Electric Potential and Potential Energy

A reformulation from a vector approach to a scalar approach

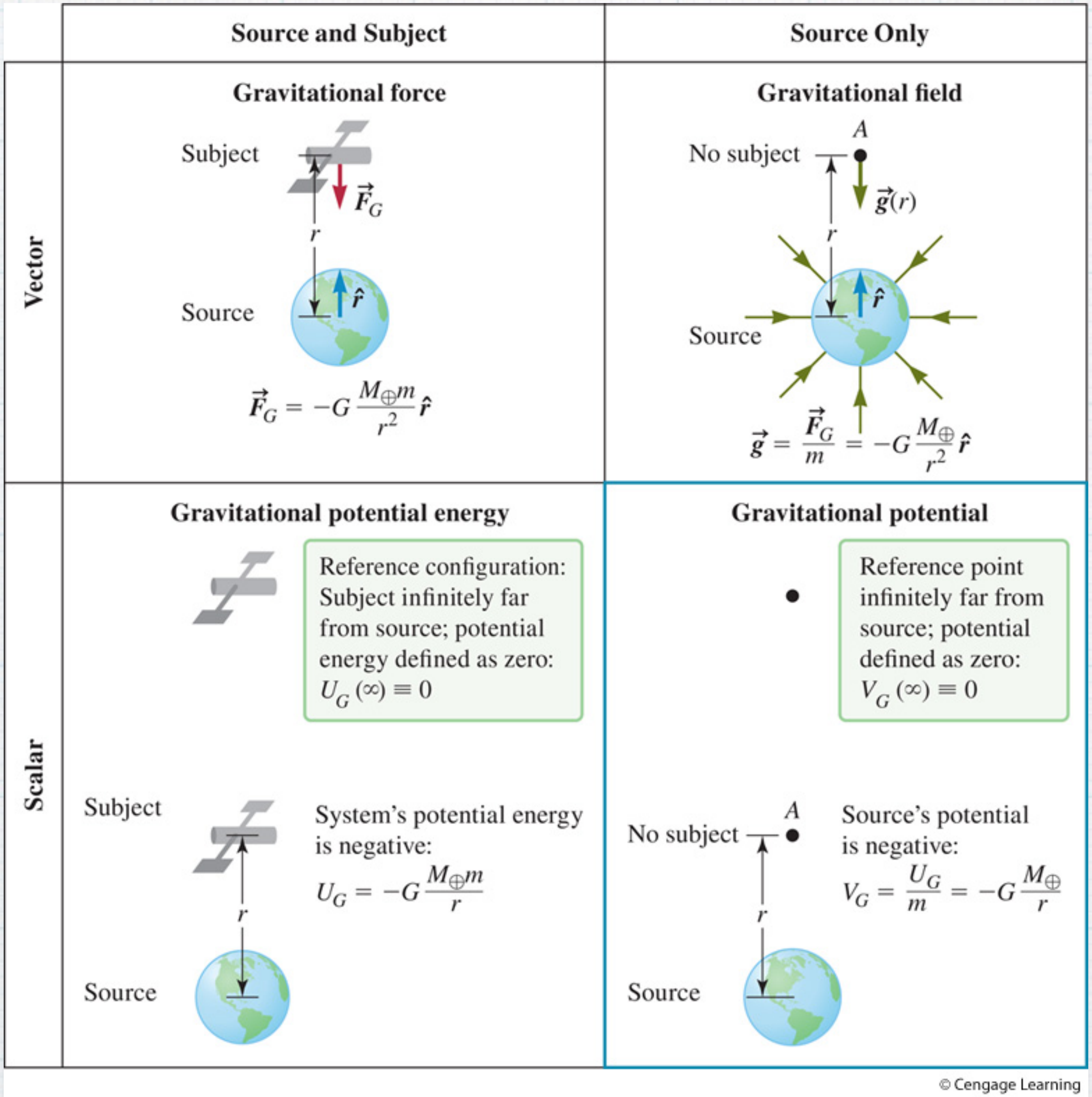
Once again, compare to gravity, be very careful though

Potential is not the same thing as potential energy

Look at the eraser in my hand

Potential has units of **volts** or **joules/coulomb** for electrical potential and **joules/kg** for gravitational potential, it is a measure of something that can happen due to external influences and does not depend on the thing it acts on

Potential Energy is associated with an object in a potential field and has units of **joules**



Let's do an example, the mass of the earth is 5.97×10^{24} kg and its radius is 6.38×10^6 m

Remember your basic energy stuff

* An object moving through a potential difference either loses or gains kinetic energy

* The net work done on an object, both conservative and non-conservative equals the change in the object's kinetic energy

$$\sum_i \oint \vec{F}_i \cdot \vec{ds} = \Delta KE$$

* If there are no non conservative forces then

$$\sum_i \oint \vec{F}_i \cdot \vec{ds} = -\sum \Delta U_i = \Delta KE$$

Energy continued

* for multiple forces (example gravity and electricity)

$$-\Delta U_{grav} - \Delta U_{electrical} = \Delta KE$$

* or more generally

$$-\Delta U_{total} = \Delta KE \rightarrow \Delta U + \Delta KE = 0 \rightarrow E_i = E_f$$

