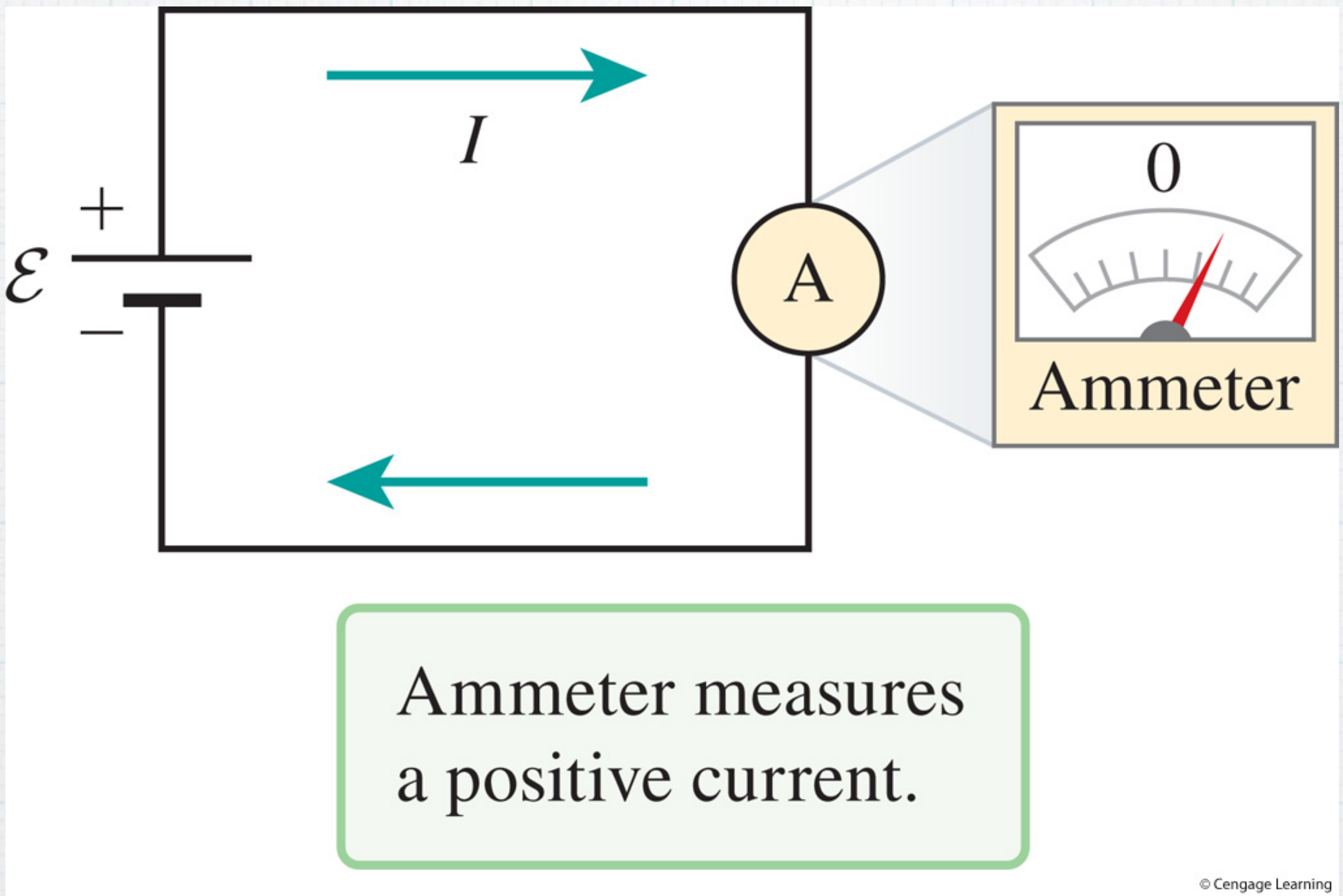
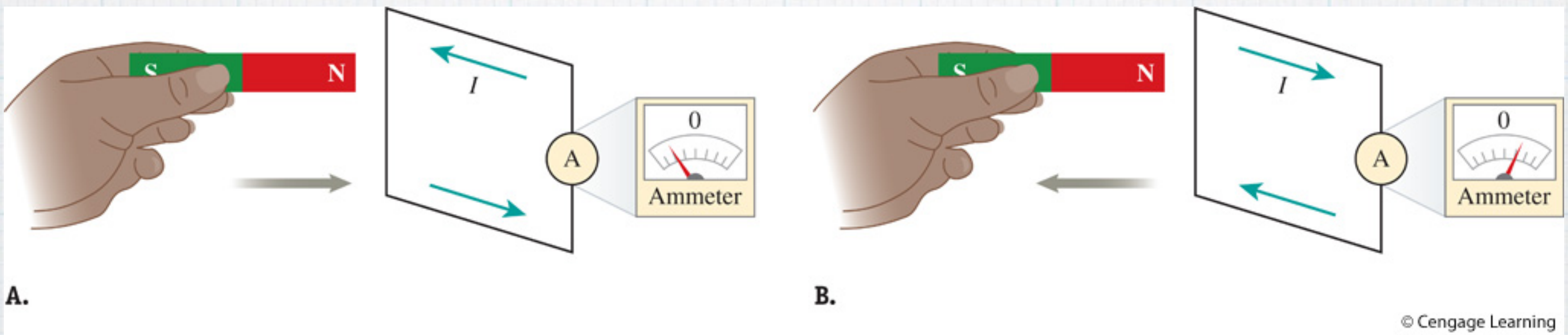
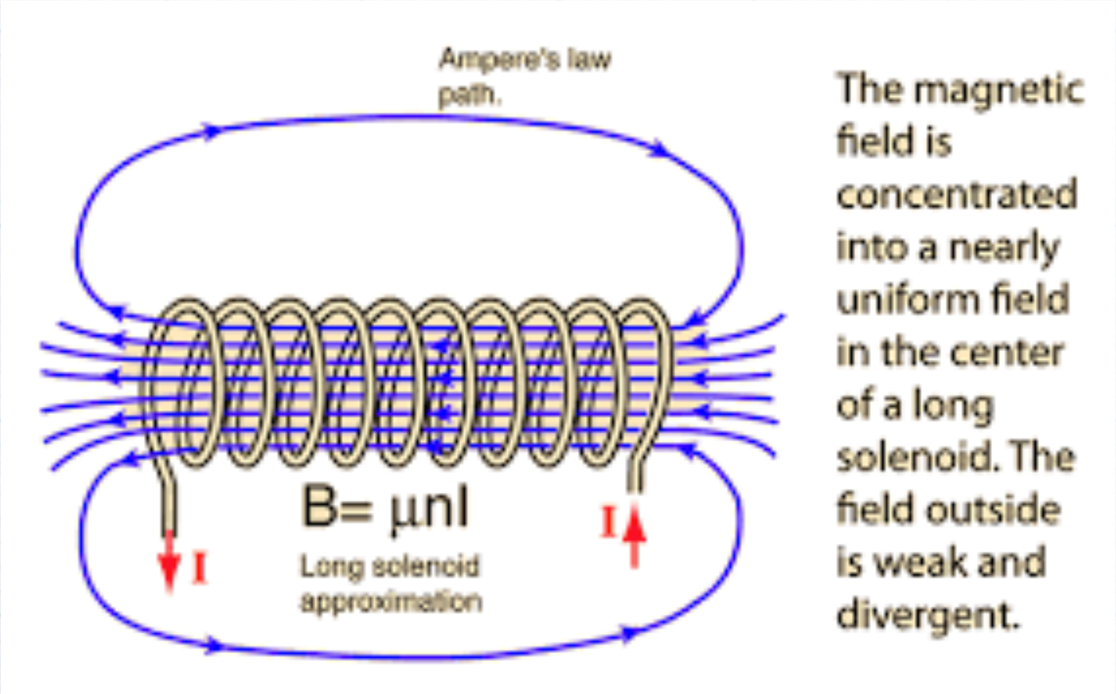
Faraday's Law of Induction

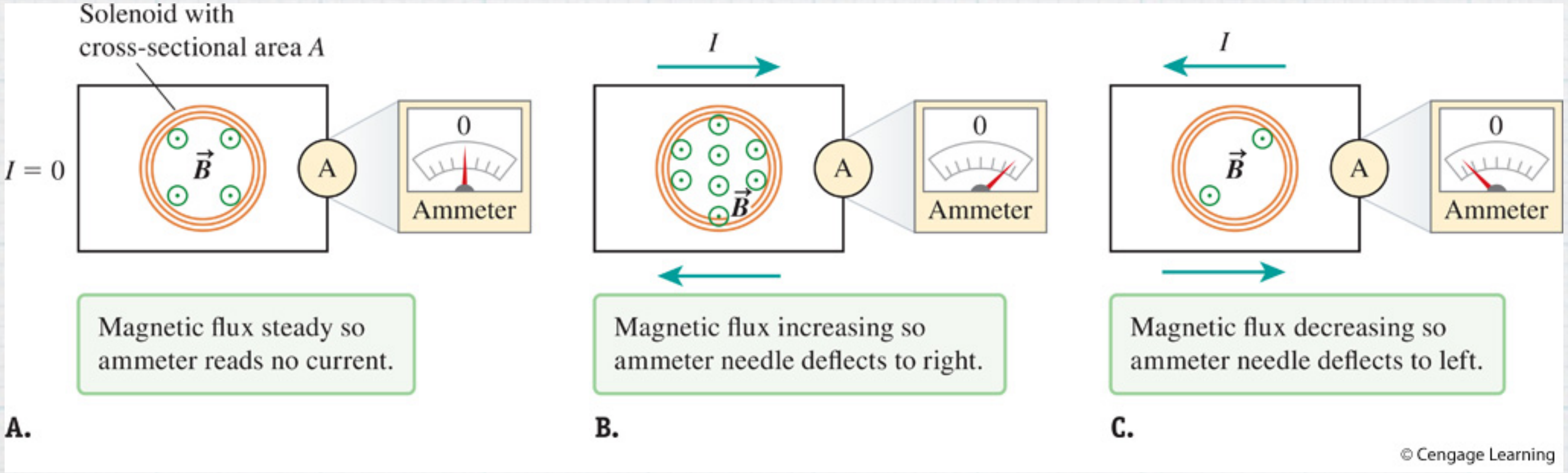
Or, how we can make generators and motors

Demo x 3


I
+
ℰ
−
A
0
Ammeter
Ammeter measures a positive current.
© Cengage Learning


S
N
I
A
0
Ammeter
A.
S
N
I
A
0
Ammeter
B.
© Cengage Learning


Ampere's law path.
B= μnI
Long solenoid approximation
I
I
The magnetic field is concentrated into a nearly uniform field in the center of a long solenoid. The field outside is weak and divergent.

