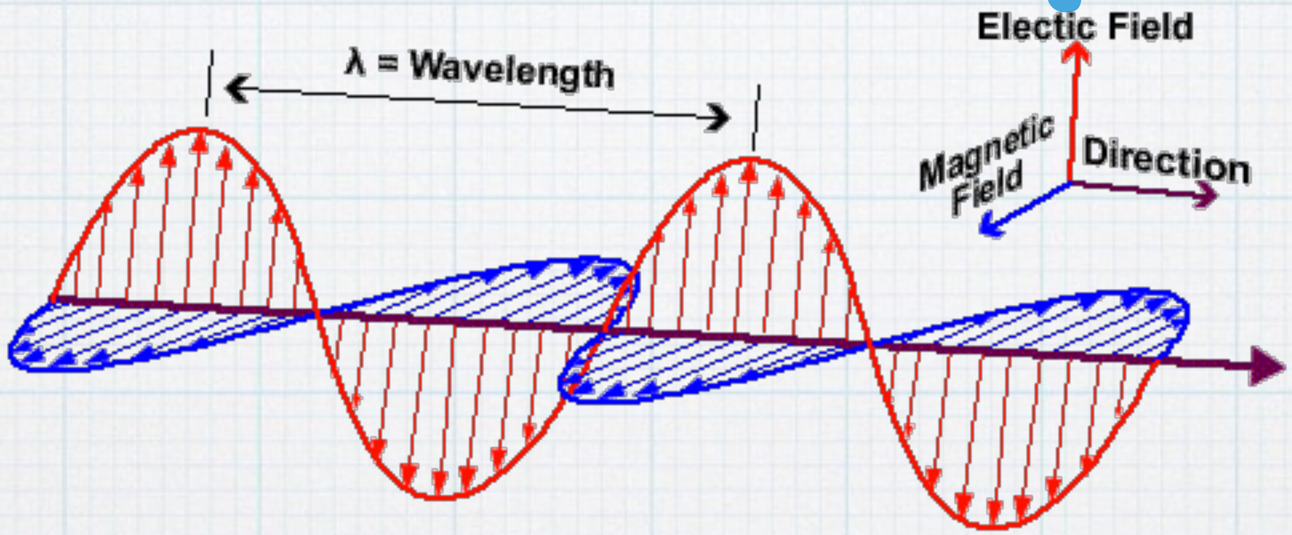
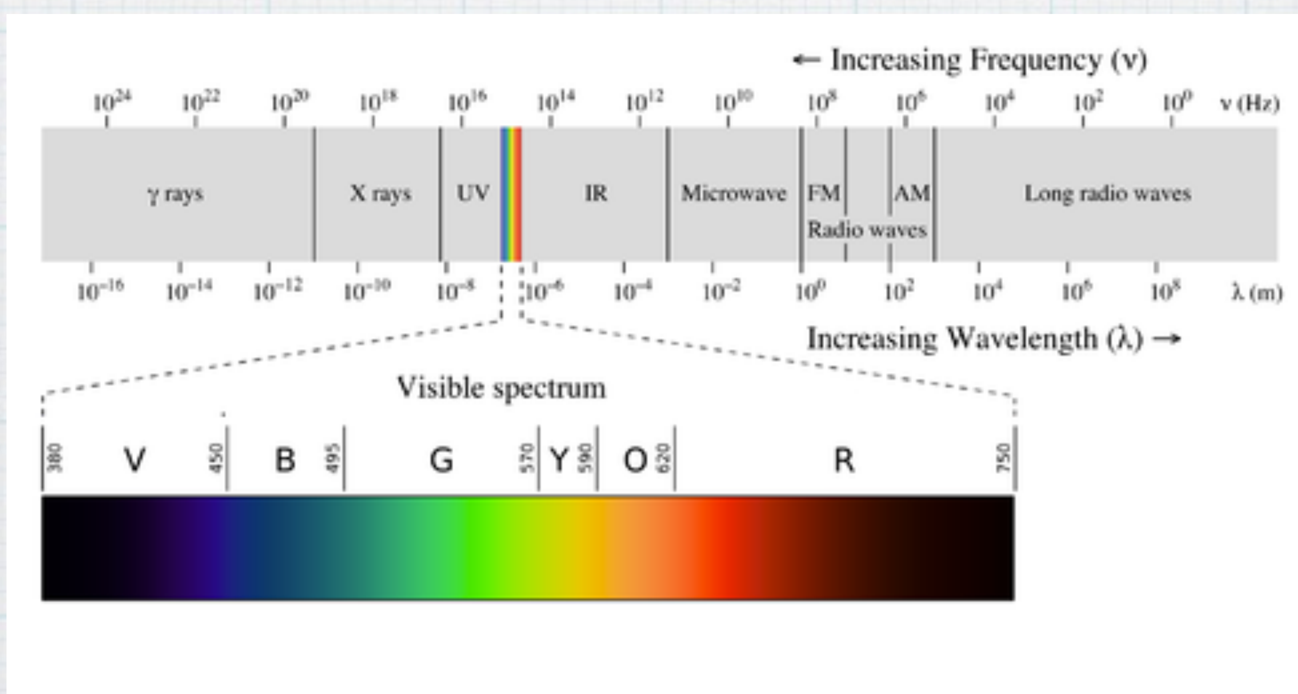
Electro Magnetic Waves and Maxwell's Equations