Reading Question 26.2

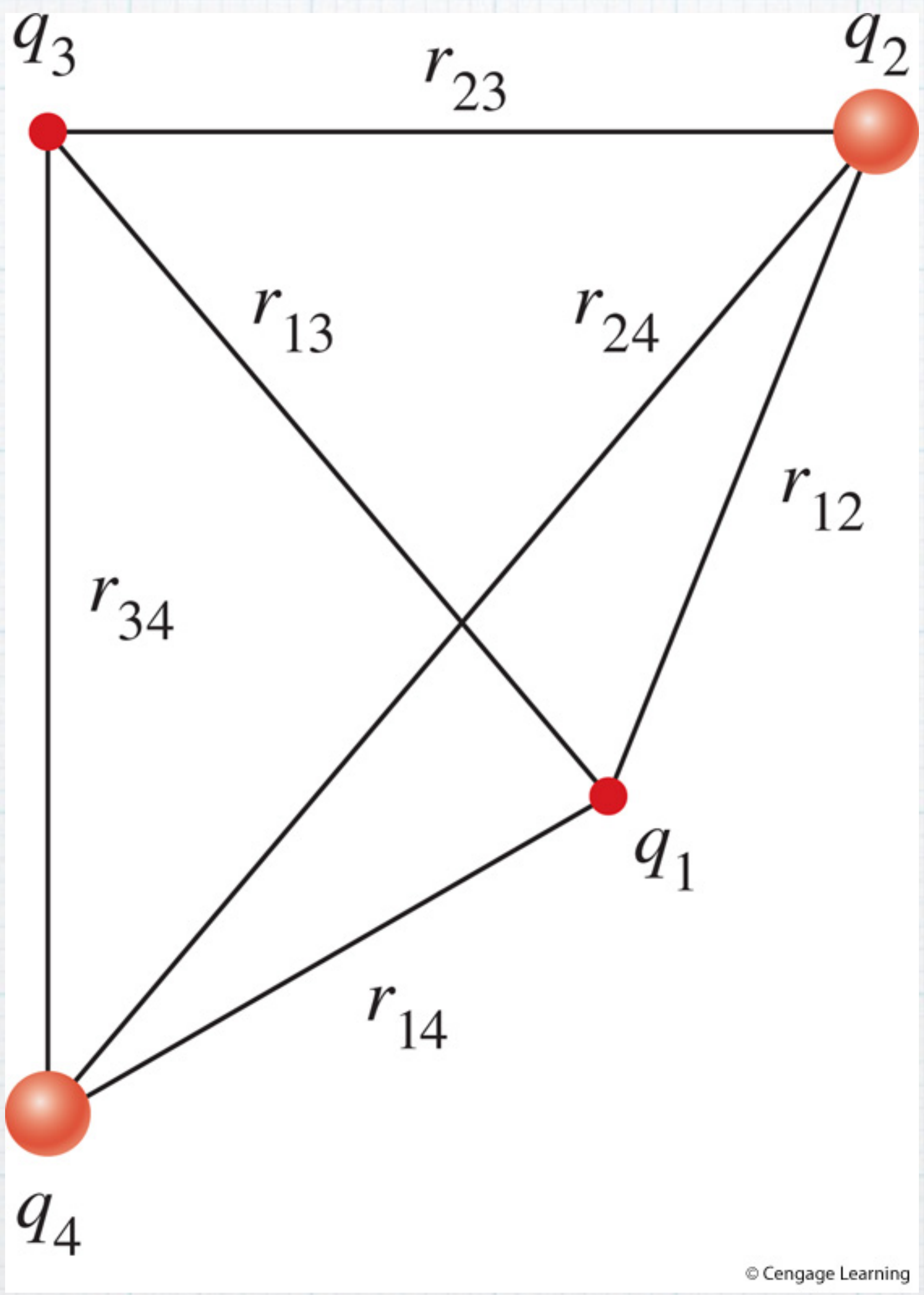
Electric potential energy is:

a. Independent of the subject's charge
b. Inversely proportional to the distance between subject and source
c. A vector quantity with direction determined by the electric flux
d. Always a negative value

Potential Energy for point charges/spheres

* The potential energy between two charged particles is given by $U_e(r) = \frac{kQq}{r}$
* For multiple particle systems add up all of the individual potentials
* Don't get confused, if you put two protons near each other they repel
* If you put a positive and negative particle near each other they attract
* Electrical potential energy and potential act just like gravity except that you fall up sometimes

Example



Examples

* Potential energy of a hydrogen atom $r=10^{-10}$ m
* This is the energy you'd need to unbind the hydrogen atom
* What happens if you use more than this much energy to liberate the hydrogen atom?
* Energy problems with various particles

Potential

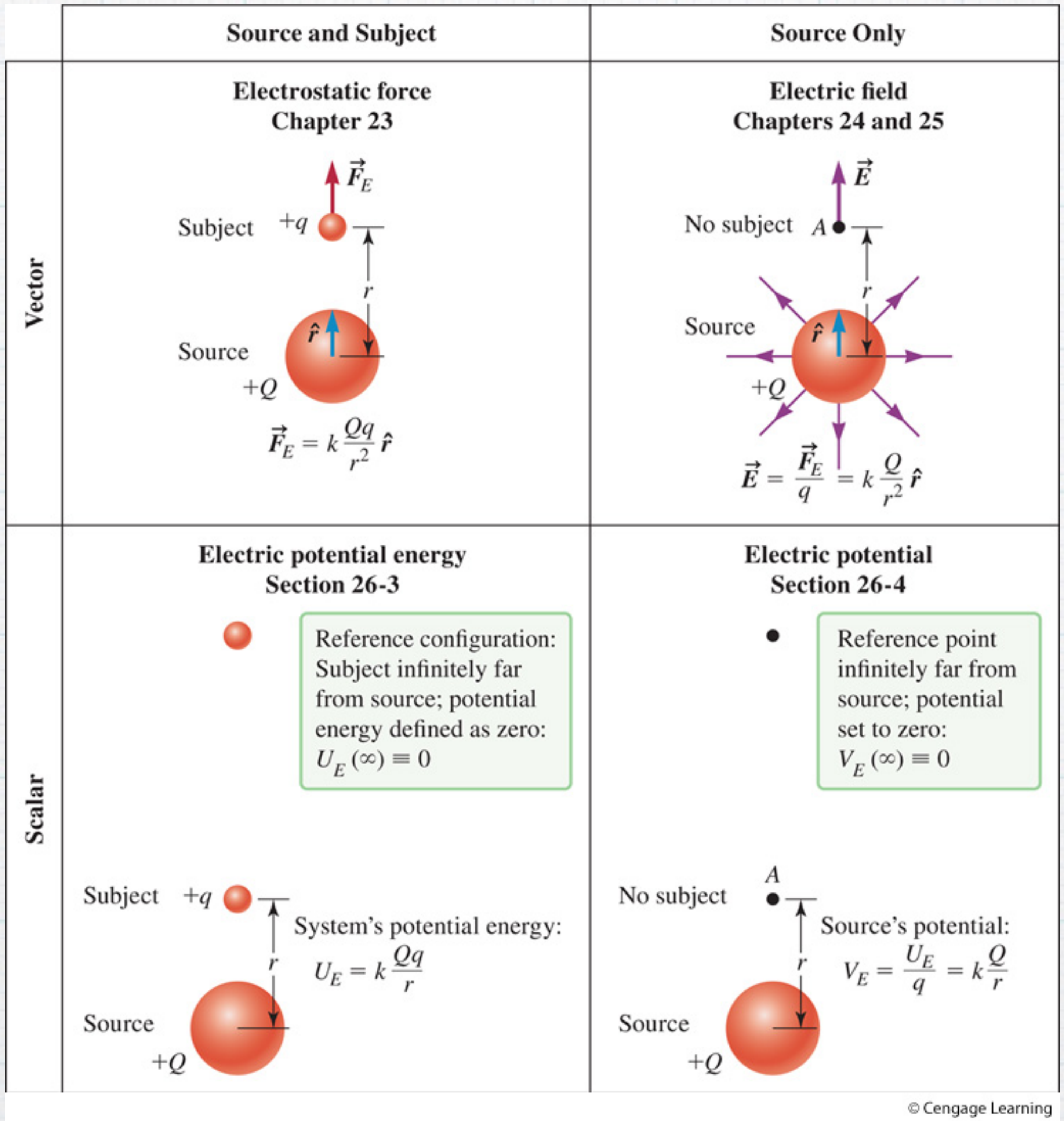
* Ok, as we saw with the electric force, one could really just look at source charges and calculate the fields they create. The electric field is a measure of the force a charged particle will experience if placed in the field
* The force experienced by a particle causes the other particles to feel equal and opposite forces
* Potential energy is a scalar reformulation of the vector force approach
* Potential is a scalar reformulation of the vector field approach