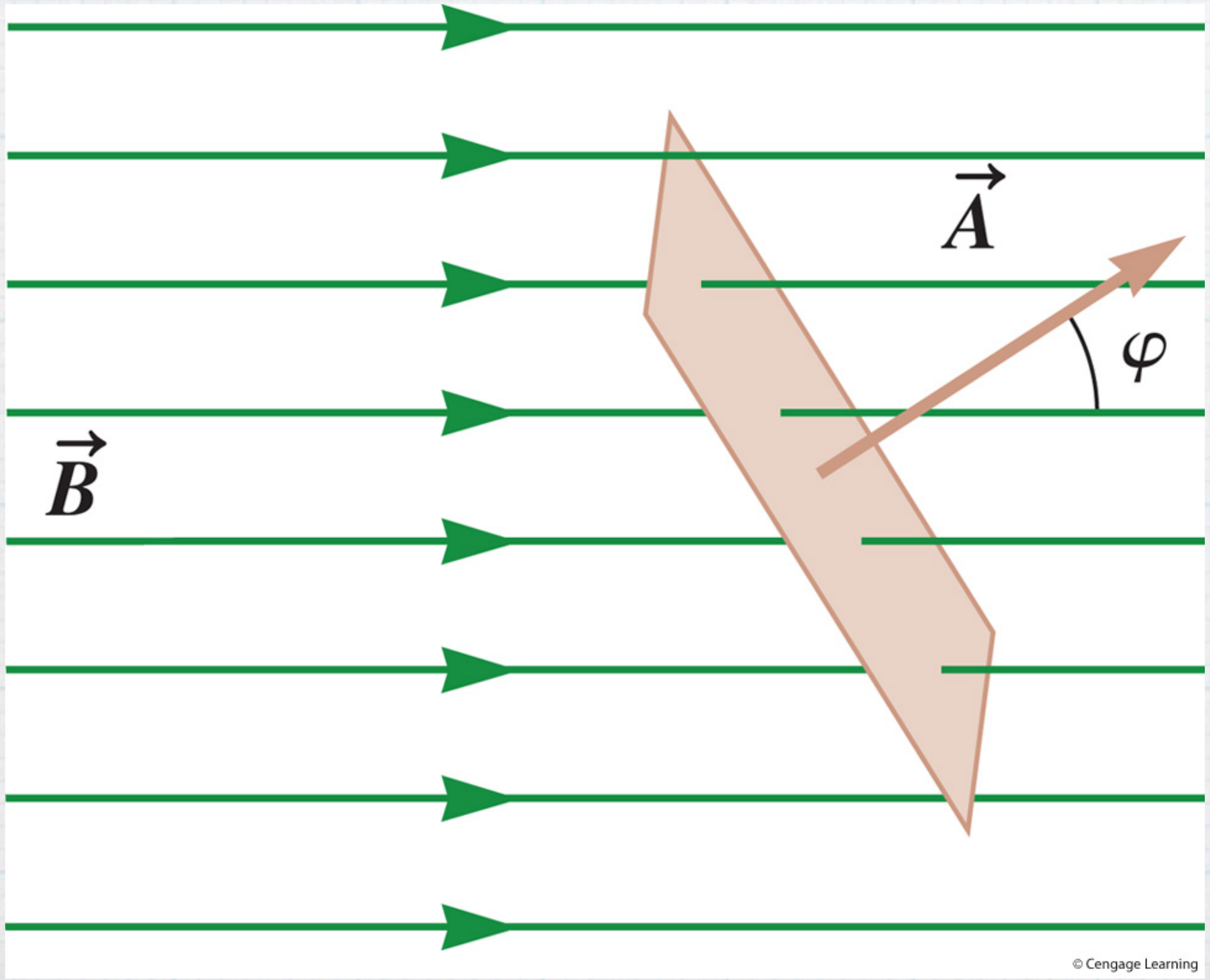

Solenoid with cross-sectional area A
I = 0
A
0
Ammeter
Magnetic flux steady so ammeter reads no current.
A.
I
Magnetic flux increasing so ammeter needle deflects to right.
B.
Magnetic flux decreasing so ammeter needle deflects to left.
C.
© Cengage Learning

