Physics 342, Homework 5

In all of these, be systematic. I want:

-coordinates defined

-kinetic energy defined

-potential energy defined

-the Lagrangian written out

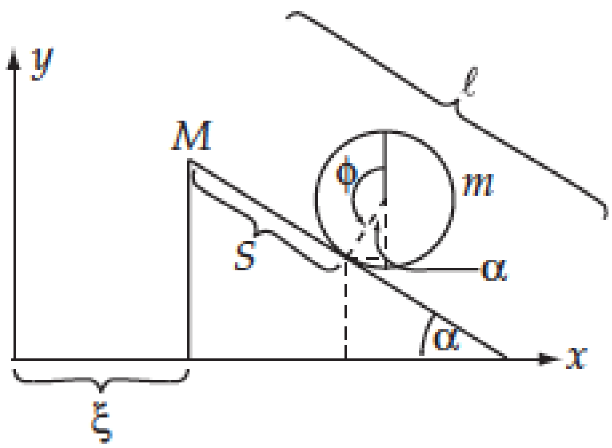
-Lagrange's equations in each coordinate

-solutions

I'll grade harshly if this prescription isn't followed

1: 7-6 Hint - The hoop has kinetic energy $\frac{1}{2}I\dot{\phi}^2$. The wedge also has kinetic energy. See the figure below. Take the center of the hoop as the generalized coordinates. Also, look at what pieces are changeable and what are not, example, R and l are fixed. You can also relate S and ϕ.

For this problem decouple the two equations and present the result for $\ddot{S}$ and $\ddot{\epsilon}$. Interpret your answer



2: A long, straight, frictionless wire is attached to the z axis a distance h above the origin. This wire rotates at a constant angular velocity ω about the z axis. A bead of mass m is threaded onto the wire such that it can move up and down it. The wire makes a constant angle α with the z axis. Denote the length along the wire that the mass is as r. Essentially α and $\omega \times t$ takes the place of θ & ϕ in the spherical coordinate system. Make sure you reduce the form for kinetic energy to something I will check. It get's down to two terms.

A) Set up the Lagrangian for this system. B) Write out Lagrange's equations. C) Determine the motion of the particle in time - essentially r(t) where r is the distance along the wire from the z axis. Ask me if you fail to understand the geometry and I'll draw it for you.

For part C, remember, this is a non-homogenous second order linear differential equation. First solve for the homogenous portion then solve for the particular solution using something like the method of undetermined coefficients. Pick up your differential equations book if you need to.

3: (LONG) Look at example 7.7 (done in class). Relax the assumption that r=R and that $\theta = \omega t$. Instead, allow the bead to have some initial angular velocity so that $\theta(t) = \theta_b + \omega t$ and $\dot{\theta}(t) = (\dot{\theta}_b + \omega t)$. Now z still = cr^2 pick c to be 1 for simplicity.

A) Construct your Lagrangian. it should be $L = \frac{m}{2}[\dot{r}^2 + 4\dot{r}^2 r^2 + r^2(\dot{\theta}_b + \omega)^2] - mgr^2$. Solve for Lagrange's equations in r and θ.

B) NOW, pick three sets of initial conditions, $r(t=0), \dot{r}(t=0), \theta(t=0)$, and $\dot{\theta}(t=0)$. (I should be letting the bead fall with no rotation or initial angular velocity. You'll also need to pick a value for ω. Plot $z(t), r(t), \theta(t)$. Is your $\theta(t)$ reasonable? Why not (if not)?

C) Does your solution agree with equation 7.50? Do you get a constant radius for the right value of ω? See what happens if you vary, by a tiny amount, the condition set for ω in equation 7.50. Is your solution stable? What happens if you pick a small time step? Do you believe your solution? Answer this question with more plots and a brief discussion.

NOTE: I would prefer you actually program this but you can use Maple. If you use Maple you will need to create a set of 4 differential equations (or maybe 2 depending on how you do it).

See the Pendel worksheet on the course site and this link for the double pendulum.

http://www.maplesoft.com/applications/view.aspx?SID=4873

or this link for the spring pendulum http://www.maplesoft.com/applications/view.aspx?SID=4897

or look through the Maplesoft applications site.

Essentially you will need to define

$V_r = \frac{dr}{dt}$

$V_\theta = \frac{d\theta}{dt}$

$A_r = F(r, \theta, \dot{r}, \dot{\theta}, \omega)$

$A_\theta = G(r, \theta, \dot{r}, \dot{\theta}, \omega)$

Then set initial conditions and use desolve and odeplot.

4: (LONG)7-15 Hint - b is the unextended length, l is the extension or contraction it can be in either direction.

A) Construct $d\vec{S} = d(l+b)\hat{r} + (l+b)d\theta\hat{\theta}$. Also, put the origin of your spring mass system at (0,0) so y is negative. remember $l = l(t)$ in the radial direction. Using the solutions manual for this book will show me you don't understand how to do these problems. Also, don't forget where explicit time dependence comes in.

B) Plot the solutions for this one as well for a set of five different initial conditions. One initial condition should be that the mass is hanging vertically with no initial velocity but some initial stretch. Another initial condition should be no stretch, no initial velocity, with the pendulum released from 90 degrees. Pick 3 more initial conditions and plot plot both $\theta(t)$ vs time AND plot $\dot{\theta}$

vs θ. These are phase plane diagrams. Do the same for l and $\dot{l}$. What do they tell you? Play around a bit to get a sense of the behaviour of this system. Is this system chaotic?

Tip, one check on the veracity of your solver should be to check whether energy is conserved or not. Using a simple spring example with the initial angle being zero you should find that your total energy is conserved to better than 100th of a percent over several cycles. Another check is to turn off k and see if you get the behaviour of a simple pendulum.