Potential

* A potential exists at a point in space due to sources
* For gravity, its mass
* For electricity, its charge
* Just like $\vec{F} = q\vec{E}$ $U = qV$
* So $U_{e,point} = qV_{e,point} \rightarrow V_{e,point} = \frac{kQ}{r}$

$$\Delta U = q\Delta V$$

* And it adds just like potential energy, just count up all the contributors


Source and Subject
Source Only
Vector
Scalar
Electrostatic force
Chapter 23
$\vec{F}_E$
Subject
$+q$
r
$\hat{r}$
Source
$+Q$
$\vec{F}_E = k\frac{Qq}{r^2}\hat{r}$
Electric field
Chapters 24 and 25
$\vec{E}$
No subject
A
Source
$+Q$
$\vec{E} = \frac{\vec{F}_E}{q} = k\frac{Q}{r^2}\hat{r}$
Electric potential energy
Section 26-3
Reference configuration: Subject infinitely far from source; potential energy defined as zero: $U_E(\infty) \equiv 0$
Subject
$+q$
System's potential energy:
$U_E = k\frac{Qq}{r}$
Source
$+Q$
Electric potential
Section 26-4
Reference point infinitely far from source; potential set to zero: $V_E(\infty) \equiv 0$
No subject
A
Source's potential:
$V_E = \frac{U_E}{q} = k\frac{Q}{r}$
Source
$+Q$
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RULE

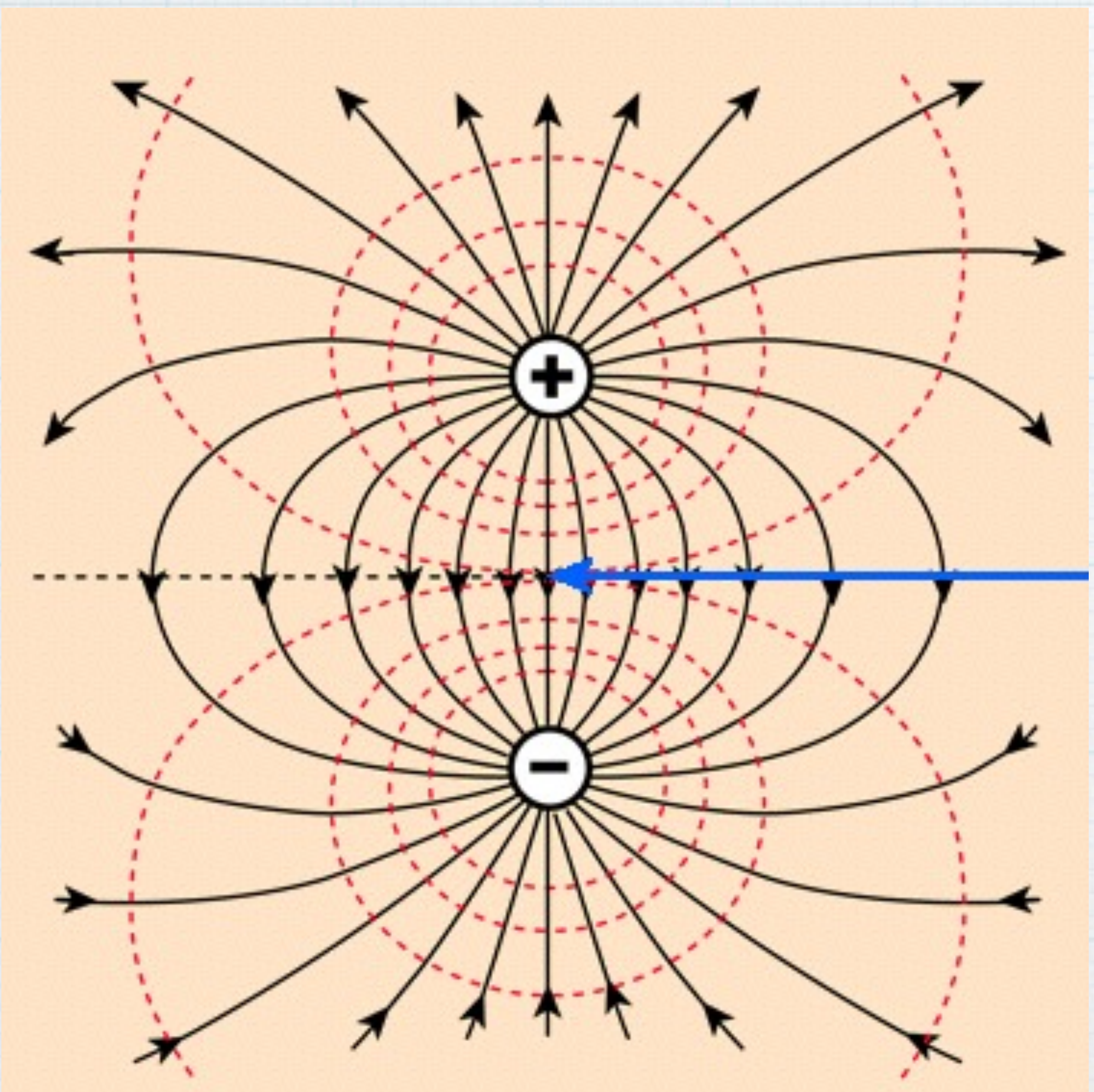
* Positively charged particles move from regions of higher to lower potential
* I will try to trick you with this
* Calculate the potential at a point due to all particles, use energy conservation if necessary to complete the problem
* Let's do a few examples
    * 2 particle system
    * 3 particle system
    * 4 particle system
    * Plot the potential in 1d and 2d for an electron and a proton