Magnetic Flux

$$\Phi = \int \vec{B} \cdot \vec{d}A$$

1 Weber = 1 T m^2

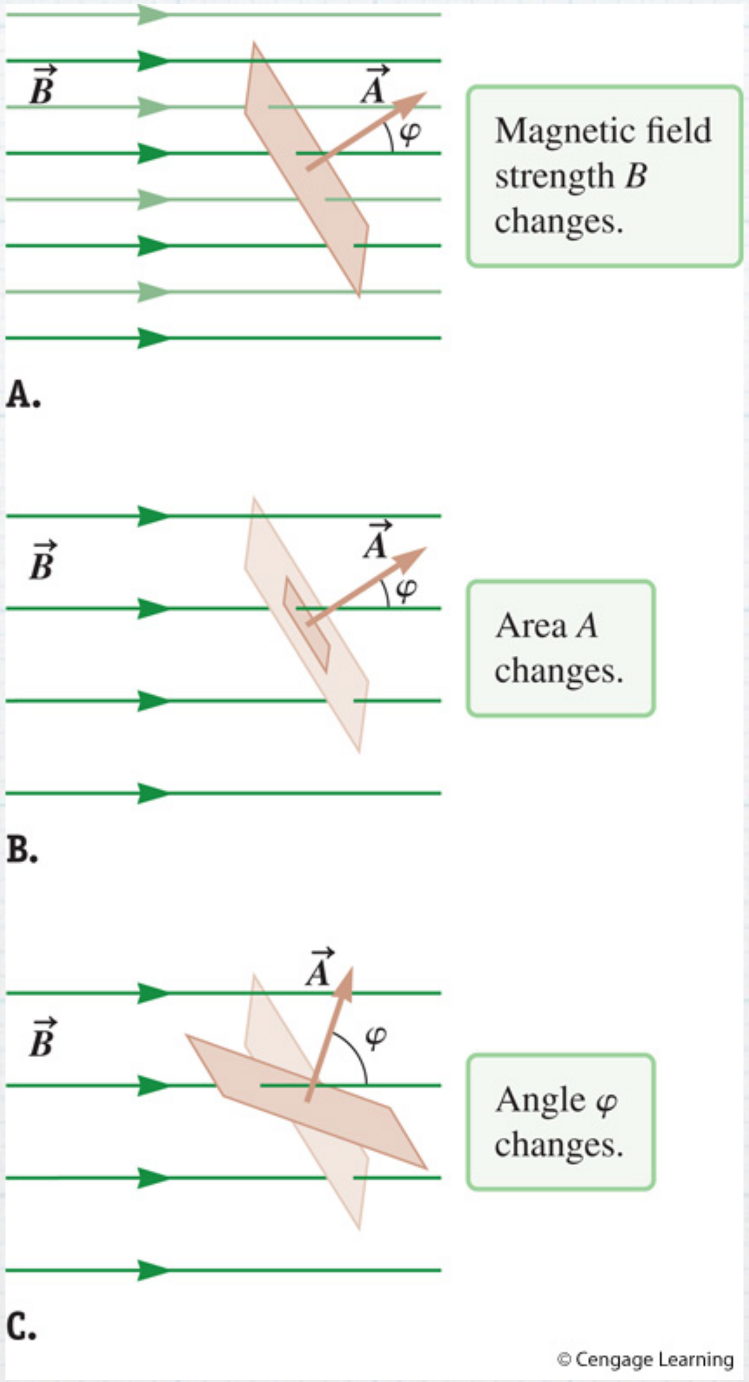
$$\Phi_B = \int B cos\psi dA$$



If the magnetic field is constant and the area is flat

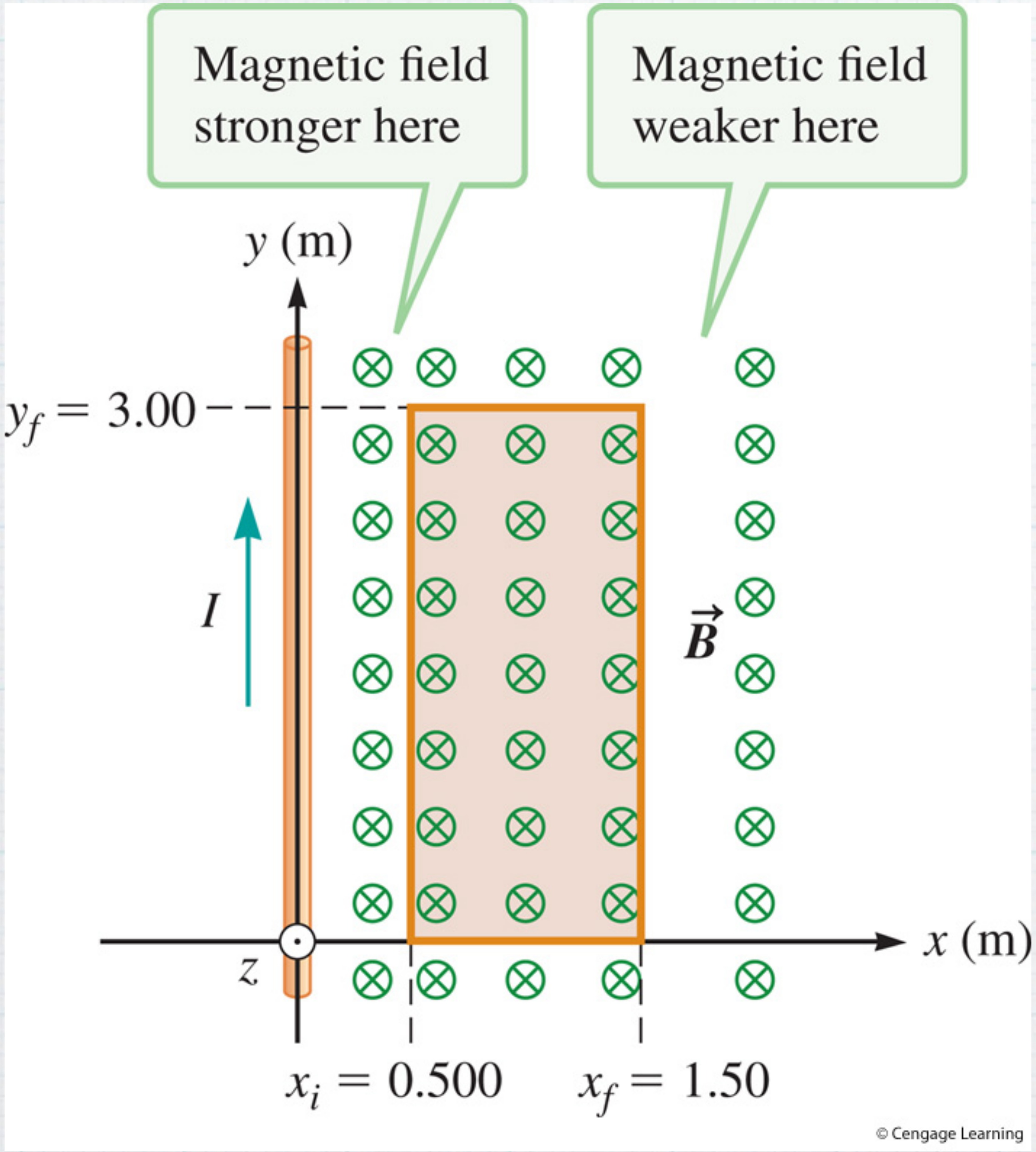
$$\Phi_B = BAcos\psi$$

Example



$\vec{B}$
$\vec{A}$
φ
Magnetic field strength B changes.
A.
$\vec{B}$
$\vec{A}$
φ
Area A changes.
B.
$\vec{B}$
$\vec{A}$
φ
Angle φ changes.
C.
© Cengage Learning

Example



$$\vec{B} = -\frac{\mu_0 I}{2\pi x}\hat{k}$$

$$d\vec{A} = -y_f dx\hat{k}$$

Faraday's Law

Faraday's Law of Induction

- Faraday identified that it was the change in the magnetic field in a coil, that induced an EMF in the circuit (coil)

$$\xi = -\frac{d\Phi_B}{dt}$$

- This is Faraday's Law of Induction
- If more than one loop is in the coil, then the induced EMF is:

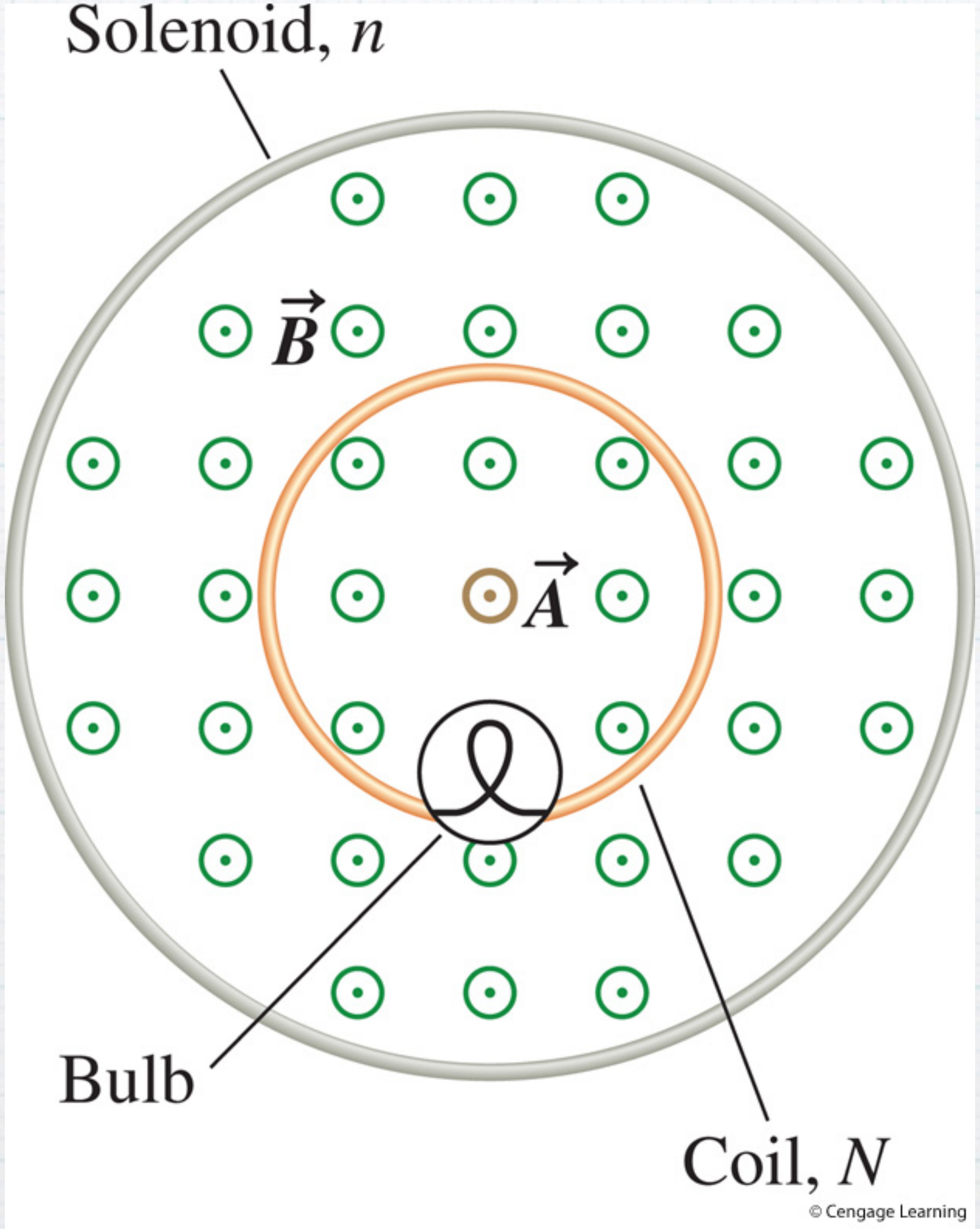
$$\xi = -N\frac{d\Phi_B}{dt}$$

- Note: $d\Phi_B = either\ dB \cdot A\ or\ B \cdot dA$
- The induced current in the circuit will therefore be:

