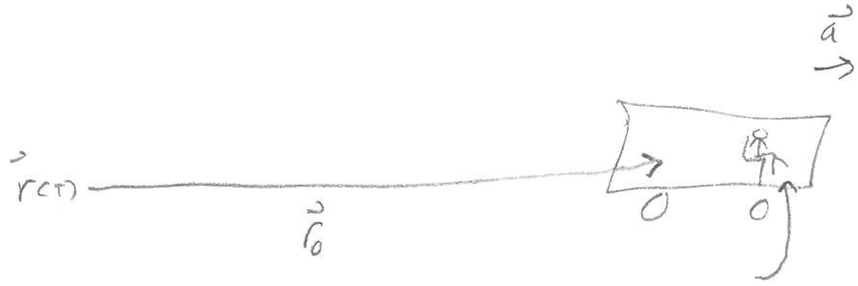
Ch. #10

Motion in a non-inertial reference frame

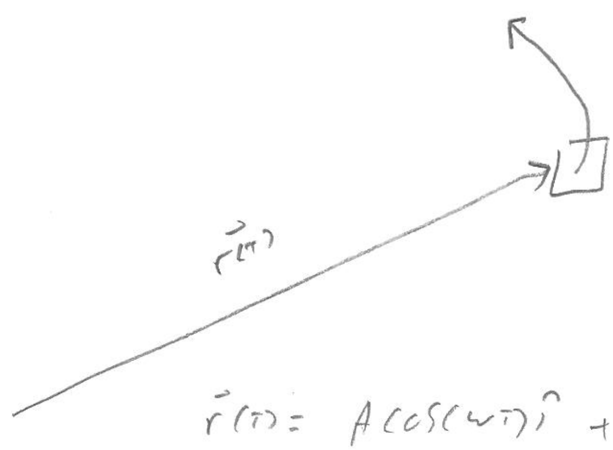
Step #1. draw r(t) in terms of a fixed inertial frame

Ex.



$\vec r(t) = \vec r_0 + \frac{1}{2} a t^2$ $\qquad \ddot{r}(t) = a$ $\qquad F = m\vec a$

Ex.