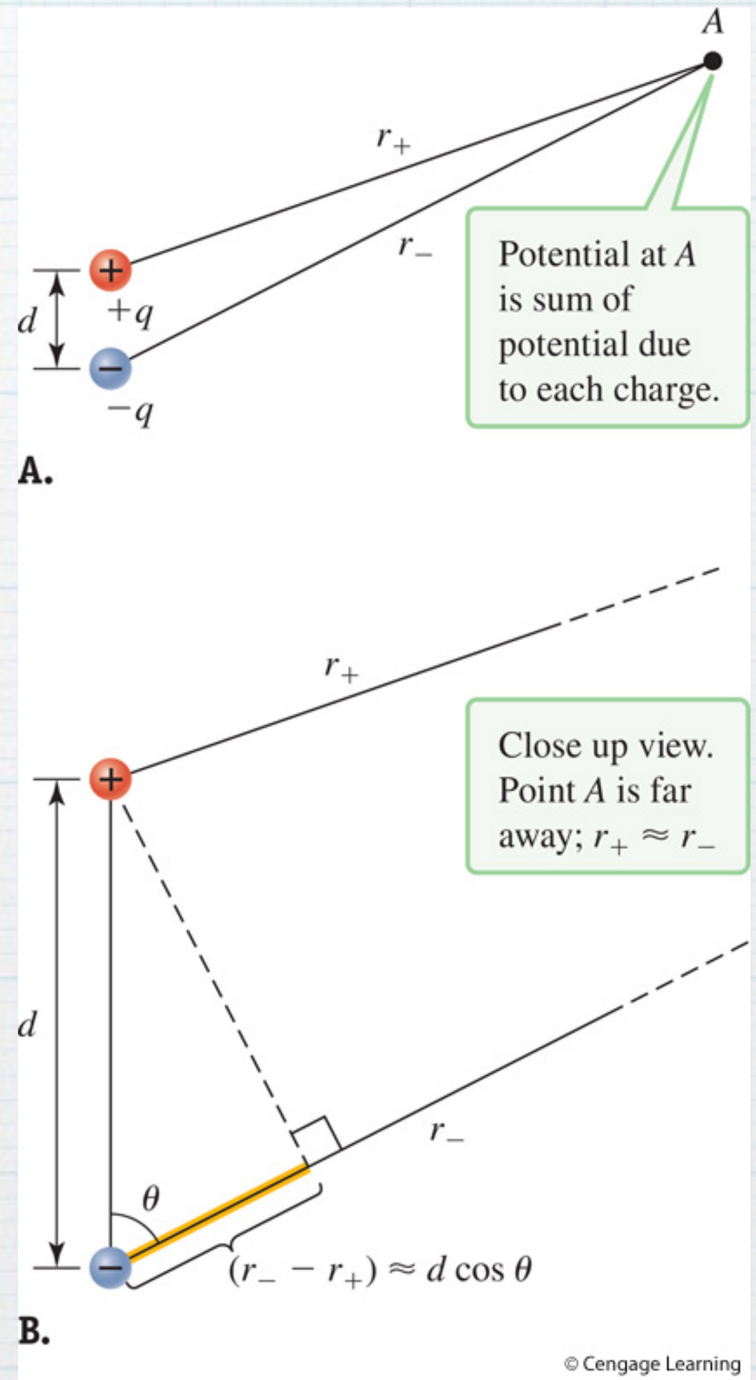
Segue

* The electron volt, eV
* A measure of energy, specifically, the energy gained or lost by an electron moving through a potential difference of 1 volt
* Lets calculate it
* Then lets do a case study with human synapses

Dipole potential

$$V_{dipole} \sim k\frac{qdcos\theta}{r^2}$$





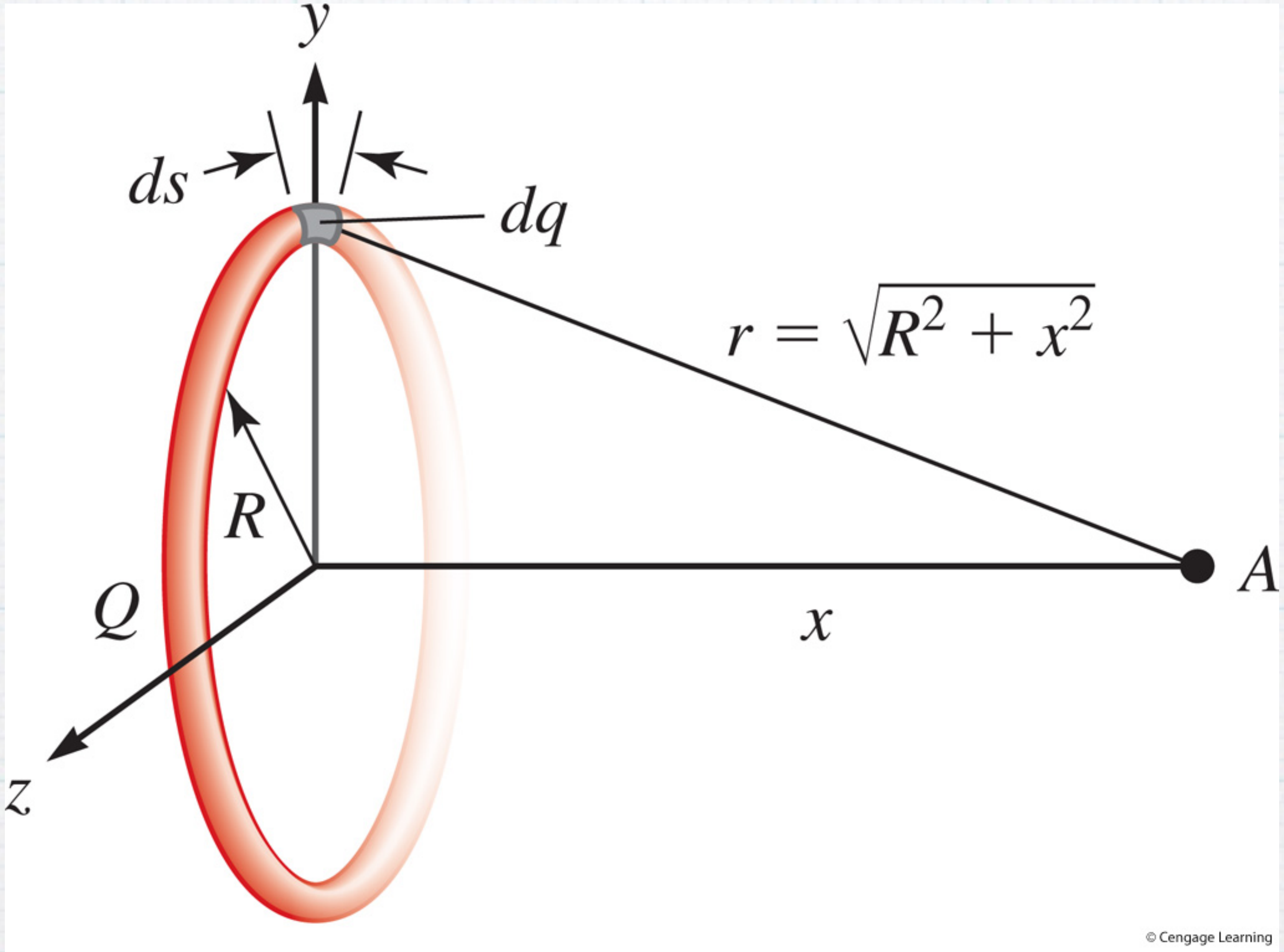
Electric Potential due to a charge distribution

