Statistical and Thermal Physics: Homework 7

Due: 6 March 2018

1 Multiple coin flips

This uses a computer simulation developed by the text authors; go to the companion site, select chapter 3 and then simulations. Run the "launcher" and select STP Coin Toss.

The simulation flips a set number of coins (e.g. 100) and records the number of heads. This constitutes a single "trial" and the simulation can do repeated trials, record the results and display these in a histogram. Each "trial" of the simulation uses many (e.g. 100) coin tosses. The histogram plots the number trials that return various numbers of heads.

a) Set the probability to 0.5 and the number of coins to 100. In one run how many heads do you predict to appear?

b) Run the simulation for 10-20 tosses. How many flips produced heads? What does the data yield for the average number of heads? Does this match your prediction?

c) Now repeat this many times by clicking Start and Stop until the simulation has done at least 500 trials. Using the Tools → Data Tools menu in the Histogram window, determine the most commonly occurring number of heads. What is the average number of heads returned over all the trials? Does this match your prediction better than the result with just one trial?

d) Now reset the simulation so that a single trial consists of 10000 flips and repeat steps a) and b). Do the results of the simulation match your predictions more or less accurately than when there were just 100 flips?

e) Repeat the previous part many times by clicking Start and Stop until the simulation has done at least 100 trials. What is the average number of heads returned over all the trials? Does this match your prediction better than the result with just one trial of 10000 flips? Does the histogram of the outcomes indicate a narrower or larger range of possibilities than when there were just 100 flips?

f) What does the animation indicate about the accuracy of statistical predictions: do more trials or fewer trials result in more accurate predictions?

2 Random walk

Consider a random walk with 6 steps.

a) If the probability of stepping right is $1/2$ in a single step is, determine the probability with which the walker ends two steps left of where he started.

b) If the probability of stepping right is $1/3$ in a single step is, determine the probability with which the walker ends two steps left of where he started.

3 Binomial distribution

Consider the binomial distribution with probabilities p and q as described in class and N trials. The the probability with which one attains n outcomes of $+1$ is

$$P(n) = \binom{N}{n} p^n q^{N-n}.$$

a) Show that the probabilities satisfy

$$\sum_{n=0}^{N} P(n) = 1.$$

b) Determine the standard deviation σ for the distribution.

c) Consider the size of the standard deviation relative to the mean. If this is small, then it means that the mean captures most of the information about the distribution. A relevant measure of the size of the standard deviation to the mean is $\sigma/\overline{n}$. Determine an expression for this in terms of p and N. As $N \to \infty$, what does this approach?

4 Gould and Tobochnik, *Statistical and Thermal Physics*, 3.30, page 140. The program is called Binomial distribution. Use Excel or something similar or this will be tedious. For part B pick N=30,60,240. Return the full width at half maximum, google this. For C use N+ 60, 120, 140. Determine the FWHM for B and C from actual plots.