$\vec r(t) = A\cos(wt)\hat i + B\sin(wt)\hat j$

$\dot{\vec r}(t) = -Aw\sin(wt)\hat i + Bw\cos(wt)\hat j$

$|\dot{\vec r}| = (A^2w^2 + B^2w^2)^{1/2} = (A^2+B^2)^{1/2} w$

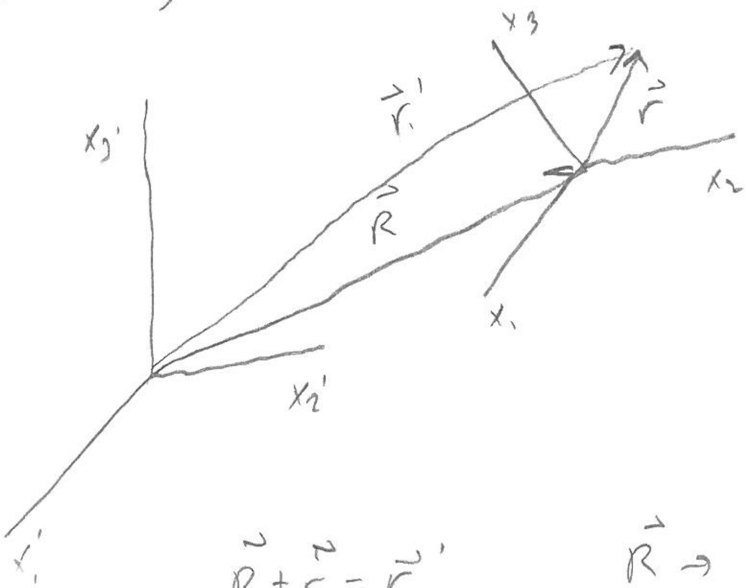
elipse else if $A=B$ circle

$\ddot{\vec r} = -Aw^2\cos(wt)\hat i - Bw^2\sin(wt)\hat j$

$|\ddot{\vec r}| = \sqrt{A^2+B^2}\, w^2 \to A=B \to Aw^2$

$\vec F = mAw^2$

More generally $\quad x' \rightarrow$ inertial



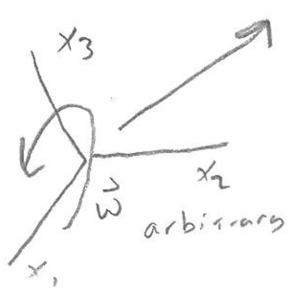
$\vec{R} + \vec{r} = \vec{r}\,'$

$\vec{R} \rightarrow$ Fixed origin to origin moving

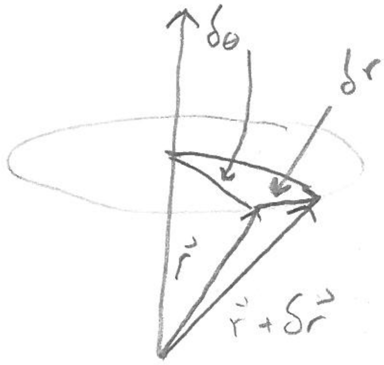
$\vec{r} \rightarrow$ wrt origin in Non inertial

$\vec{r}\,' \rightarrow$ Fixed to point in Non-Inertial

Now, allow



If non inertial is rotating then an infinitesimal



$\delta\theta$ corresponds to a δr

to $\vec{r}$ to $\vec{r} + d\vec{r}$

where $d\vec{r} = \delta\vec{\theta} \times \vec{r}$

or $\dfrac{d\vec{r}}{dt} = \vec{\omega} \times \vec{r}$

1: CALL $\vec{r}' = \vec{R} + \vec{r} = \hat{Q}$ Then For $\vec{r} = \hat{Q}$ or any Fixed Vs non-inertial quantity

For $\vec{R}$ fixed $\left.\frac{d\vec{Q}}{dt}\right|_{\text{Fixed}} = \left.\frac{d\vec{Q}'}{dt}\right|_{\text{rotating}} + \vec{\omega} \times \vec{Q}$

$\uparrow$

$\vec{V}_r(t)$

Note $\left.\frac{d\vec{\omega}}{dt}\right|_{\text{Fixed}} = \left(\frac{d\vec{\omega}}{dt}\right)_{\text{rotating}} + \underbrace{\vec{\omega} \times \vec{\omega}}_{0} = \hat{\omega}$

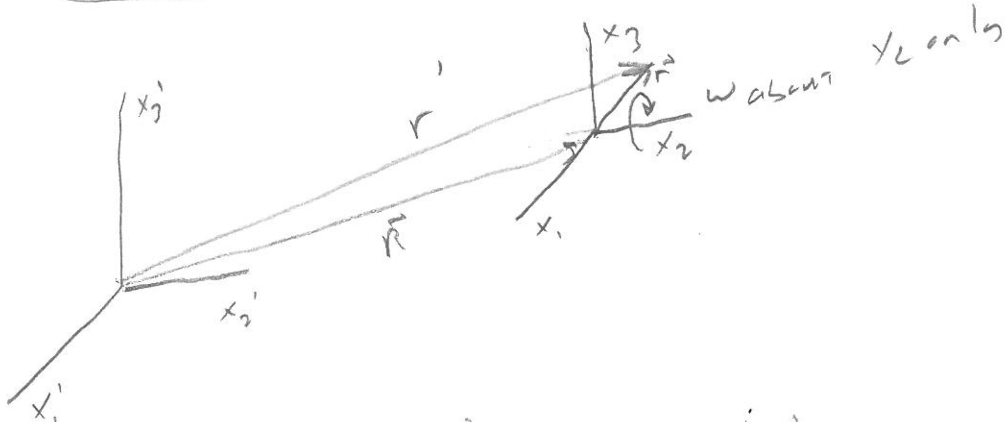
Now $\frac{d\vec{r}'}{dt} = \frac{d\vec{R}}{dt} + \left.\frac{d\vec{r}}{dt}\right| + \vec{\omega} \times \vec{r}$

$\nearrow$ Fixed

$\frac{d\vec{R}}{dt}$ $\downarrow$ motion of origin of Non inertial Coordinate System

