Physics 342, Homework 1

1: 1-9

2: 1-10

3: 1-13 (this is either really easy or really hard, maybe try crossing vector A with vector B)

4: 1-31 - don't forget that $r = \sqrt{x^2 + y^2 + z^2}$ and $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial r}\frac{\partial r}{\partial x}$ Also, you really only need to take the first partial derivative with respect to x or the 2nd derivative with respect to x to figure this out since all taking the derivatives with respect to y and z changes is to make x go to y or z. This can be long or relatively short if you don't overdo the work.

5: An equation often occurring in mechanics describes the behavior of a simple mass connected to a spring where $\vec{F} = m\ddot{x} = -kx$. The motion of a pendulum displaced from equilibrium is harmonic. Assume k = 4 and m =1 for simplicity. If you get complex exponentials recall Eulers formula. You will also need to apply the initial conditions to position and velocity to solve for your two unknown constants. Two useful identities are $\frac{e^{\alpha it}+e^{-\alpha it}}{2} = cos(\alpha t)$, where α is a number, and $\frac{e^{\alpha it}-e^{-\alpha it}}{2i} = sin(\alpha t)$. Hint, divide through by m.

A) Using the initial conditions $x(0) = 1$ and $\dot{x}(0) = 0$ put this equation into the general SOLDE form shown in class and solve it and then graph it using any method you like. Approximate the square root portion to two digits and remember, you can not combine the real and imaginary components. Comment on the behavior.

B)If one allows for friction that is velocity dependent this equation may be rewritten as $m\ddot{x} = -kx - b\dot{x}$ Leave everything as variables in this case and do not apply initial conditions to determine a numerical solution. Your solution should look like $C_1 e^{[\frac{-b+\sqrt{b^2-4mk}}{2m}]t} + C_2 e^{[\frac{-b-\sqrt{b^2-4mk}}{2m}]t}$. Assume $C_1 = C_2$ and recall $e^{-\alpha t+\beta t} = e^{-\alpha t}e^{\beta t}$ Simplify your solution. Three possible cases exist. What happens if $b^2 = 4mk$? What happens if $b^2 > 4mk$? What happens if $b^2 < 4mk$?

6:Integrate $\int_0^1 \frac{1-e^x}{\sqrt{x}}dx$ using the 4 terms of the Taylor expansion of the integrand. Compare this result to one done numerically. You only have to find the taylor expansion of one thing here then do some algebra and simple integrals. Exams about x = 0. This should give you an idea of how to write a simple computer program to evaluate tricky integrals. You get a number larger than the real one. What does this tell you about your choice of x=0 for the expansion?

7: Derive a series representation for the $ln(1 + x)$ about x = 0. Can you do the same for $ln(x)$?

Extra Credit

1-25 The unit vectors in the spherical coordinate system are given by

$\hat{e}_\phi = -\sin\phi\hat{i} + \cos\phi\hat{j}$
$\hat{e}_\theta = \cos\theta\cos\phi\hat{i} + \cos\theta\sin\phi\hat{j} - \sin\theta\hat{k}$
$\hat{e}_r = \sin\theta\cos\phi\hat{i} + \sin\theta\sin\phi\hat{j} + \cos\theta\hat{k}$

Following the procedure used in class for plane cylindrical coordinates do this problem assuming your initial vector is $r\hat{e}_r$. Hint $\hat{e}_\phi$ depends on $\hat{e}_r$ and $\hat{e}_\theta$ in a way that is not as straightforward to see as the time derivatives of the other unit vectors. See what you can do with factors of $-\dot{\phi}$ with trig functions to make this work.