$$\int dV = k \int \frac{dq}{r}$$

$$dq = \lambda ds, \sigma da, \rho dv$$

* Easier, scalar integral
* Use symmetry where possible
* Easier way to get electrical field

Example



$$V = \frac{kQ}{\sqrt{R^2 + x^2}}$$

Connection between field and potential

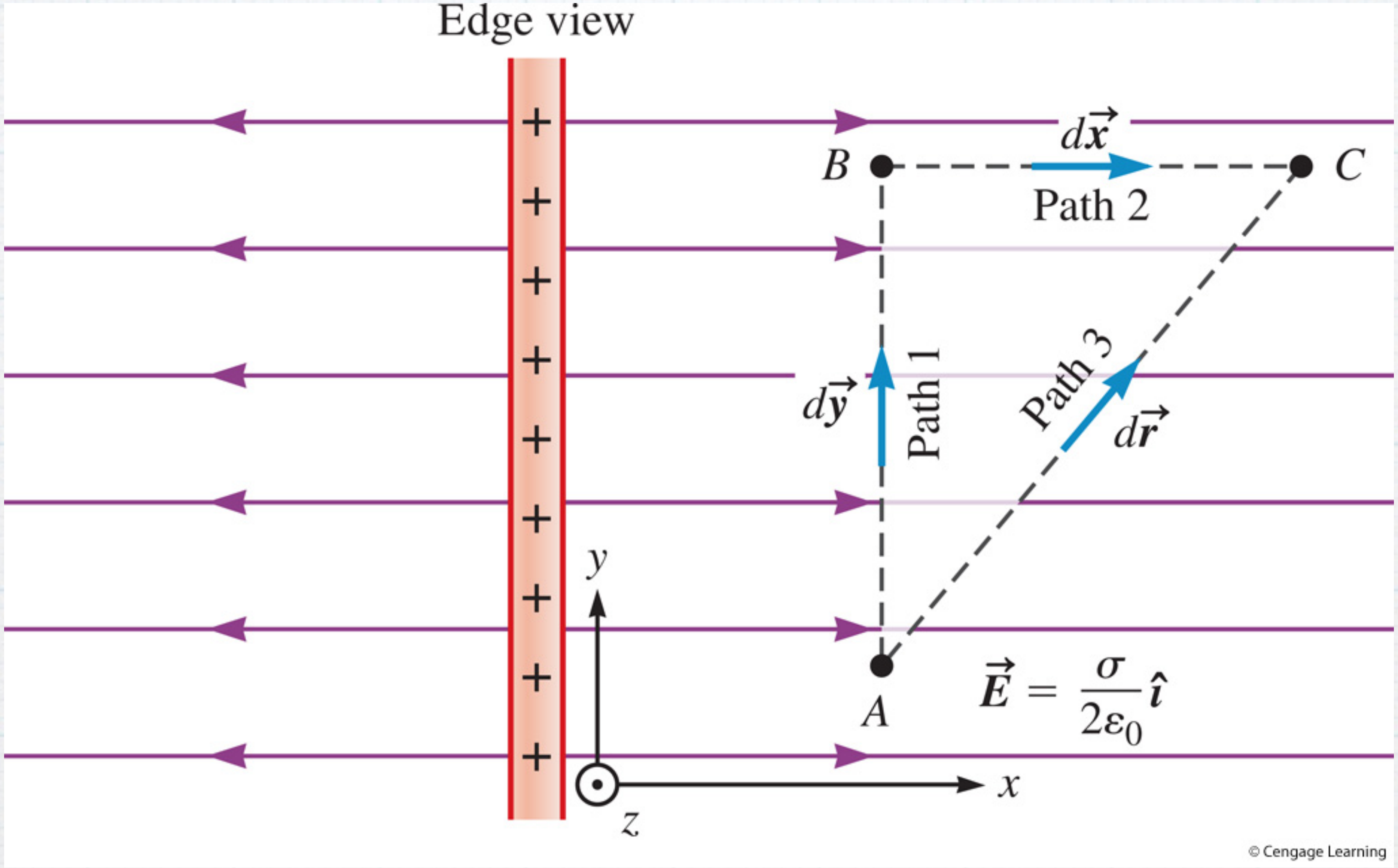
$$\Delta U = -\int_{r_i}^{r_f} \vec{F} \cdot \vec{dr} = q\Delta V$$

$$\frac{\Delta U}{q} = -\int_{r_i}^{r_f} \frac{\vec{F}}{q} \cdot \vec{dr} = \Delta V$$