$$I = \frac{\xi}{R}$$

where R = the resistance of the coil of wire

Examples



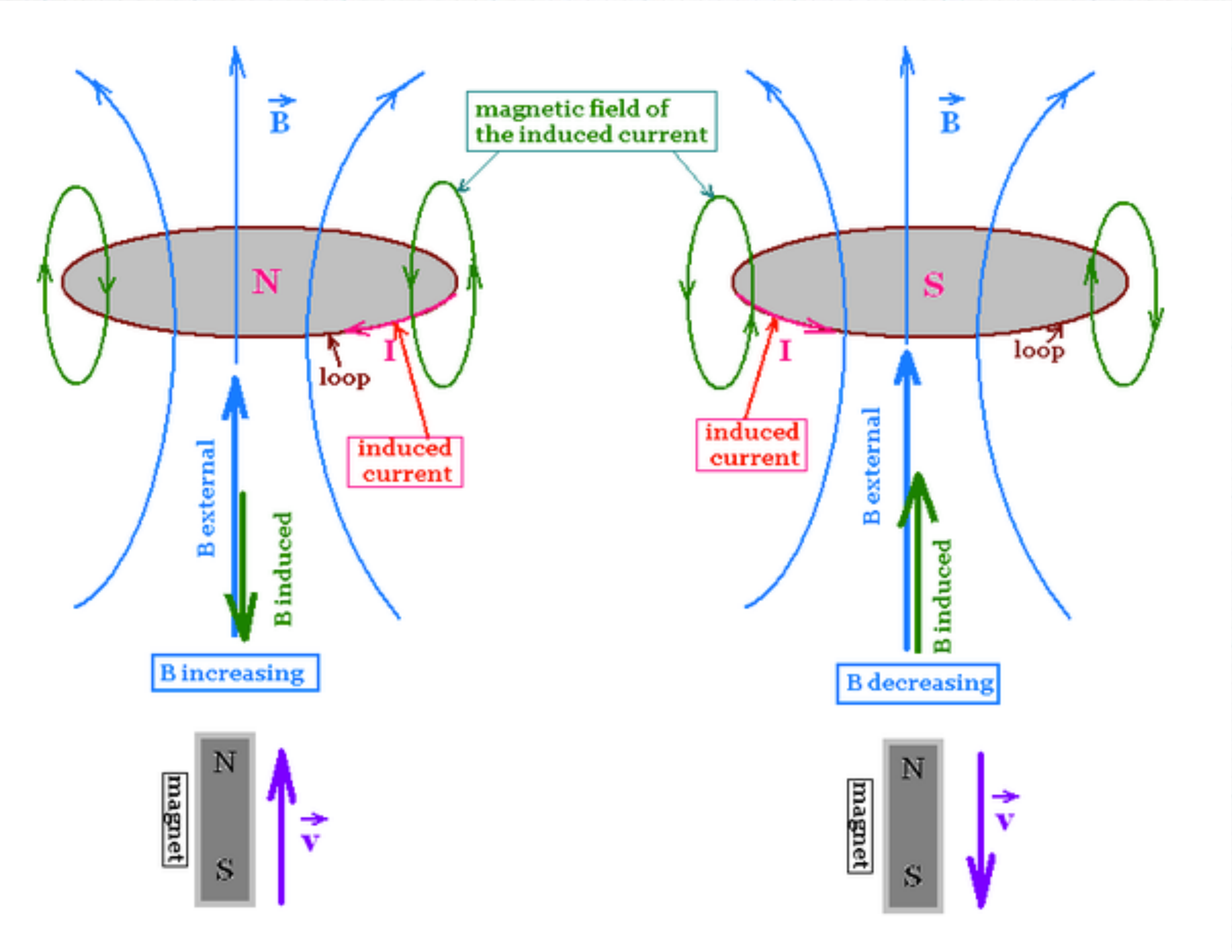
I=1/2 t

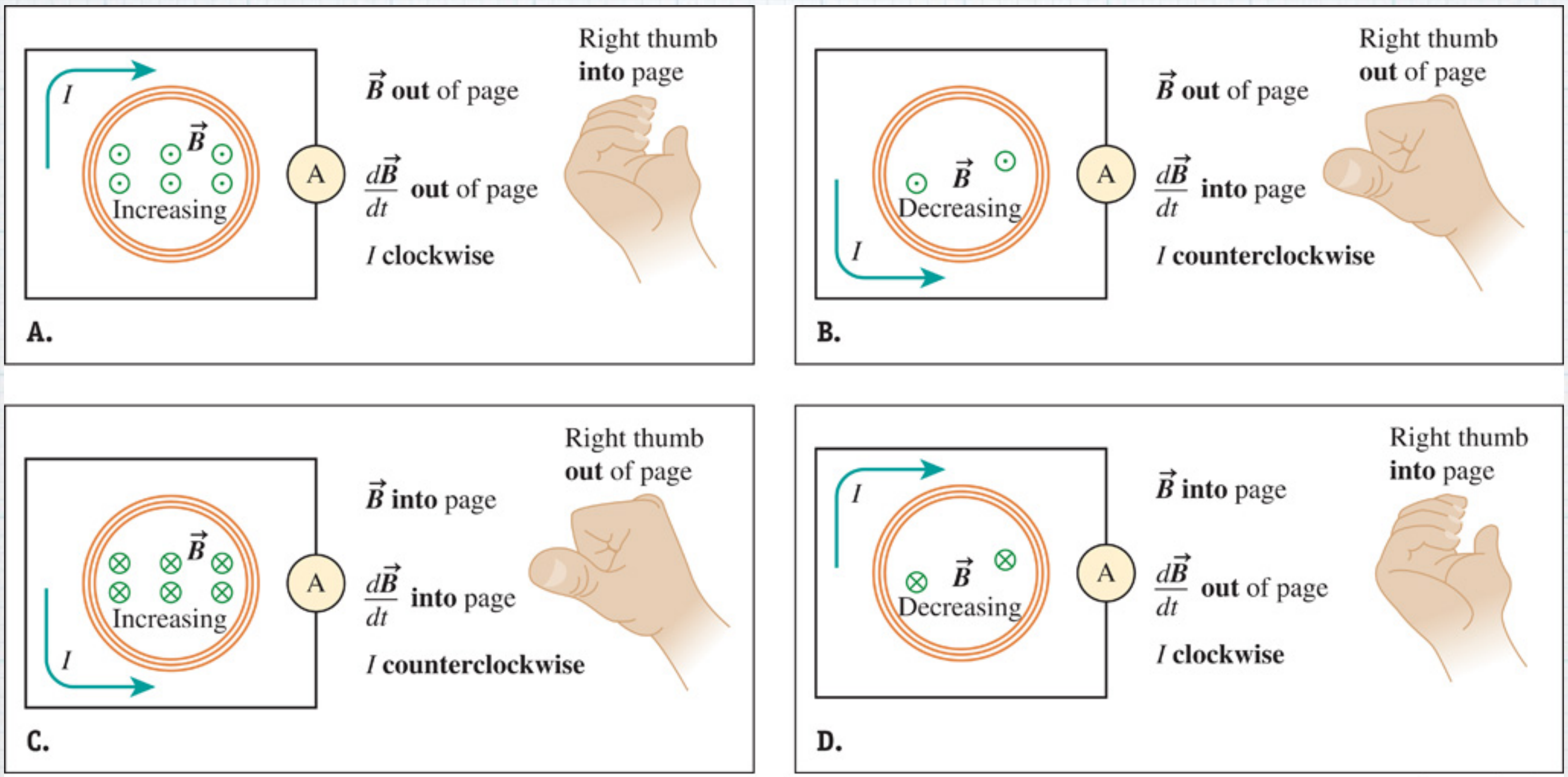
Here's the tricky part

* Direction of induced current and field
* What is happening is that there is relative motion between the conductor and the field
* The electrons "feel" a magnetic force
* They move in response to it
* A moving charge IS a current
* A current generates an electrical field

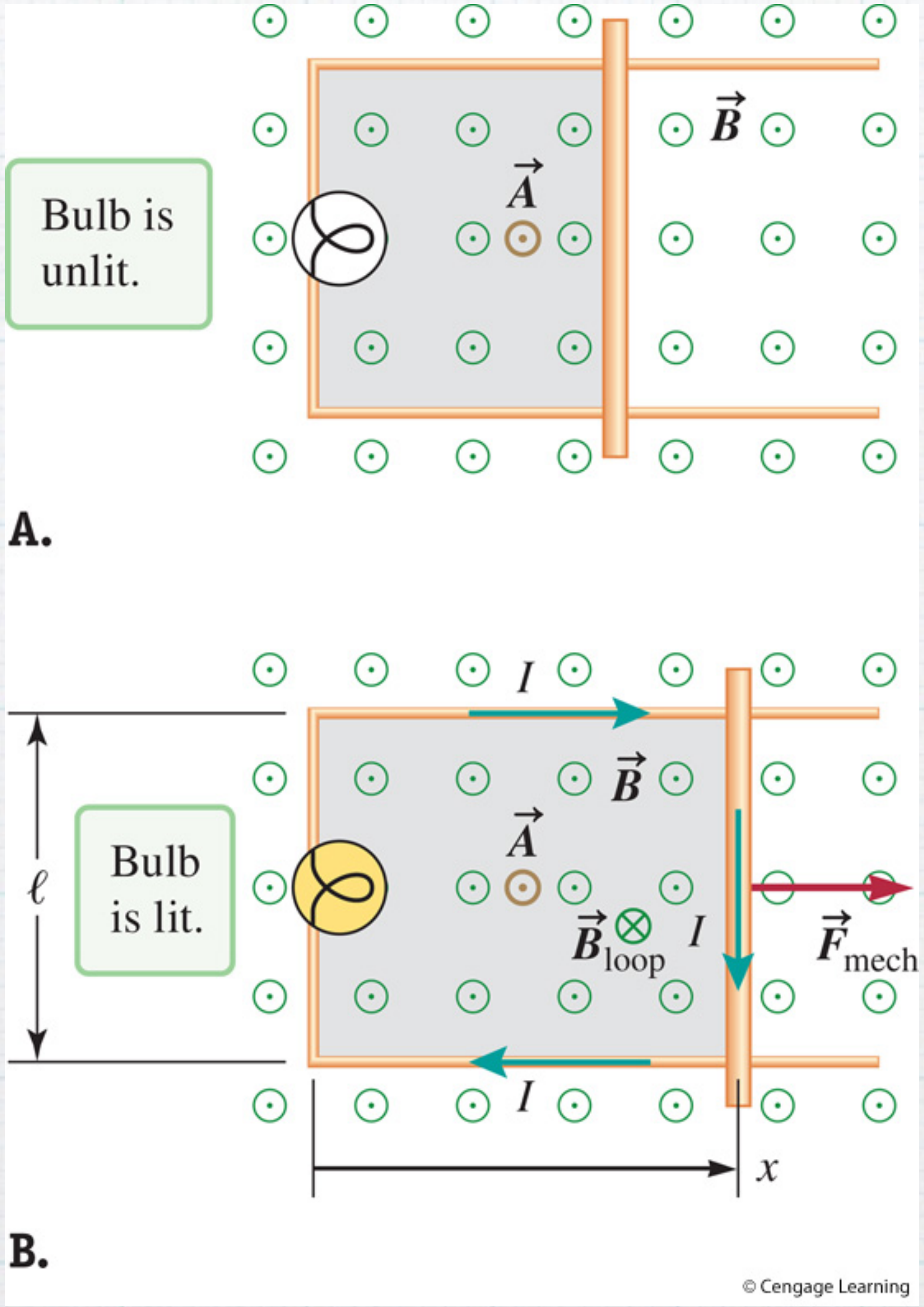
Lenz's Law

* Nature abhors a change in flux
* It will ALWAYS act to oppose it
* Do you understand why?
* Many examples


B
magnetic field of the induced current
B
N
S
I
loop
I
loop
induced current
induced current
B external
B induced
B external
B induced
B increasing
B decreasing
magnet
N
S
v
magnet
N
S
v


I
Increasing
A
$\vec{B}$ out of page
$\frac{d\vec{B}}{dt}$ out of page
I clockwise
Right thumb into page
A.
I
Decreasing
A
$\vec{B}$ out of page
$\frac{d\vec{B}}{dt}$ into page
I counterclockwise
Right thumb out of page
B.
I
Increasing
A
$\vec{B}$ into page
$\frac{d\vec{B}}{dt}$ into page
I counterclockwise
Right thumb out of page
C.
I
Decreasing
A
$\vec{B}$ into page
$\frac{d\vec{B}}{dt}$ out of page
I clockwise
Right thumb into page
D.

