Phys 362
Spring 2018

Statistical and Thermal Physics: Homework 9

Due: 10 April 2018

1 Spin system microstates

Consider an isolated system of spins in a magnetic field. A spin up particle has energy $-\mu B$ and a spin down particle μB. If there are five particles in the ensemble consider the following microstates: EASY 2 lines

State A: $\uparrow\uparrow\uparrow\downarrow\downarrow$
State B: $\downarrow\uparrow\uparrow\downarrow\downarrow$
State C: $\downarrow\downarrow\uparrow\uparrow\uparrow$
State D: $\downarrow\downarrow\downarrow\uparrow\uparrow$

According to the fundamental postulates of statistical physics which of these states are guaranteed to occur with the same probability as each other? For which will the probabilities probably be different? Explain your answer

2 Interacting spin systems

Consider two spin systems, A and B, that interact. For each the magnetic field is such that $\mu B = 1$. Let E_A be the energy of A, N_A be the number of particles in A with similar variables representing B. Suppose that $N_A = 8$ and $N_B = 12$ are fixed and the total energy $E = E_A + E_B$ is also fixed. MEDIUM - HARD

a) The macrostate for the system can be described using the total particle number N, the total energy E and the energy of either subsystem, E_A or E_B. Determine an expression for the multiplicity of the macrostate in the usual form $\Omega(E, E_A, N)$. Note that the only variables that are allowed to appear in the expression are: E and E_A, write out the multiplicity in terms of factorials.

b) Suppose that the total energy is fixed at 10 units. Determine an expression for the entropy; the only variable that should appear is E_A.

c) Explain how you would use your expression to determine the equilibrium state.

3 Gould and Tobochnik, *Statistical and Thermal Physics*, 4.38, page 234. In part b), list the probability for each possible value of E_A. Work in terms of N_+. TEDIOUS, skip E_B part for c).

4 Interacting Einstein solid and spin system HARD

Consider a spin system in a magnetic field interacting with an Einstein solid. The frequency for the solid and the external magnetic field are adjusted so that the energy lost or gained when a single spin changes state is the same as the gap between energy levels for the solid. Thus the two systems can exchange energy in the same way as two Einstein solids.

In a conventional Einstein solid with N particles the energy of the macrostate with q energy units is $E = \hbar\omega\,(q + N/2)$. The rightmost term adds an overall constant to the energy that is irrelevant for the dynamics of the system. This can be removed and the frequency can be set so that $\hbar\omega = 1$. Thus the energy of this "modified" Einstein solid is $E = q$.

In a conventional spin system, the spin up state has energy $-\mu B$ and the spin down state has $+\mu B$. The magnetic field can be adjusted so that $\mu B = 1/2$ and thus the spin up state has energy $-1/2$ and the spin down state has energy $1/2$. These modifications simplify the analysis.

Consider such a spin system with 8 particles and an Einstein solid with 8 particles. The two systems are initially isolated and each has 4 energy units. They are then placed in thermal contact and allowed to reach equilibrium.

a) Determine the entropy of the entire system prior to contact.

b) List the possible macrostates after contact and determine the multiplicity of each. Describe the equilibrium state. Don't forget your macrostates, they are N_+,q,E_{spin}. Also, bear in mind that the total spin energy may be negative.

c) Determine the entropy of the system after it has interacted and reached equilibrium. Did the entropy increase or decrease?

d) Determine the average energy per particle for the spin system after interaction. Repeat this for the Einstein solid. Which way did energy flow?

5 Gould and Tobochnik, *Statistical and Thermal Physics*, 4.11, page 196. VERY EASY

6 Einstein solid: high temperature limit EASY

For an Einstein solid where $q \gg N$, we found that

$$\Omega(N, q) \approx \left(\frac{eq}{N}\right)^N \frac{1}{\sqrt{2\pi N}}.$$

Recall that the energy of the solid is

$$E = \hbar\omega\left(q + \frac{N}{2}\right).$$

a) Determine an expression for the temperature of the Einstein solid in terms of the total energy of the solid.

b) Use this to determine an expression for the energy of the solid in terms of temperature.

7 Volumes of hyperspheres EASY

The volume of a sphere of radius R in n dimensions is given by:

$$V_n(R) = \frac{2\pi^{n/2}}{n\Gamma(n/2)}\, R^n.$$

Verify that this is correct for $n = 1, 2$, and 3.