$\left.\frac{d\vec{r}}{dt}\right|$ linear wrT noninertial

1: only rotation



$$\frac{d\vec{r}'}{dt} = \frac{d}{dt}\left(\vec{R} + \vec{r}\right) = \left.\frac{d\vec{r}}{dt}\right|_{\text{rotational}} = \vec{\omega} \times \vec{r}$$

2: $R(t)$ + rotation

how about $\vec{R} = \vec{R}(t)$ so some linear velocity WRT inertial

Frame.

$$\frac{d\vec{r}'}{dt} = \frac{d\vec{R}}{dt} + \left.\frac{d\vec{r}}{dt}\right|_{\text{rotational}} = \underset{\text{Translation}}{V(t)} + \underset{\text{rotation}}{\vec{\omega} \times \vec{r}}$$

3. $\vec{R}(t)$, linear $\vec{r}(t)$, rotational $r(t)$

how about $\vec{r}(t)$, $\vec{R}(t)$, and rotation

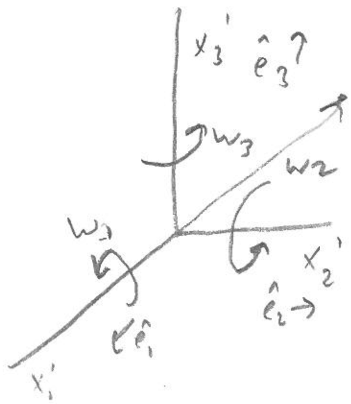
$$\frac{d\vec{r}'}{dt} = \frac{d\vec{R}}{dt} + \left.\frac{d\vec{r}}{dt}\right|_{\text{linear}} + \vec{\omega} \times \vec{r} = \vec{V}(t) + \vec{v}_r(t) + \vec{\omega} \times \vec{r}$$

rotational

relative to moving coordinate

relative to inertial frame

Physically what is $\vec{\omega} \times \vec{r}$?



Make $\vec{R}$ fixed

but non zero

$\omega_1, \omega_2, \omega_3$

$$\frac{d\vec{r}\,'}{dT} = \frac{d\vec{r}}{dT} = \frac{d}{dT}\sum x_i' e_i = \frac{dx_i'}{dT}\hat{e}_i + x_i'\frac{d\hat{e}_i'}{dT} = \dot{\vec{r}}\,' + \sum x_i'\dot{\hat{e}}_i$$

hmm what is $\frac{d\hat{e}_i'}{dT}$?

$$\frac{d\hat{e}_1'}{dT} = w_3\hat{e}_2' - w_2\hat{e}_3'$$

dogon see this?

$$\frac{d\hat{e}_2'}{dT} = w_1\hat{e}_3' - w_3\hat{e}_1'$$

$$\frac{d\hat{e}_3'}{dT} = w_2\hat{e}_1 - w_1\hat{e}_2$$

or $\dot{\hat{e}}_i' = w \times \hat{e}_i$ so $\frac{d\vec{r}\,'}{dT} = \dot{\vec{r}} + \sum \vec{\omega} \times \hat{e}_i x_i$

$$\frac{d\vec{r}\,'}{dT} = \dot{\vec{r}} + \vec{\omega} \times \vec{r}$$

How about

$$\left.\frac{d}{dT}\right|_{Fixed}\left(\dot{\vec{r}}'\right)=\left.\frac{d}{dT}\right|_{Fixed}\left[\vec{V}(T)+\vec{V}_r(T)+\vec{\omega}\times\vec{r}\right]$$

$$\ddot{\vec{r}}' = \vec{A}(T) + \left.\frac{d}{dT}V_r(T)\right|_{Fixed} + \left.\frac{d}{dT}\right|_{Fixed}(\vec{\omega}\times\vec{r})$$

$$\ddot{\vec{r}}' = \vec{A}(T) + \vec{a}_r(T) + \vec{\omega}\times\vec{V}_r + \dot{\vec{\omega}}\times\vec{r} + \vec{\omega}\times\left.\frac{d}{dT}\right|_{Fixed}\vec{r}$$

$$\ddot{\vec{r}}' = \vec{A}(T) + \vec{a}_r(T) + \vec{\omega}\times\vec{V}_r + \dot{\vec{\omega}}\times\vec{r} + \vec{\omega}\times\left[V_r + \vec{\omega}\times\vec{r}\right]$$

