Physics 342, Homework 6

In all of these, be systematic. I want:

-coordinates defined

-kinetic energy defined

-potential energy defined

-the Lagrangian written out

-Lagrange's equations in each coordinate

-solutions

I'll grade harshly if this prescription isn't followed

1: (easy) Use the method of constrained multipliers to determine the Lagrangian and Lagrange's equations for a particle of mass m, that is constrained to roll along the surface of a paraboloid with $az = x^2 + y^2 = r^2$. The only external force is gravity. You should get a system of 4 equations, 3 equations of motion and one equation of constraint.

2: (Potentially tricky) Using the results of problem 1

A) Prove that the particle will undergo a circular path at a constant heigh h if it is given an angular velocity $\dot{\theta} = \omega = \sqrt{2g/a}$ Hint - in this case both the height and the radius are constant.

B) Show that a particle displaced from this path will undergo small oscillations with a frequency given by $\frac{1}{\pi}\sqrt{2g/a}$ and a period given by $P = \pi\sqrt{\frac{a}{2g}}$.

Hint - use the equation for conservation of angular momentum derived in problem 1 and assume angular velocity as in part A

Hint - assume oscillations take place almost in the plane z = h so we can leave $\lambda = -\frac{mg}{a}$.

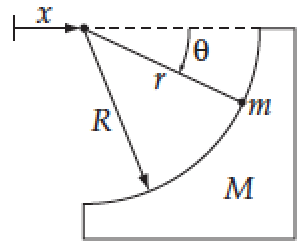
Finally - Derive an equation for the radial motion from the equations in problem 1 then assume small perturbations about r given by $r_0 + u$ where u are small perturbations about a fixed value. Derive the governing equation of motion for u then taylor expand your equation to first order in u recalling $u << r_0$. This procedure should give you a integrable differential equation.

3: 7-22 (Easy) By discuss, explain whether or not the energy of the system is conserved and whether or not the Hamiltonian represents the total energy.

4: 7-26 (Medium) You can just write the lagrangians down if you know them. You may not ignore the pulley mass here.

5: 7-28 (Easy)

6: 7-34 (Medium) Use lag ranges method. See Figure Below for hint. Note the reaction asked for is the normal force from the generalized force of constraint (eq 7.66). Assume all the potential is in the sliding mass. Your equation of constraint is r=R or r-R=0. SKIP part B too annoying.



The coordinates of the wedge and the particle are

$$x_M = x \qquad x_m = r\cos\theta + x$$

$$y_M = 0 \qquad y_m = -r\sin\theta$$