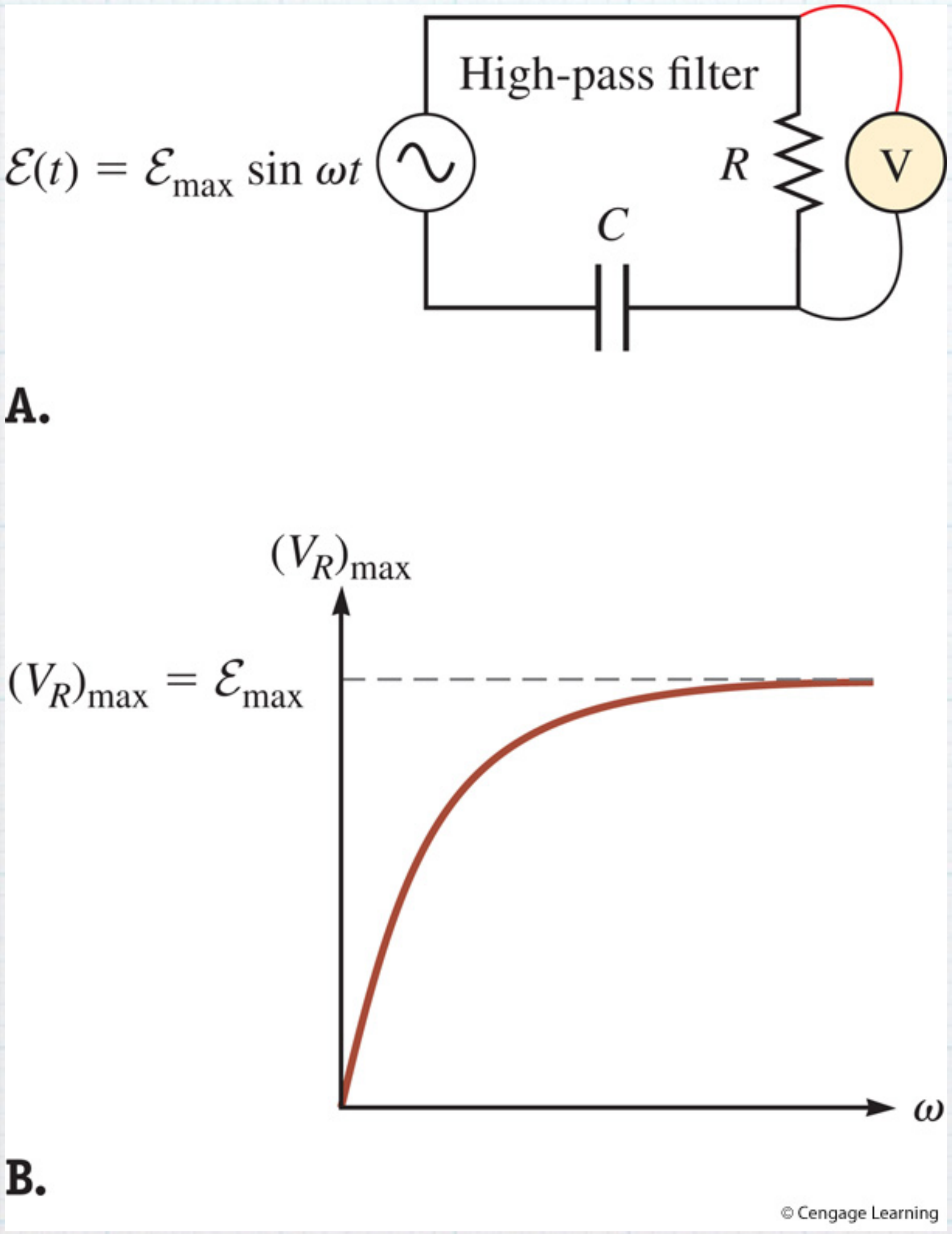
$$V_C(T) = X_C I_{max} sin(\omega t - \frac{\pi}{2})$$

$$I_{max} = \frac{\xi_{max}}{\sqrt{X_C^2 + R^2}}$$

$$V_{C,max} = X_C I$$

woofer

RC Filters



$$V_R(t) = RI_{max} sin(\omega t)$$

$$I_{max} = \frac{\xi_{max}}{\sqrt{X_C^2 + R^2}}$$

$$V_{R,max} = RI_{max}$$

tweeter

Example