$$\ddot{\vec{r}}' = \vec{A}(T) + \vec{a}_r(T) + \dot{\vec{\omega}}\times\vec{r} + 2\vec{\omega}\times\vec{V}_r + \vec{\omega}\times\vec{\omega}\times\vec{r}$$

acceleration of origin

acceleration of $\vec{r}$ wrt origin in non-inertial

$\dot{\vec{\omega}}$ Term

Coriolis effect

centripetal acceleration

Now $\vec{F}_{ext} = m\ddot{\vec{r}}'$ what we really want is $\vec{a}_r(T)$

what do <u>you</u> feel?

$$m\vec{a}_r(T) = \vec{F}_{EXT} - m\vec{A}(T) - m\dot{\vec{\omega}}\times\vec{r} - 2m\vec{\omega}\times\vec{V}_r - m(\vec{\omega}\times(\vec{\omega}\times\vec{r}))$$

$\ddot{\vec{R}}$

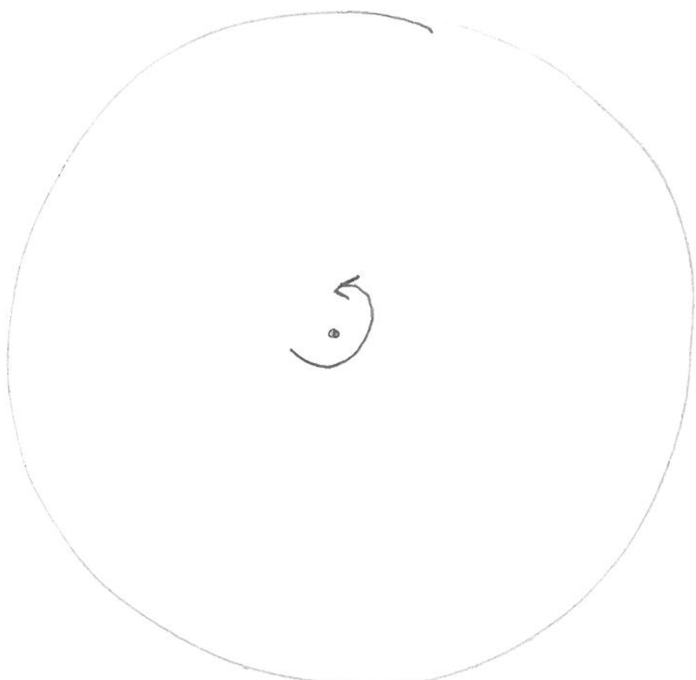
Force YOU FEEL!!

or $\vec{F}_{EXT} = m\vec{a}_r(T)$ iff $\vec{\omega} = \vec{A} = 0$ iff your frame doesn't turn or accelerate $\rightarrow$ inertial

2 Important Forces

$$-2\vec{\omega} \times \vec{v} \quad \text{and} \quad -\vec{\omega} \times \vec{\omega} \times \hat{r}$$

1st 2D



$$\vec{\omega} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & \omega \\ v_x & v_y & 0 \end{vmatrix} = -\omega v_y \hat{i} + \omega v_x \hat{j}$$

$$-2\vec{\omega} \times \vec{v} = 2\omega \left[v_y \hat{i} - v_x \hat{j} \right]$$

$\xrightarrow{V_x}$ goes to $\downarrow$ $\quad$ $\uparrow$ v_y goes to $\longrightarrow$

or

$$\vec{v} \nearrow = \searrow \vec{a} \quad \rightarrow \text{Turning Force}$$

$$\vec{\omega} \times \vec{\omega} \times \vec{r} = \vec{\omega} \times \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & \omega \\ x & y & 0 \end{vmatrix} = \vec{\omega} \times [-\omega y \hat{i} + \omega_x \hat{j}]$$