Motional EMF

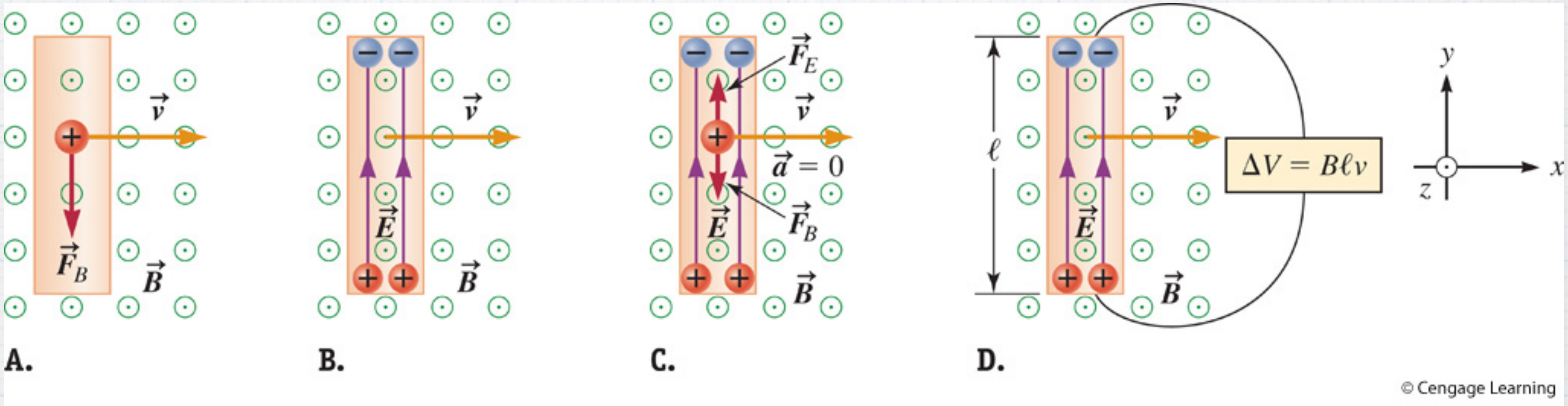


$$\Phi = Blx$$

$$\xi = Blv$$

$$P = IlBv = \frac{(Blv)^2}{R}$$

Show



In equilibrium

$$\vec{F}_E = \vec{F}_B$$

and

$$\Delta V = El$$

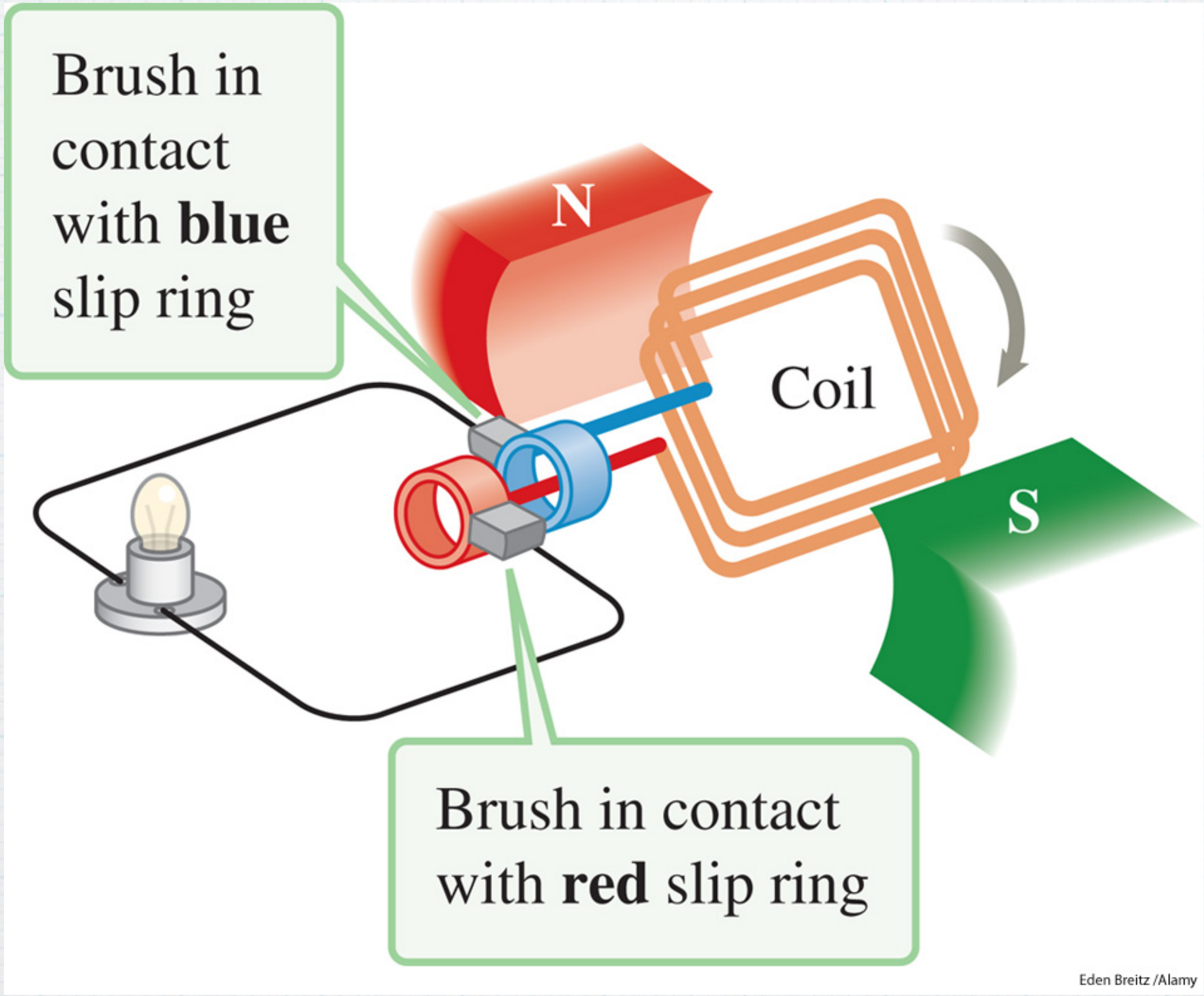
Reading Question 32.1

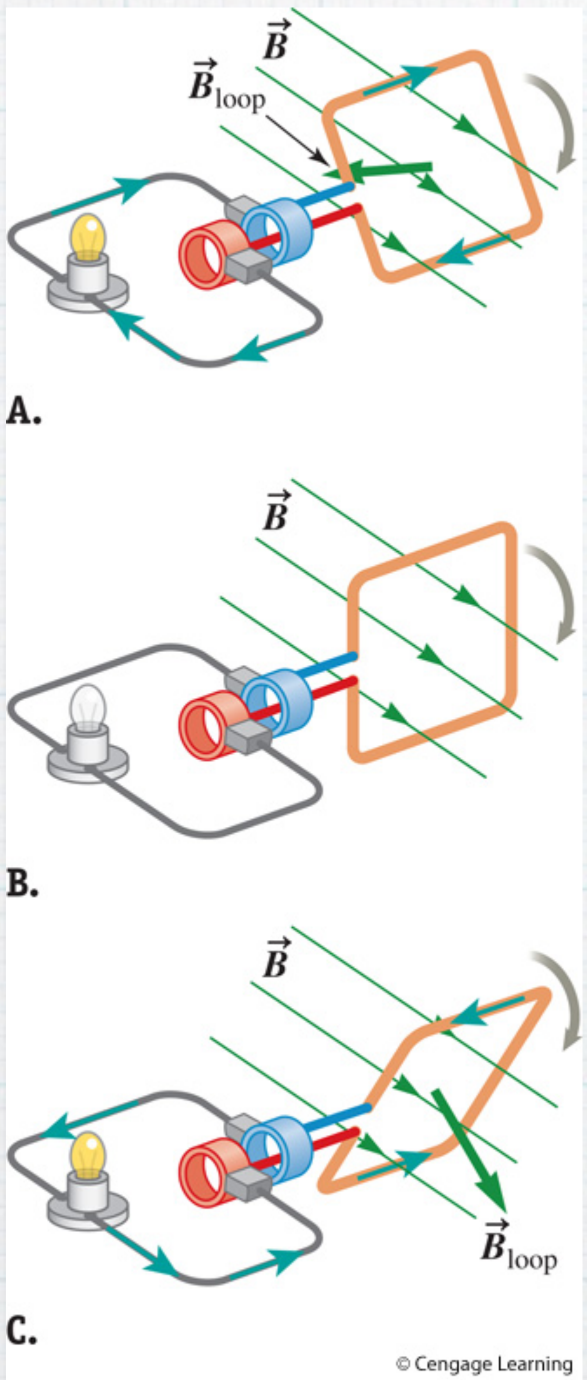
The source of an induced current is

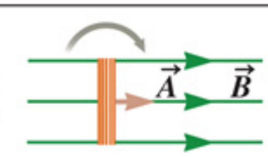
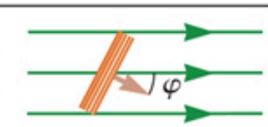
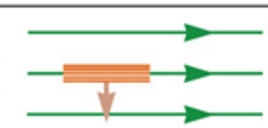
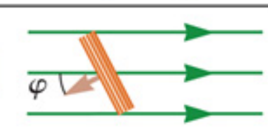
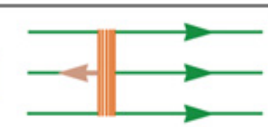
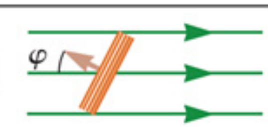
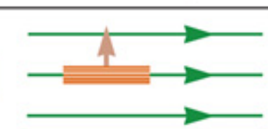
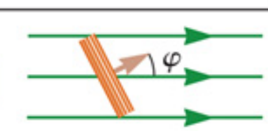
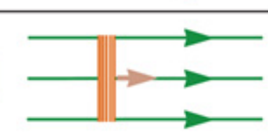
a. A changing magnetic flux
b. A changing electric flux
c. A discharging battery
d. A discharging capacitor