$$-\int_{r_i}^{r_f} \vec{E} \cdot \vec{dr} = \Delta V$$

Show for point charge, describe what this all means

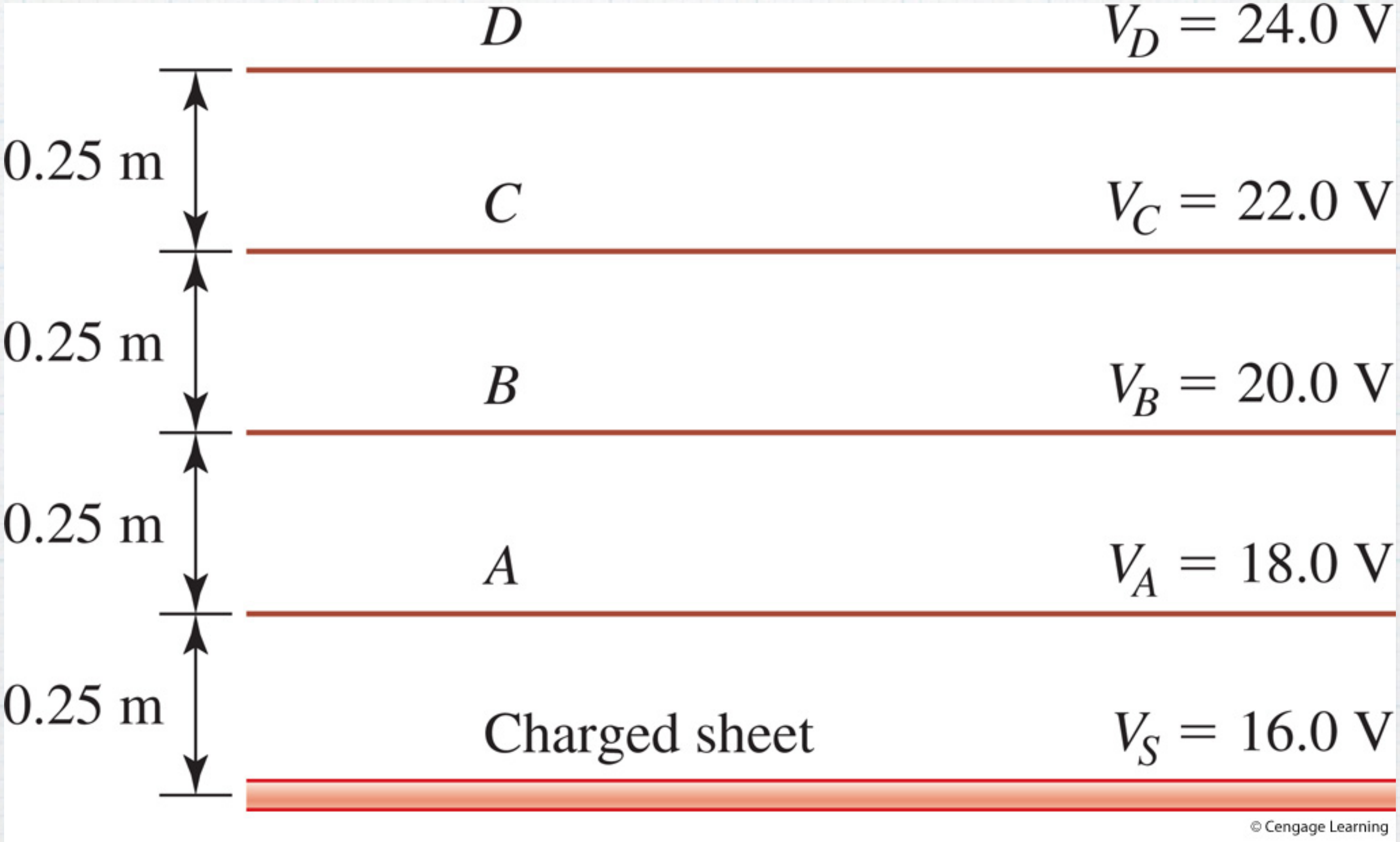
Potential due to charged sheet



$$\Delta V_{i,f} = V_f - V_i = \frac{\sigma}{2\epsilon_0}(x_f - x_i)$$

This is path independent, the very definition of a conservative force or potential

Is this sheet negatively or positively charged?