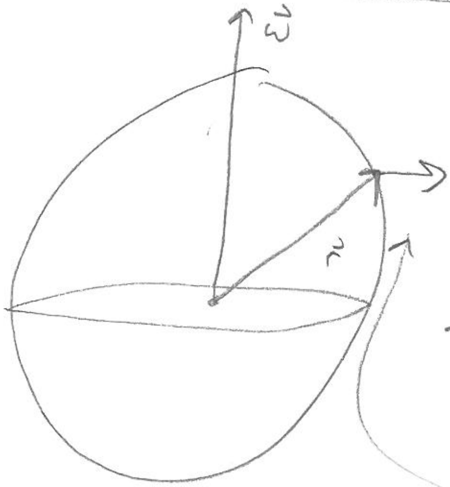
$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & \omega \\ -\omega y & \omega_x & 0 \end{vmatrix} = -\omega^2 x \hat{i} - \omega^2 y \hat{j}$$

$$-\vec{\omega} \times \vec{\omega} \times \vec{r} = \omega^2 [x\hat{i} + y\hat{j}]$$

or $\quad \rightarrow + \uparrow = \nearrow \quad \underline{\underline{\text{OUT}}}$

In frame of person ON merry go round

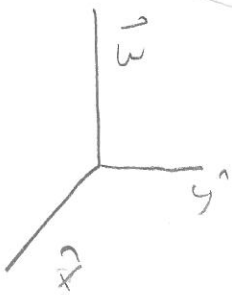
3 D



$\vec{\omega} \times \vec{r} = \otimes$

$\vec{\omega} \times (\vec{\omega} \times \vec{r}) = \uparrow \times \otimes$

$-\vec{\omega} \times (\vec{\omega} \times \vec{r}) =$ OUT

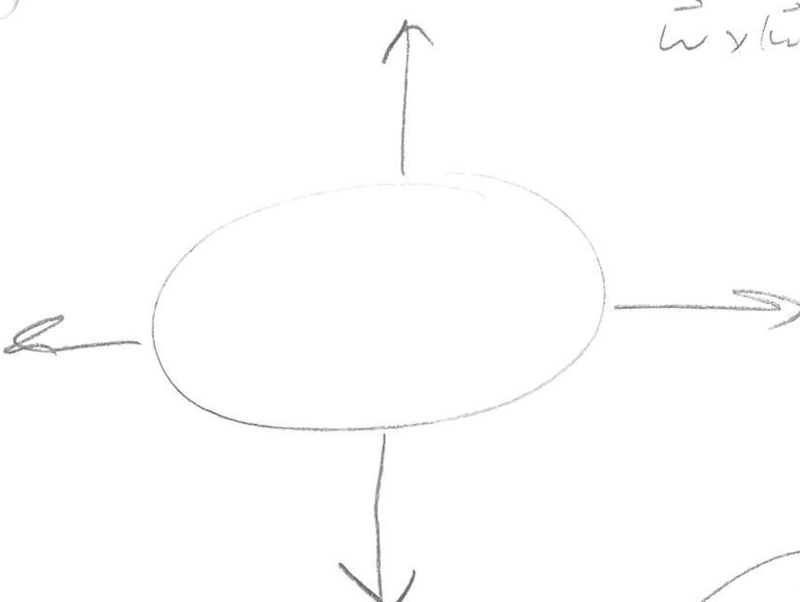


$\vec{\omega} \times \hat{y} = -|\omega|\hat{x}$

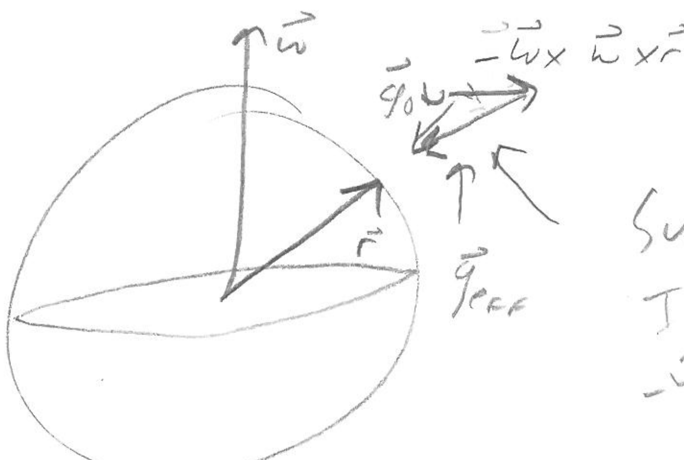
$\vec{\omega} \times (\vec{\omega} \times \hat{y}) = -\hat{y}|\omega|^2$