Example

AC Generators





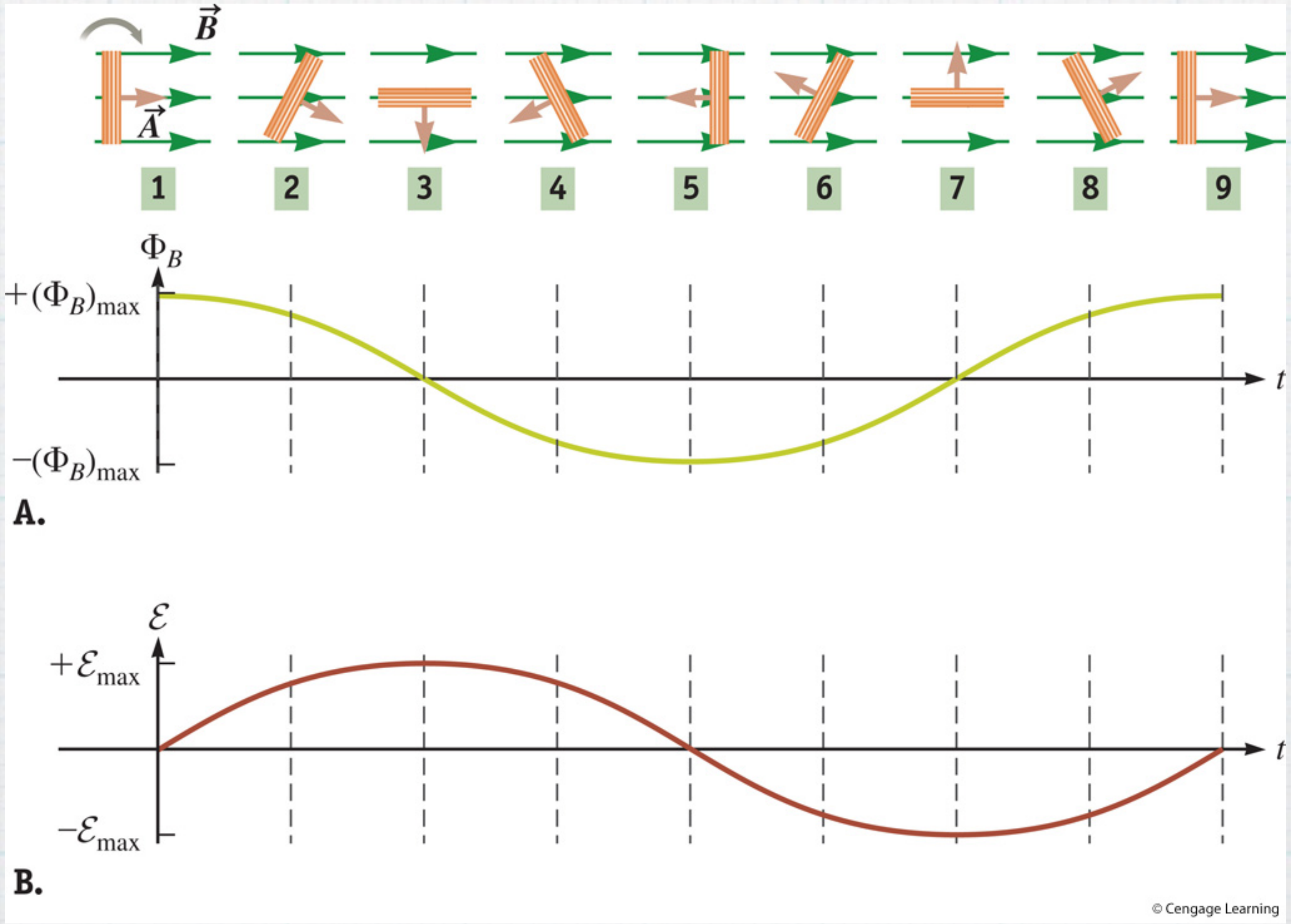
Configuration	Magnetic flux Φ_B	Change in flux $\frac{d\Phi_B}{dt}$	Induced emf $\mathcal{E}$
1 	Positive, maximum	Momentarily zero (constant flux)	Zero
2 	Positive	Decreasing (negative)	Positive
3 	Zero	Decreasing (negative)	Positive
4 	Negative	Decreasing (negative)	Positive
5 	Negative, maximum	Momentarily zero (constant flux)	Zero
6 	Negative	Increasing (positive)	Negative
7 	Zero	Increasing (positive)	Negative
8 	Positive	Increasing (positive)	Negative
9 	Return to positive, maximum	Momentarily zero (constant flux)	Zero

Power and Current

$$\xi = NBA\omega sin(\omega t) = \xi_{max} sin(\omega t)$$

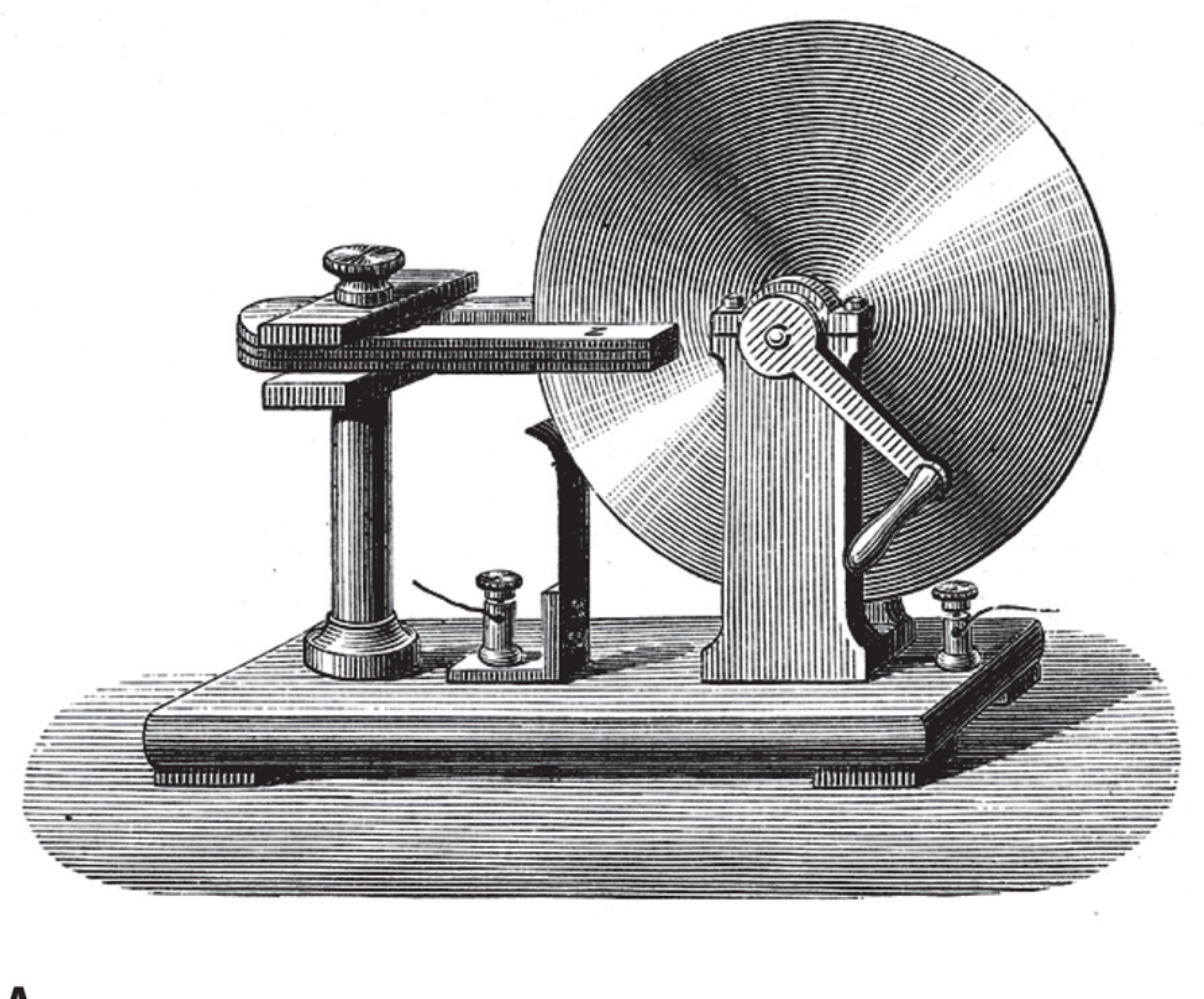
$$I = \frac{NBA\omega sin(\omega t)}{R} = I_{max} sin(\omega t)$$