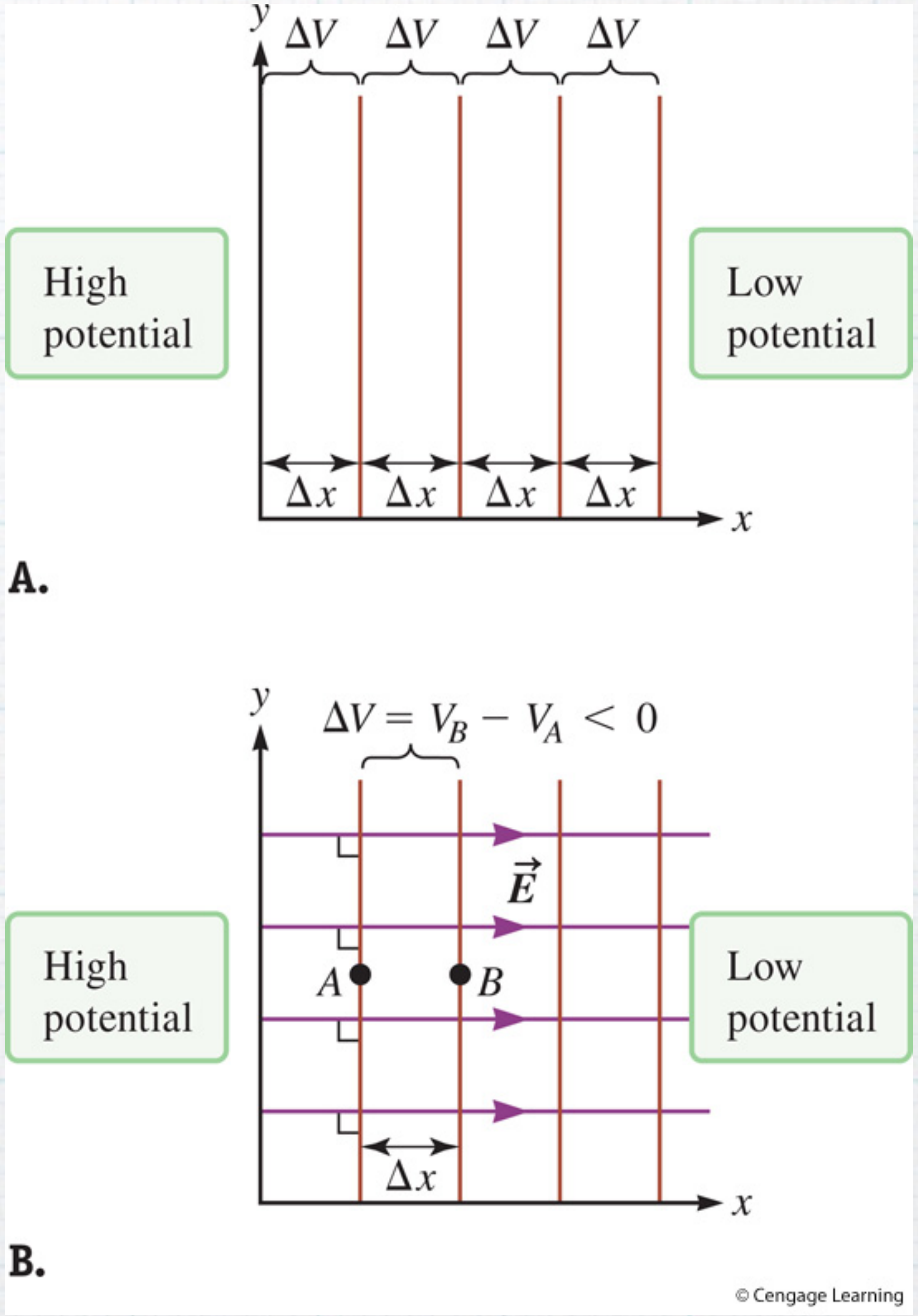
Electric field vs potential again

Simple Case $\vec{E} = C\hat{i}$
This is what a battery or parallel plate capacitor field looks like

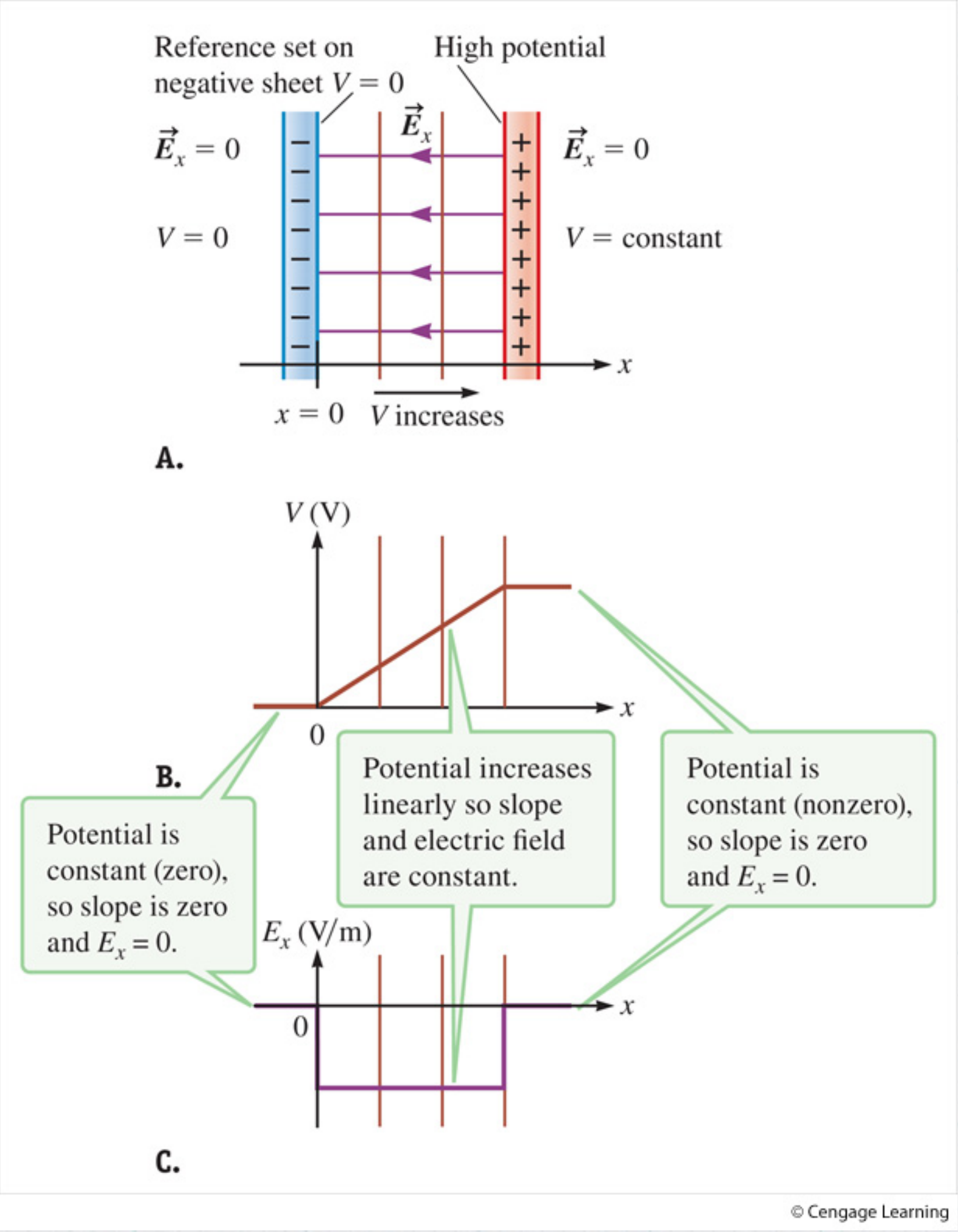
* Look at points A and B, which way will a proton move? Which way would an electron?

* For constant electric field

* Gives us the relation that 1 volt /meter = 1 newton/coulomb



$$\Delta V_{constant\vec{E}} = -C\Delta x = -|\vec{E}|\Delta x \rightarrow -\frac{\Delta V}{\Delta x} = \vec{E}$$



Example, proton and electron dynamics in parallel plate system

Reading Question 26.4

The electric potential difference between two points a distance d apart in a uniform electric field E is:

a. 0
b. - E d
c. kqQ/d
d. We do not have enough information

Reading Question 26.6

The electrical potential energy of a particle of mass *m* and charge *q* that is a distance *d* from a charged sheet of charge *q* is ΔU. The potential difference between the particle and the sheet is?