$\vec{\omega} \times \hat{x} = \hat{y}|\omega|$

$\vec{\omega} \times (\vec{\omega} \times \hat{y}) = -\hat{x}|\omega|^2$

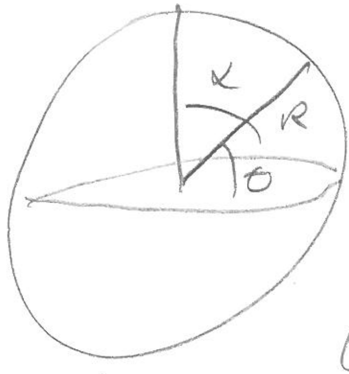


Locally



Super Inflated $-\vec{\omega} \times \vec{\omega} \times \vec{r}$

$\vec{g} = \vec{g}_0 - \vec{\omega} \times \vec{\omega} \times \vec{r}$



$\vec{\omega} \times (\vec{\omega} \times \vec{r}) =$

$\omega^2 r \, SIN(\alpha)$

Calculate ?

Where do you weigh less ?

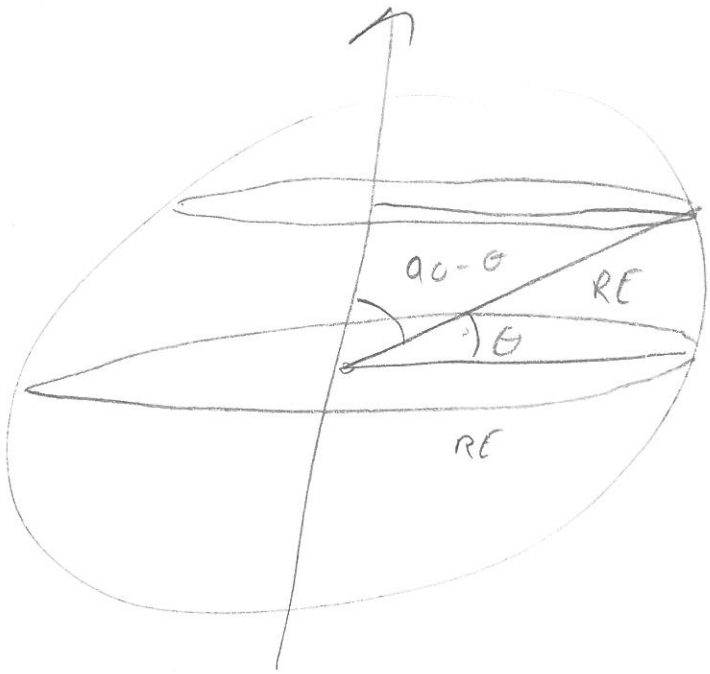
The Earth is BIGGER AT THE EQUATOR

Coriolis Vs Centripetal

$\frac{|\vec{\omega} \times \vec{v}|}{|\vec{\omega} \times \vec{\omega} \times \vec{r}|} = \ldots\ldots\ldots$ what $|\vec{v}|$?

Coriolis ?

$V = R\,W$



R < RE

$\frac{R}{RE} = SIN(90-\theta)$

ΔV 1 Second $\sim 5m$ $\frac{V_1}{V_2} = \frac{6{,}317{,}000}{6{,}317{,}005}$ = Small difference