Classical Physics

* Newton's Laws - forces, momentum, energy
* Newton's Law of Gravitation
* Thermodynamics
* Maxwell's Equations

Maxwell's Equations

$$\oint_A \vec{E} \cdot \vec{d}A = \frac{q_{in}}{\epsilon_0}$$

Gauss's Law, charges produce electric fields

$$\oint_A \vec{B} \cdot \vec{d}A = 0$$

Gauss's Law for magnetism, no magnetic monopoles

$$\oint_r \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

Faraday's Law, induction, changing B makes E

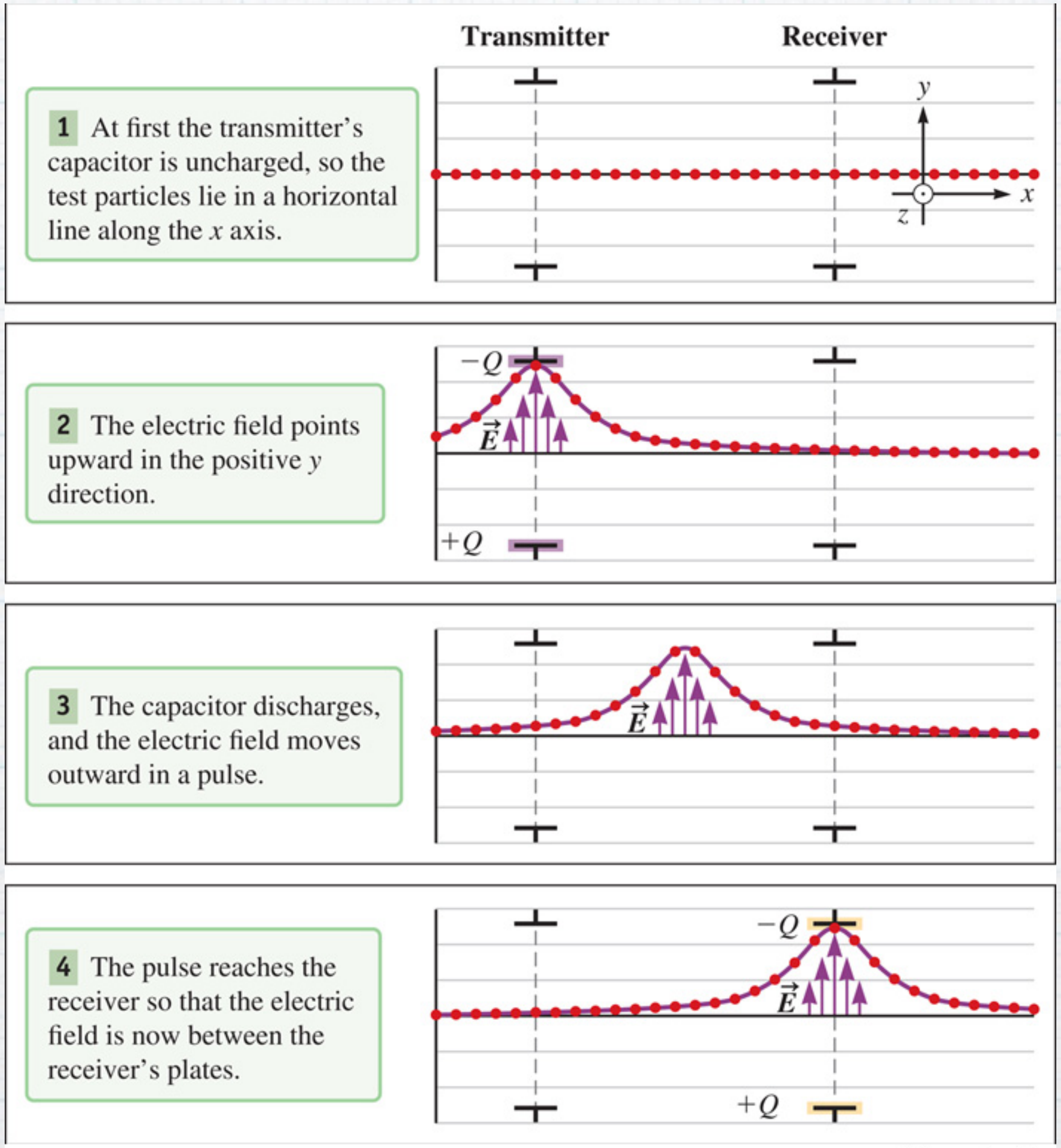
$$\oint_r \vec{B} \cdot d\vec{l} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