Show (hint) $\Phi = BAcos(\psi)$ $\quad cos(\psi) = cos(\omega t)$



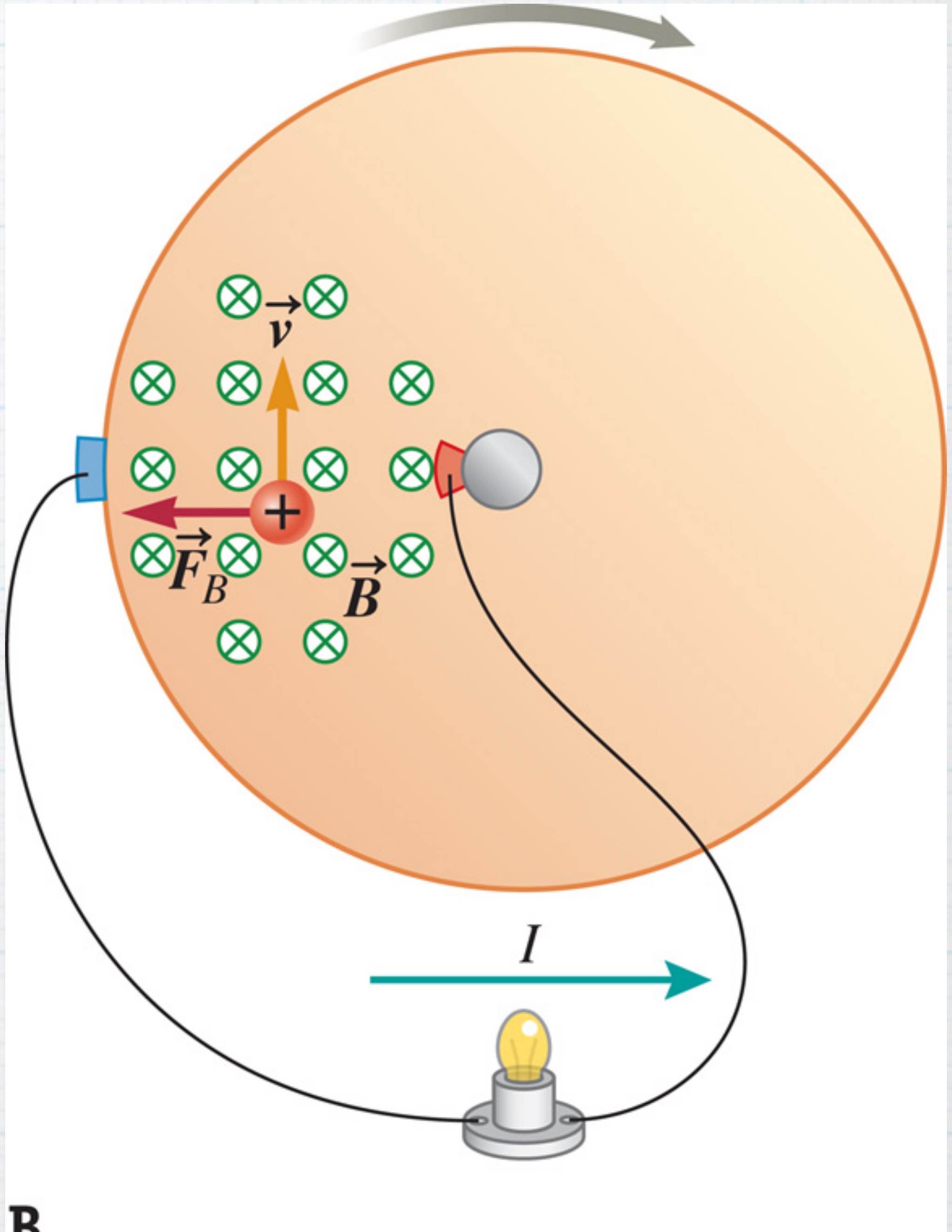
Examples

DC Generator

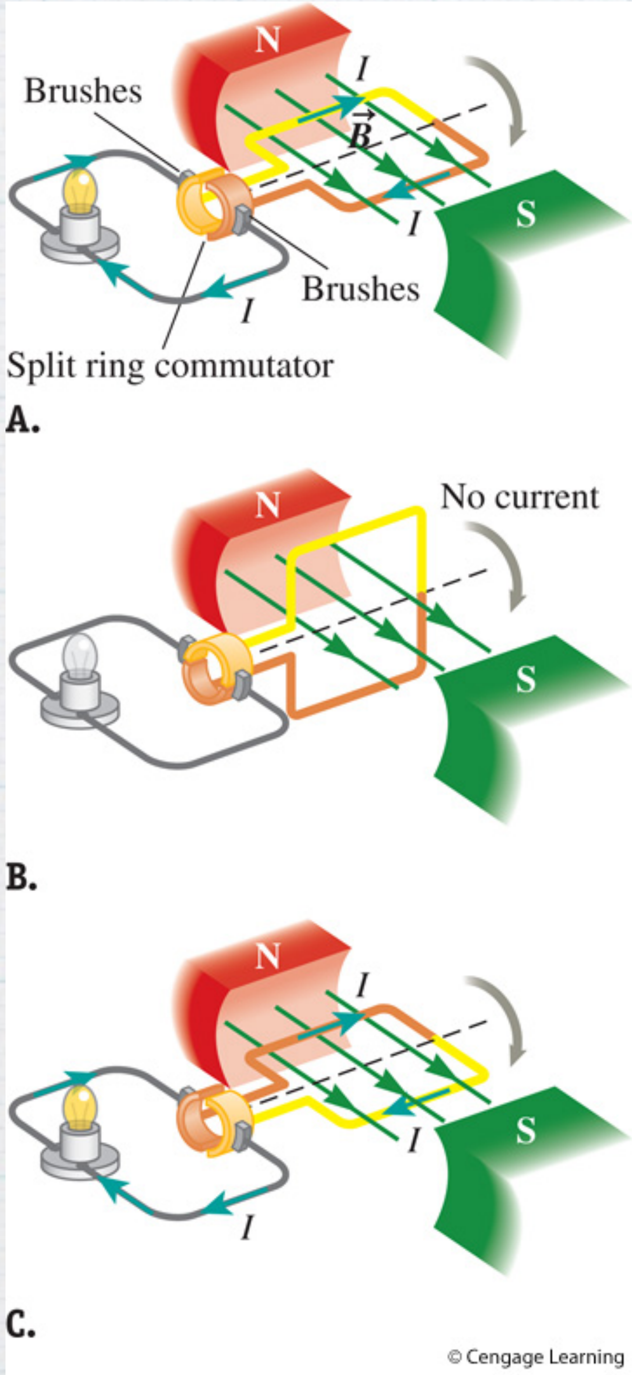


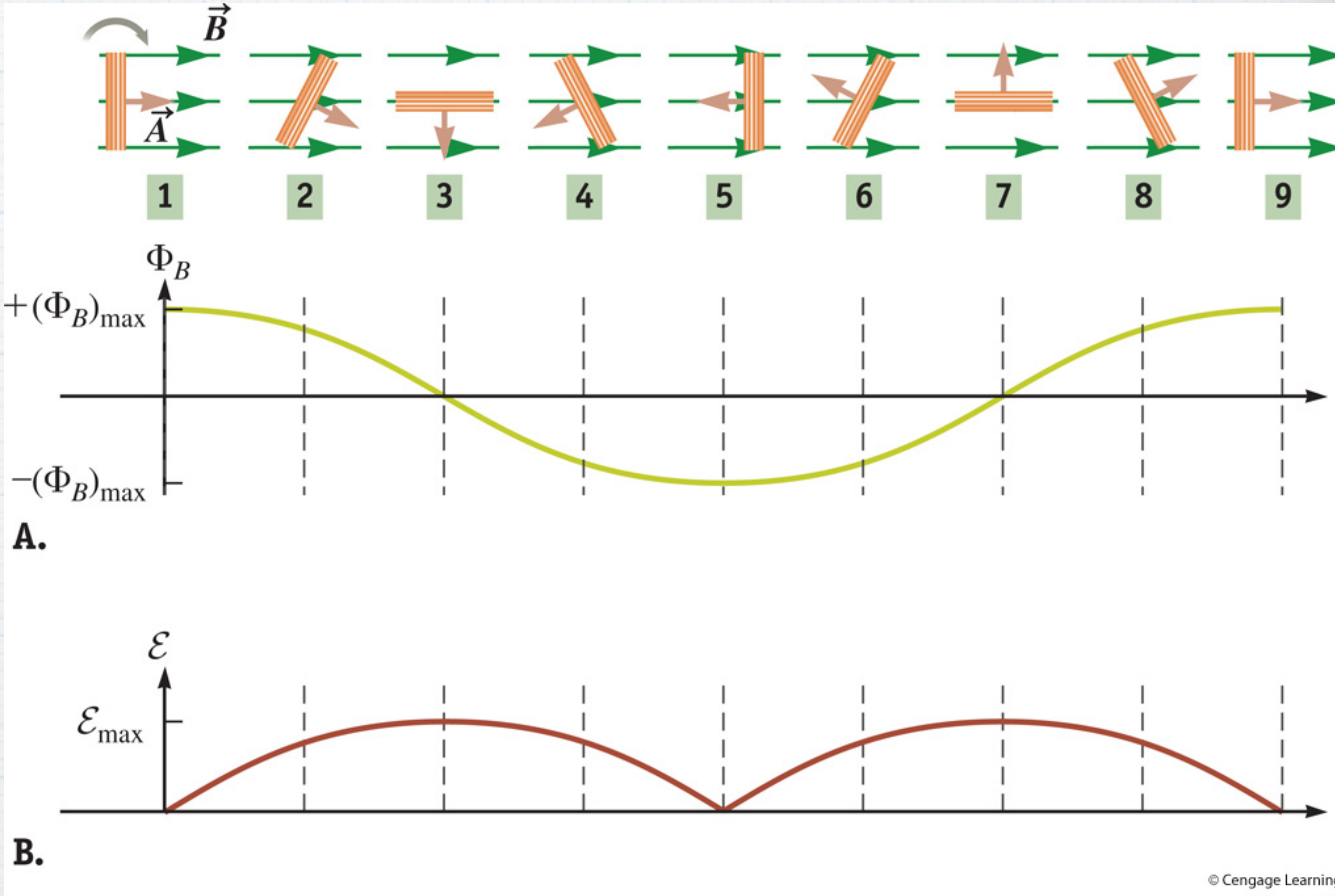
A.