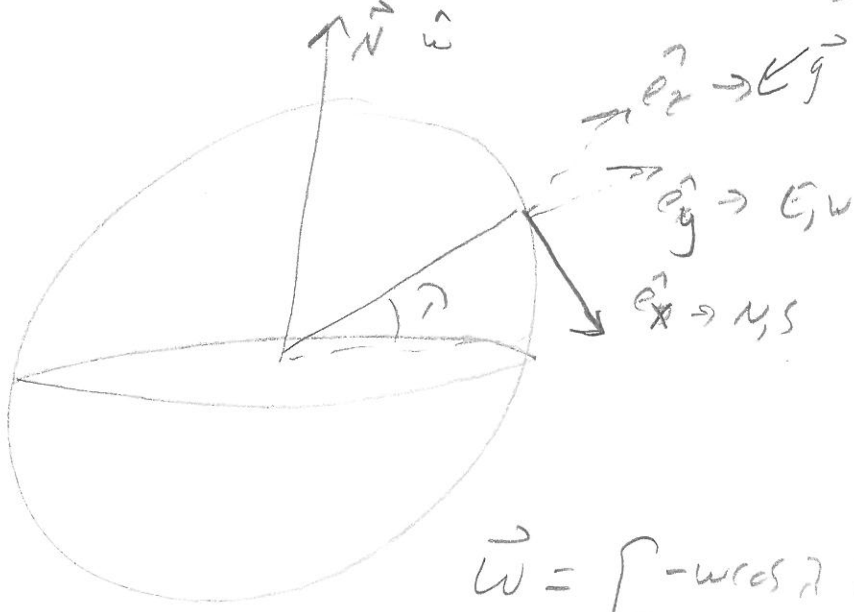
Earth has rotated 8 ten thousandth of a meter

↑

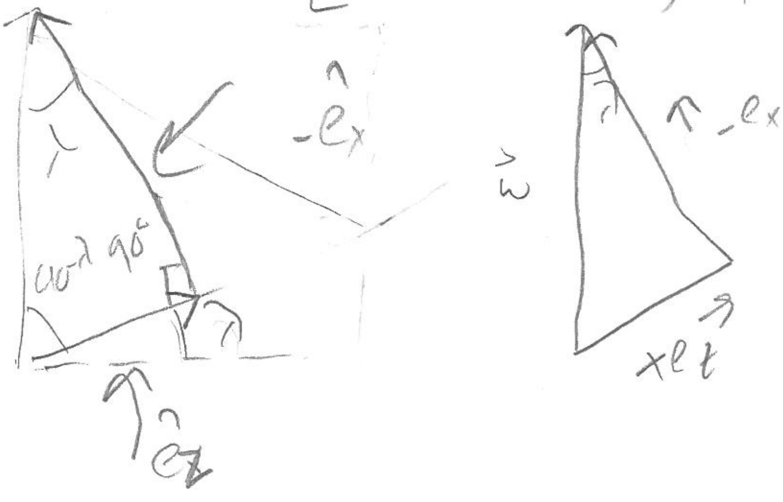
That's your deflection

Iff you jump due North

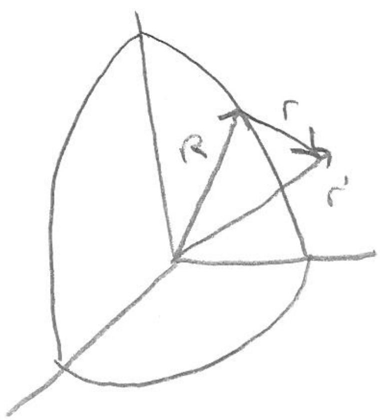
deflection → distance and velocity



$\vec{\omega} = [-\omega\cos\lambda\,\hat{i} + 0\hat{j} + \omega\sin\lambda\,\hat{k}]$



$$\vec{F}_{eff} = \vec{S} + m\vec{g}_0 - m\vec{\omega} \times (\vec{\omega} \times \vec{r}) - 2m(\vec{\omega} \times \vec{v}) - m\underbrace{\dot{\vec{\omega}}}_{0} \times \vec{r} - m\ddot{\vec{R}}$$



$$\left.\frac{d\dot{R}}{dt}\right|_{Fixed} = \left.\frac{d\dot{R}}{dt}\right|_{moving} + \vec{\omega} \times \dot{R}$$

$\downarrow$ 0

$$\dot{R} = \frac{dR}{dt} \qquad \left.\frac{d\vec{R}}{dt}\right|_{Fixed} = \frac{d\vec{R}}{dt} + \vec{\omega} \times (\vec{\omega} \times R)$$

$\downarrow$ 0

$$\vec{F}_{eff} = \vec{S} + m\vec{g}_0 - m(\vec{\omega} \times (\vec{\omega} \times [\vec{r} + \vec{R}])) - 2m(\vec{\omega} \times \vec{v})$$