Ampere-Maxwell Law, currents and changing E makes B

Lorentz Force

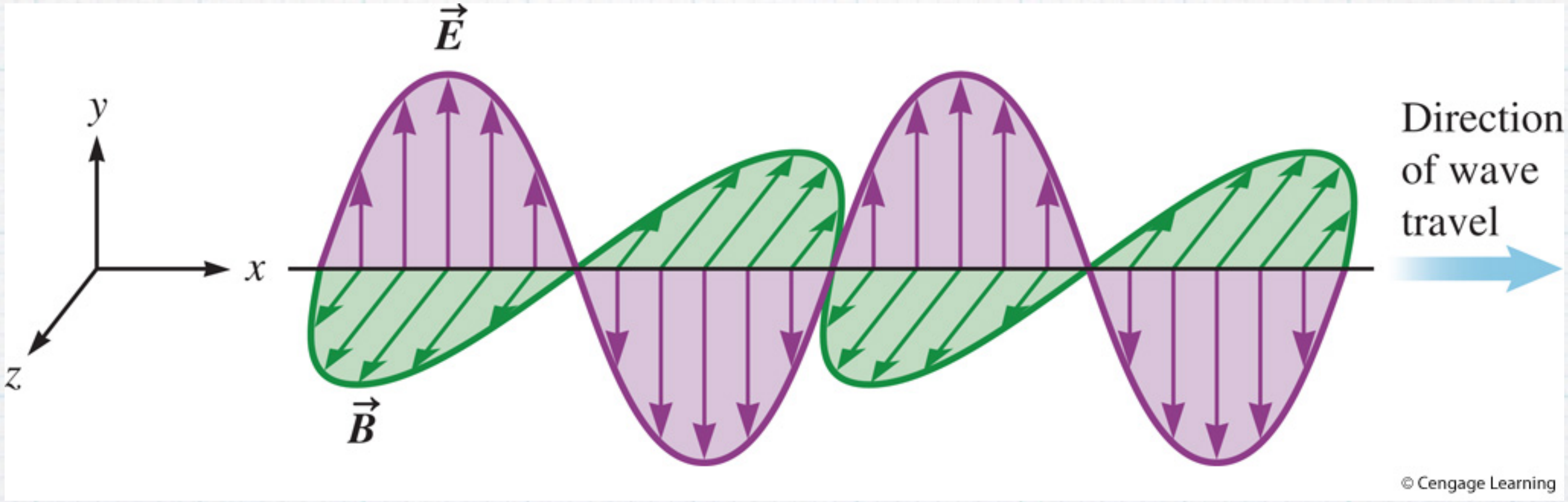
$$\vec{F}_L = \vec{F}_E + \vec{F}_B = q(\vec{E} + \vec{v} \times \vec{B})$$

With Some Math, We Get Electromagnetic Waves or LIGHT



Transmitter
Receiver
1 At first the transmitter's capacitor is uncharged, so the test particles lie in a horizontal line along the x axis.
y
x
z
2 The electric field points upward in the positive y direction.
−Q
+Q
E
3 The capacitor discharges, and the electric field moves outward in a pulse.
E
4 The pulse reaches the receiver so that the electric field is now between the receiver's plates.
−Q
+Q
E