Stock Montage/Getty Images



B.





Same results as for AC but always positive

Examples

Power Transmission and Transformers

* Power is transmitted at low current and high voltage
* It is converted to high current and low voltage for your use
* Joule Heating losses

Root Mean Square Emf, current, power

$$\xi = NBA\omega sin(\omega t) = \xi_{max} sin(\omega t)$$

$$\xi^2 = (NBA\omega sin(\omega t))^2$$

$$\xi_{avg}^2 = (NBA\omega)_{avg}^2 (sin(\omega t))_{avg}^2 = \frac{1}{2}(NBA\omega)^2$$

$$(sin(\omega t))_{avg}^2 = \int_0^T \frac{(sin(\omega t))^2}{T} dt \rightarrow T = \frac{2\pi}{\omega}$$

$$\xi_{rms} = \sqrt{\frac{1}{2}(NBA\omega)^2}$$

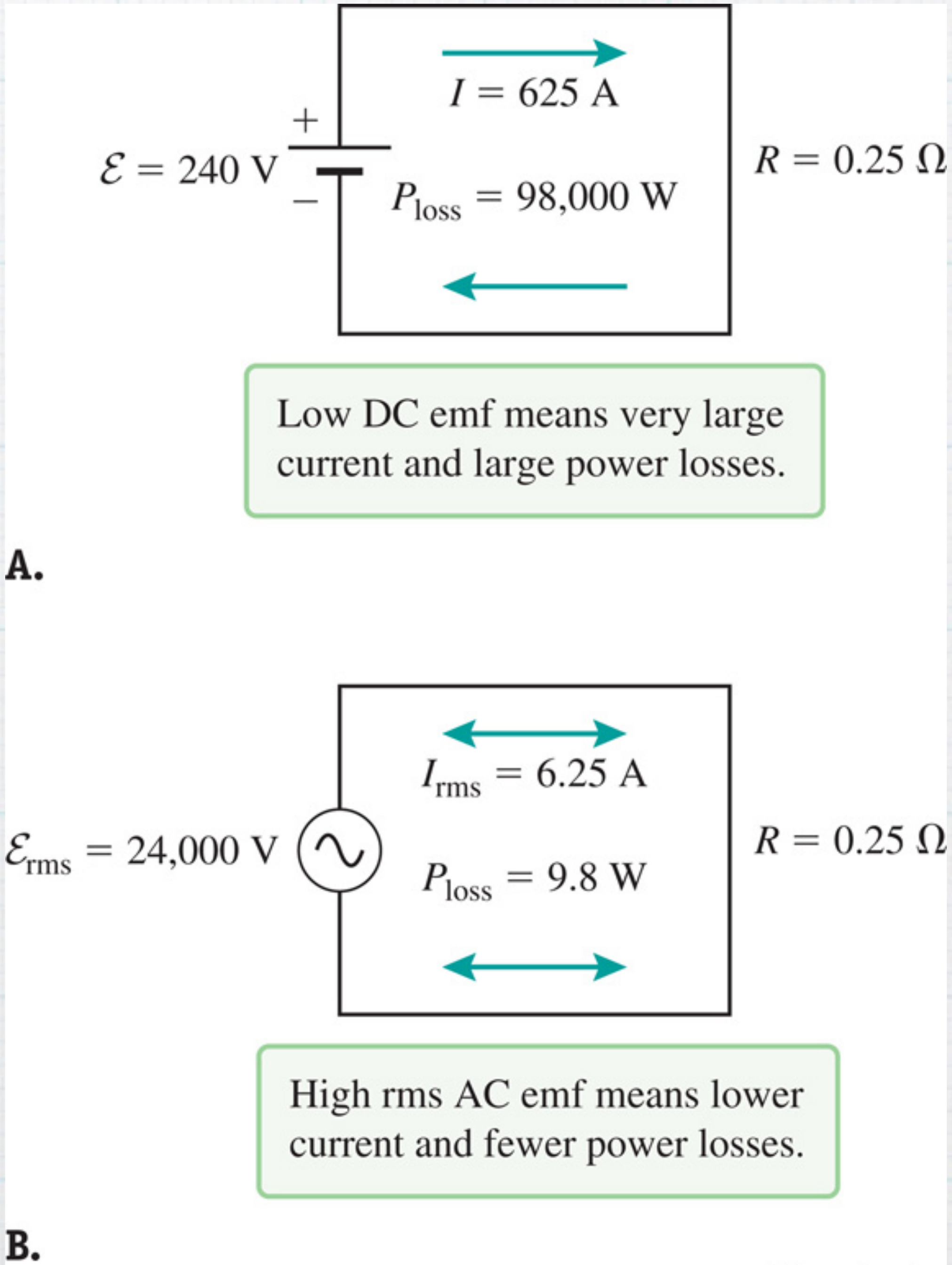
$$\xi_{rms} = \frac{1}{\sqrt{2}} NBA\omega = \frac{\xi_{max}}{\sqrt{2}}$$

$$I_{rms} = \frac{I_{max}}{\sqrt{2}}$$

$$P = I_{max}\xi_{max} sin^2(\omega t)$$

$$P_{avg} = I_{rms}\xi_{rms}$$

$$P_{lost} = I_{rms}^2 R$$

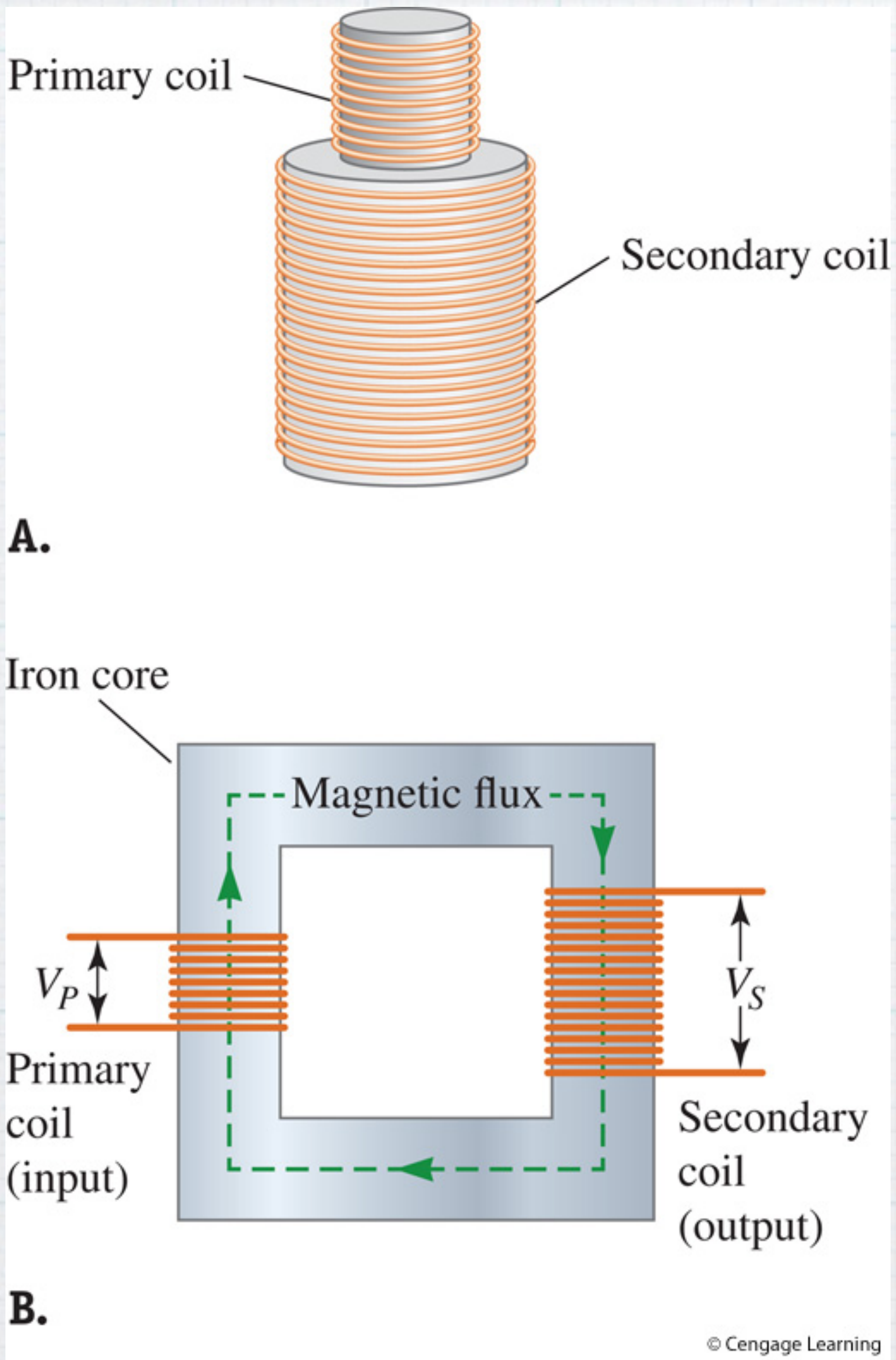


Examples

A.

Mark Williamson/Photo Researchers

Transformers



$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

$$P_{in} = P_{out} \rightarrow I_p V_p = I_s I_p$$

$$\frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s}$$

Examples