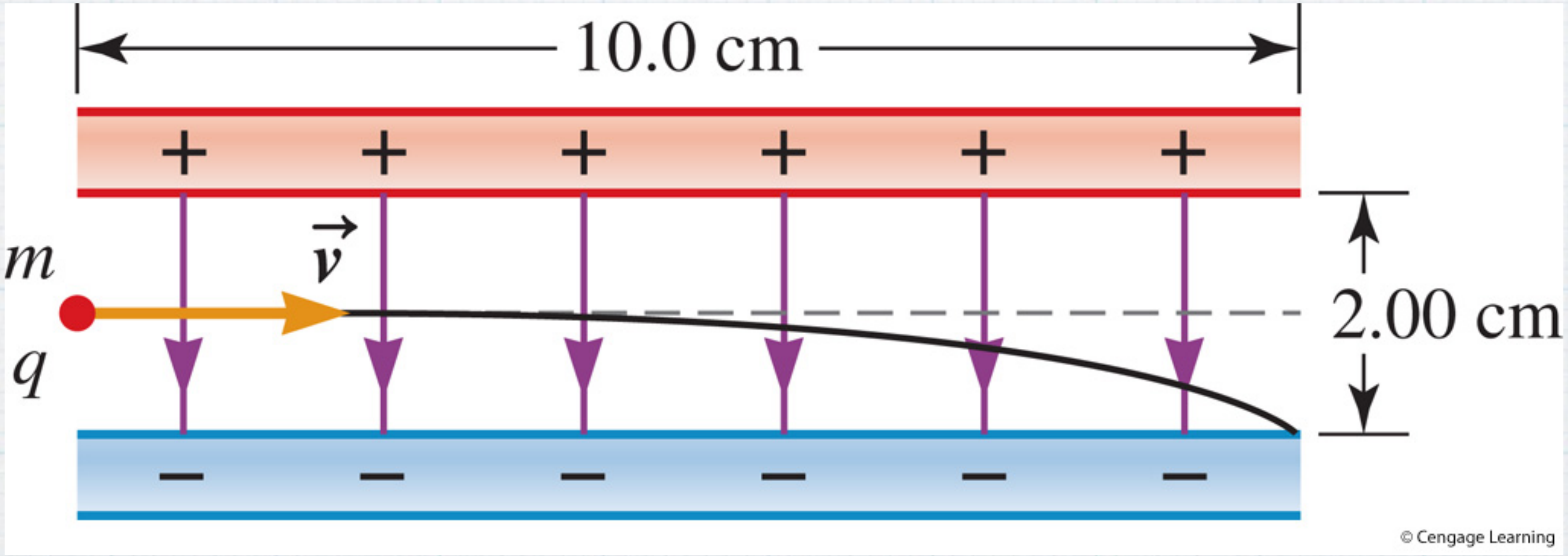
a. $\Delta U/md$
b. $qQm/r\uparrow 2$
c. $\Delta U/q$
d. $Qm/r\uparrow 2$

This is a trick question

Reading Question 26.5

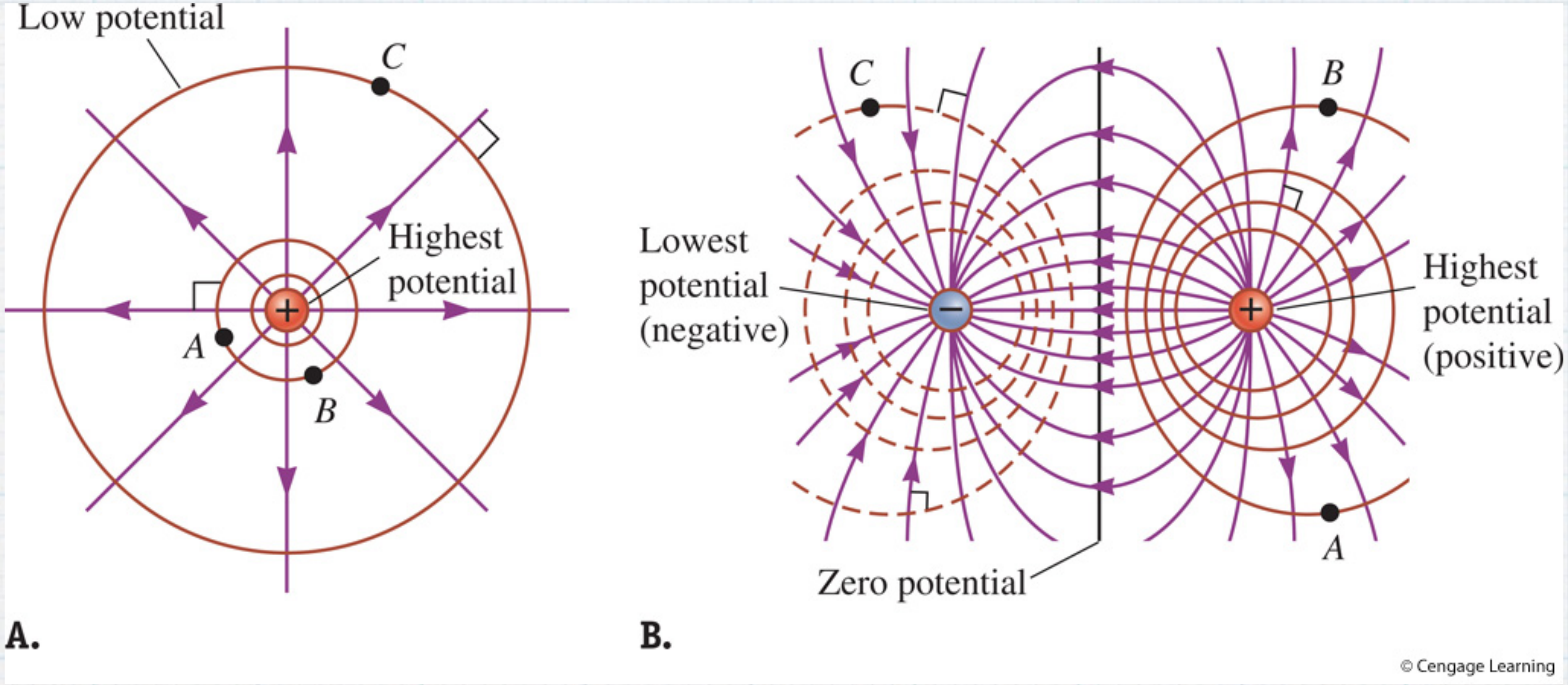
You measure the potential difference between two points a distance d apart to be -V. What is the electric field between these two points?

a. - V d
b. V d
c. d / V
d. We do not have enough information.



Potential is 300 Volts at top, who minimum velocity does it need to make it out

Trickier



- For non-constant electrical fields, the electrical field is the gradient of the potential

$$\vec{E} = -\vec{\nabla}V = -(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k})$$

Examples

$$V = 2\pi k\sigma[\sqrt{R^2 + x^2} - x] \rightarrow \vec{E}?$$

* The potential for a circular plate is given below, find E