Waves



Calc 3 math and differential equations gives

$$\frac{\partial^2(\vec{E},\vec{B})}{\partial x^2} = \mu_0\epsilon_0\frac{\partial^2(\vec{E},\vec{B})}{\partial t^2}$$

Solutions

$$\vec{E}(x,t) = E_{max}sin(kx-\omega t)\hat{j}$$

$$\vec{B}(x,t) = B_{max}sin(kx-\omega t)\hat{k}$$

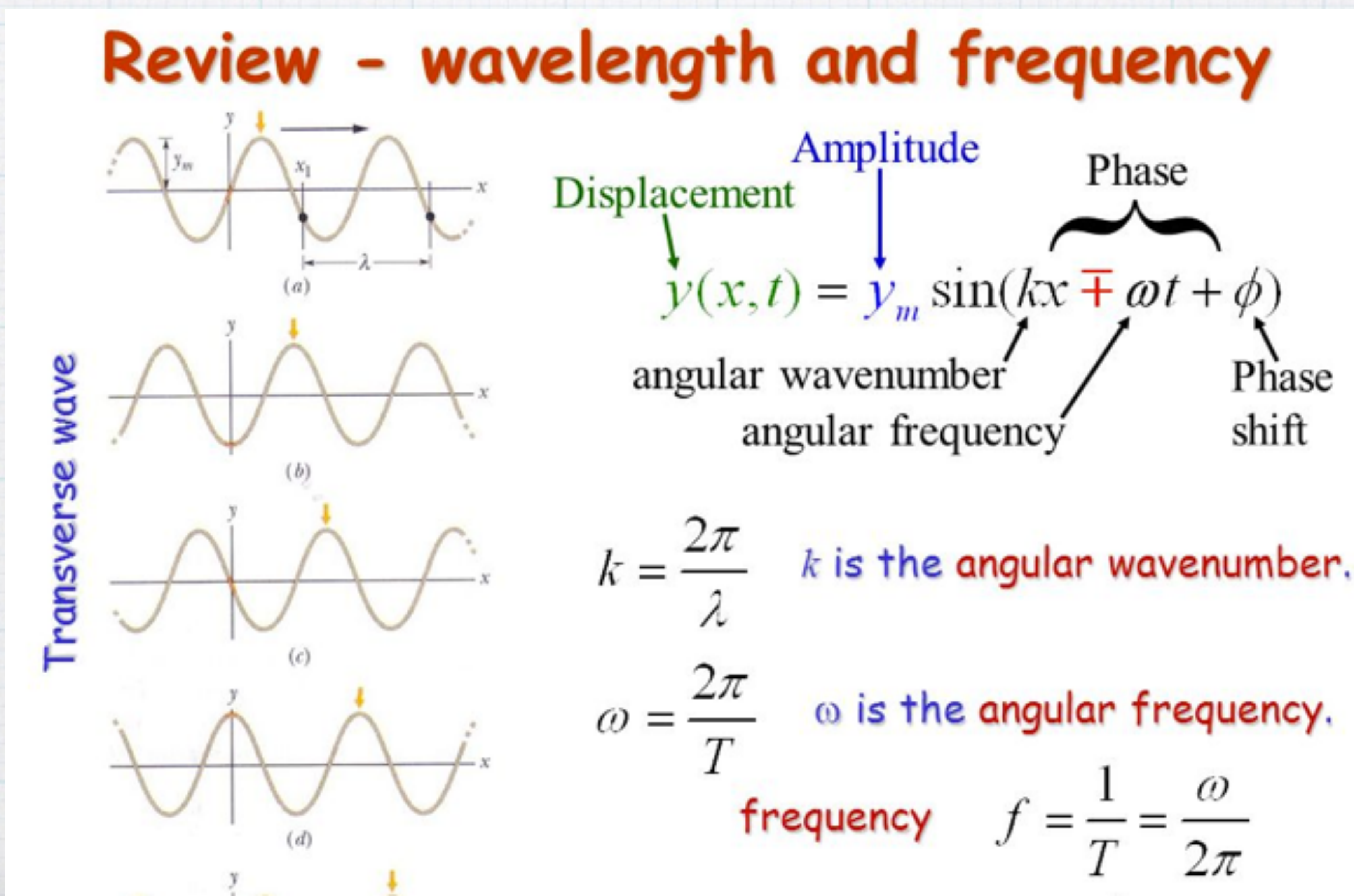
Orthogonally propagating E and B

Light Waves

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$\omega = 2\pi f \qquad k = \frac{2\pi}{\lambda}$$

$$v = \frac{\omega}{k} = \lambda f = c$$



Relationship Between E and B

$$E_{max} = cB_{max}$$

energy density

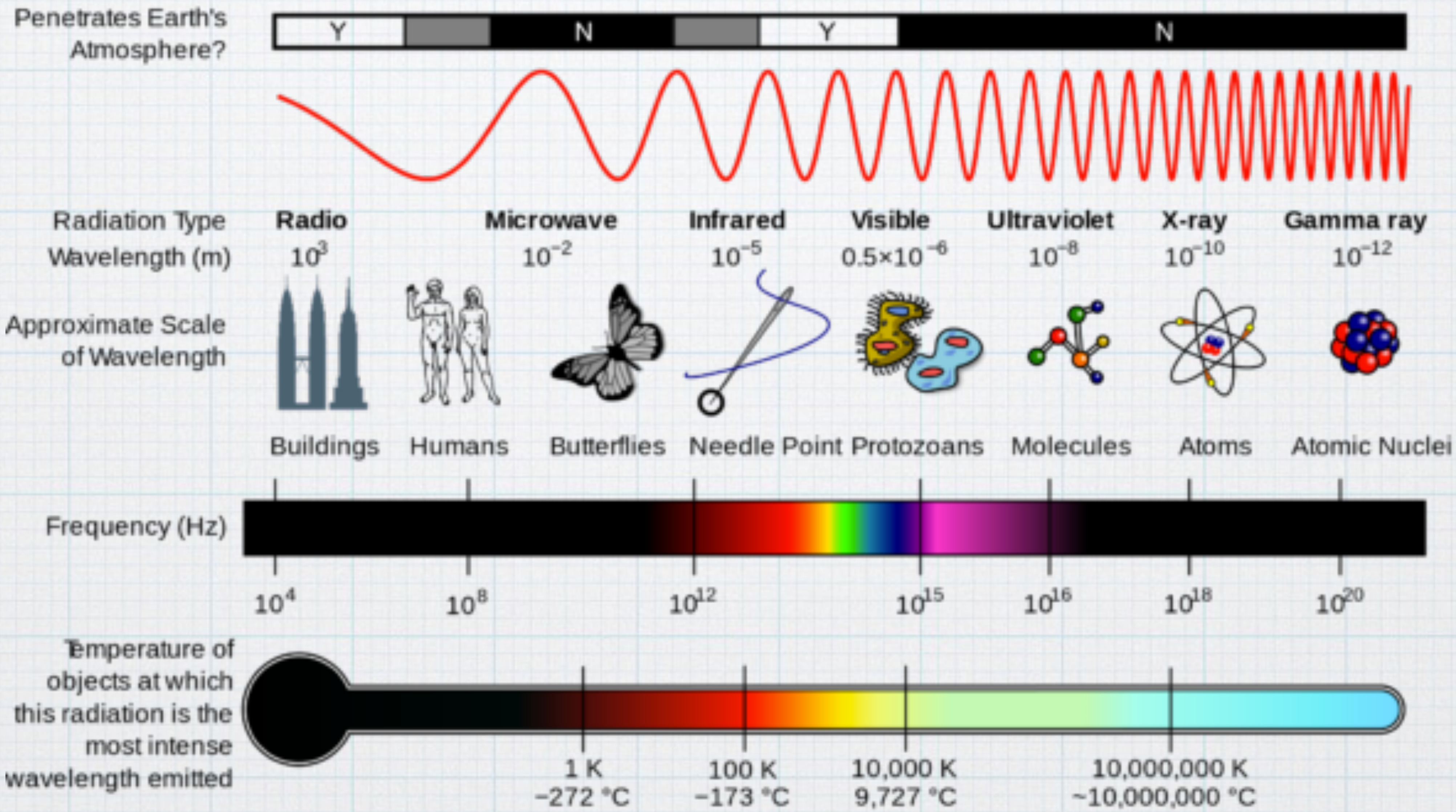
$$u_E = \frac{1}{2}\epsilon_0 E^2 \rightarrow E = cB \rightarrow = \frac{1}{2}\epsilon_0 c^2 B^2 = u_B = \frac{1}{2}\frac{B^2}{\mu_o}$$

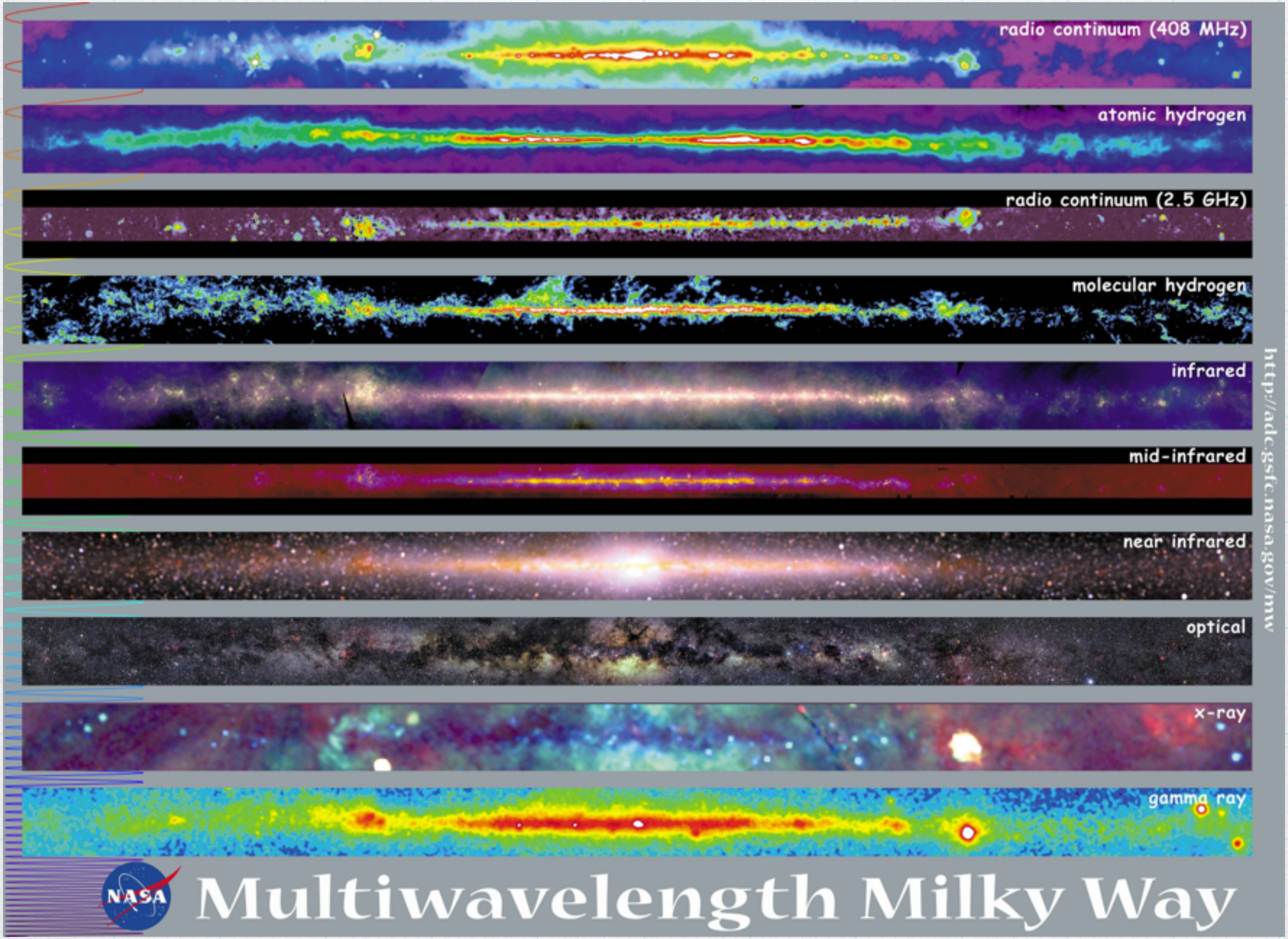
E and B contribute same energy density

Energy switches from E to B and back again

Example

Electromagnetic Spectrum



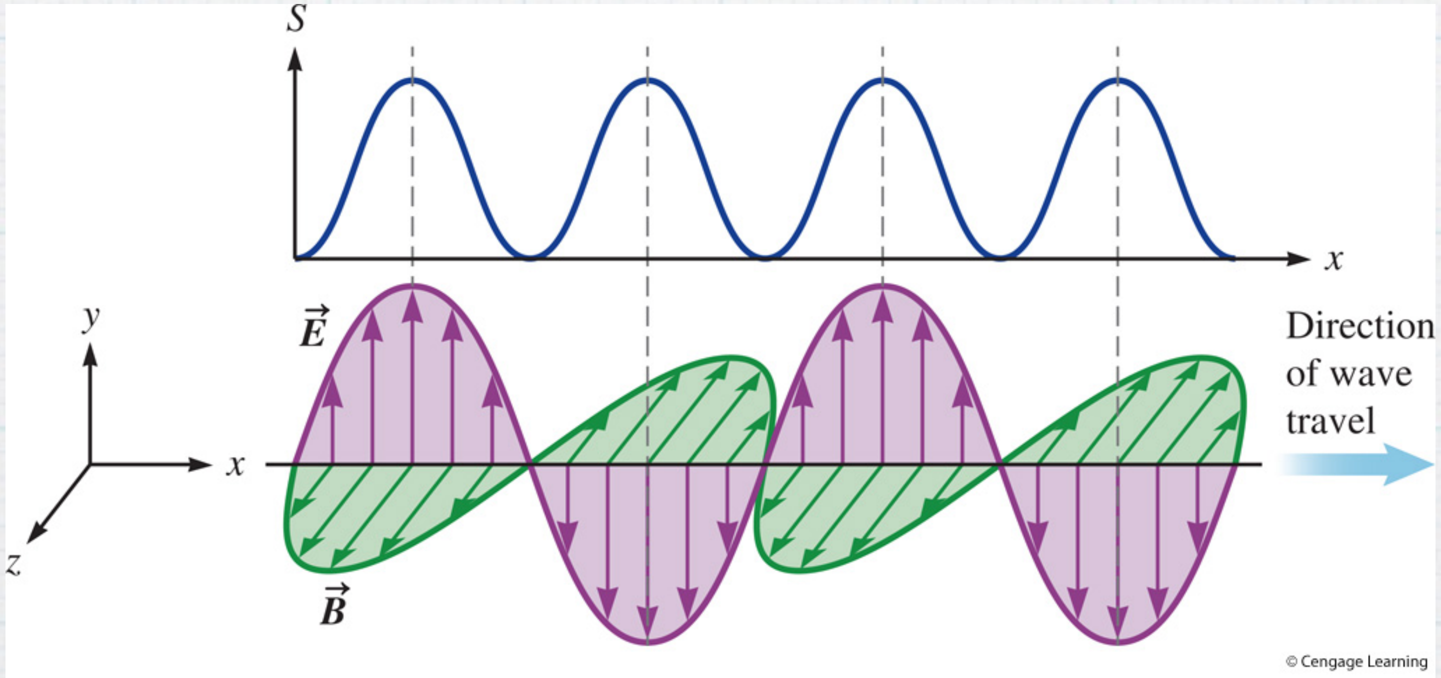

radio continuum (408 MHz)
atomic hydrogen
radio continuum (2.5 GHz)
molecular hydrogen
infrared
mid-infrared
near infrared
optical
x-ray
gamma ray
http://adc.gsfc.nasa.gov/mw
NASA
Multiwavelength Milky Way

Energy Carried by Light

Poynting Vector   Power flux, energy per time per unit area

$$\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B})$$

$$\vec{S} = \frac{1}{\mu_0}[E_{max}B_{max}sin^2(kx - \omega t)]\hat{i}$$



Example

Average Intensity and Energy density

Time averaged Poynting Flux

$$I = \frac{P_{av}}{4\pi r^2} = \frac{1}{2\mu_0} E_{max} B_{max} = \frac{E_{max}^2}{2\mu_0 c} = \frac{c}{2\mu_0} B_{max}^2$$

Average energy density

$$I = c u_{avg} = \frac{1}{2\mu_0} B_{max}^2 = \frac{1}{2} \epsilon_0 E_{max}^2$$

Recalling

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

Example

Radiation Pressure

$$P = \frac{|\vec{F}|}{A} = \frac{1}{A}\frac{|d\vec{p}|}{dt}$$ Pressure

$$|\vec{p}| = \frac{E}{c}$$ Absorbed

reality, in between the two

$$|\vec{p}| = 2\frac{E}{c}$$ Reflected

$$I = \frac{1}{A}\frac{dE}{dt} = \frac{power}{area}$$

$$P = \frac{I}{c}$$ absorbed

Examples

$$P = 2\frac{I}{c}$$ reflected

Intensity