$\vec{R} \gg \vec{r}$

$$-m(\vec{\omega} \times (\vec{\omega} \times \vec{R})) + m\vec{g}_0 = m\vec{g}_{eff}$$

$-m(\vec{\omega} \times (\vec{\omega} \times R))$

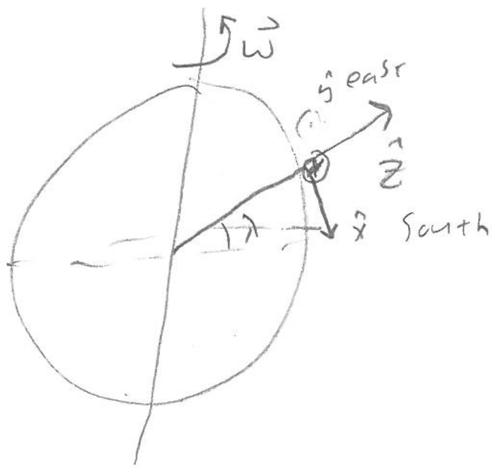
$\vec{g}_{eff}$

exagerated by 300 or so

$$\vec{F}_{eff} = \vec{S} + m\vec{g}_{eff} - 2m\vec{\omega} \times \vec{v}$$

Example align z with $\vec{g}_{eff}$

$$\vec{a}_r = \vec{g} - 2\vec{\omega} \times \vec{v}$$



$\omega_x = -\omega\cos\lambda$

$\omega_y = 0$

$\omega_z = \omega\sin\lambda$

Free Fall $\dot{z} = -gT$

$$m\ddot{r} = m\vec{g} - 2m\,\vec{\omega}\times\vec{v} = m\vec{g} - 2m\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \omega_x & 0 & \omega_y \\ 0 & 0 & -gT \end{vmatrix}$$

$$m\ddot{r} = m\vec{g} - 2m\,[gT\omega_x\,\hat{j}]$$

$$m\ddot{r} = [0\hat{i} + 2mgT\omega\cos\lambda\,\hat{j} - m\vec{g}\,\hat{k}]$$

$m\ddot{x} = 0$ $\qquad\qquad \dot{x} = \dot{x}_0$

$m\ddot{y} = 2mgT\omega\cos\lambda$ $\qquad \dot{y} = gT^2\omega\cos\lambda + \dot{y}_0$

$m\ddot{z} = -m\vec{g}$ $\qquad\qquad \dot{z} = -g\vec{T} + \dot{z}_0$

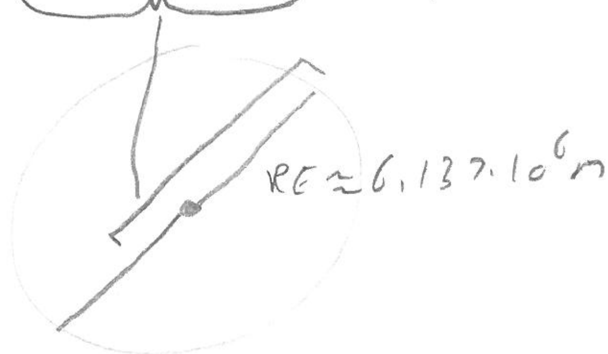
$\dot{y}$? $\quad |gT^2\omega\cos\lambda| = |gT|$

$T = \dfrac{1}{\omega\cos\lambda}$ $\quad T = \dfrac{24.3600}{2\pi\,[0,1]} = 13751$ Seconds

Silly 4 hours

$\frac{1}{10^{th}} g$? $1.376 \cdot 10^3$ seconds $\rightarrow$ 22 minutes

$(y) = \frac{1}{2} g T^2 = \underbrace{9.26 \cdot 10^6 \, m}$ in perspective <u>Silly</u>



So $\ddot{x} = 0$ $\ddot{y} = 2\omega g T \cos \lambda$ $\ddot{z} = -g$

Take $\dot{x}_0 = 0 = \dot{y}_0 = \dot{z}_0$

Then $\vec{x}(T) = 0$

$$y(T) \cong \frac{1}{3} \omega g T^3 \cos(\lambda)$$

$$z(T) = z(0) - \frac{1}{2} g T^2 \qquad z(0) = h$$

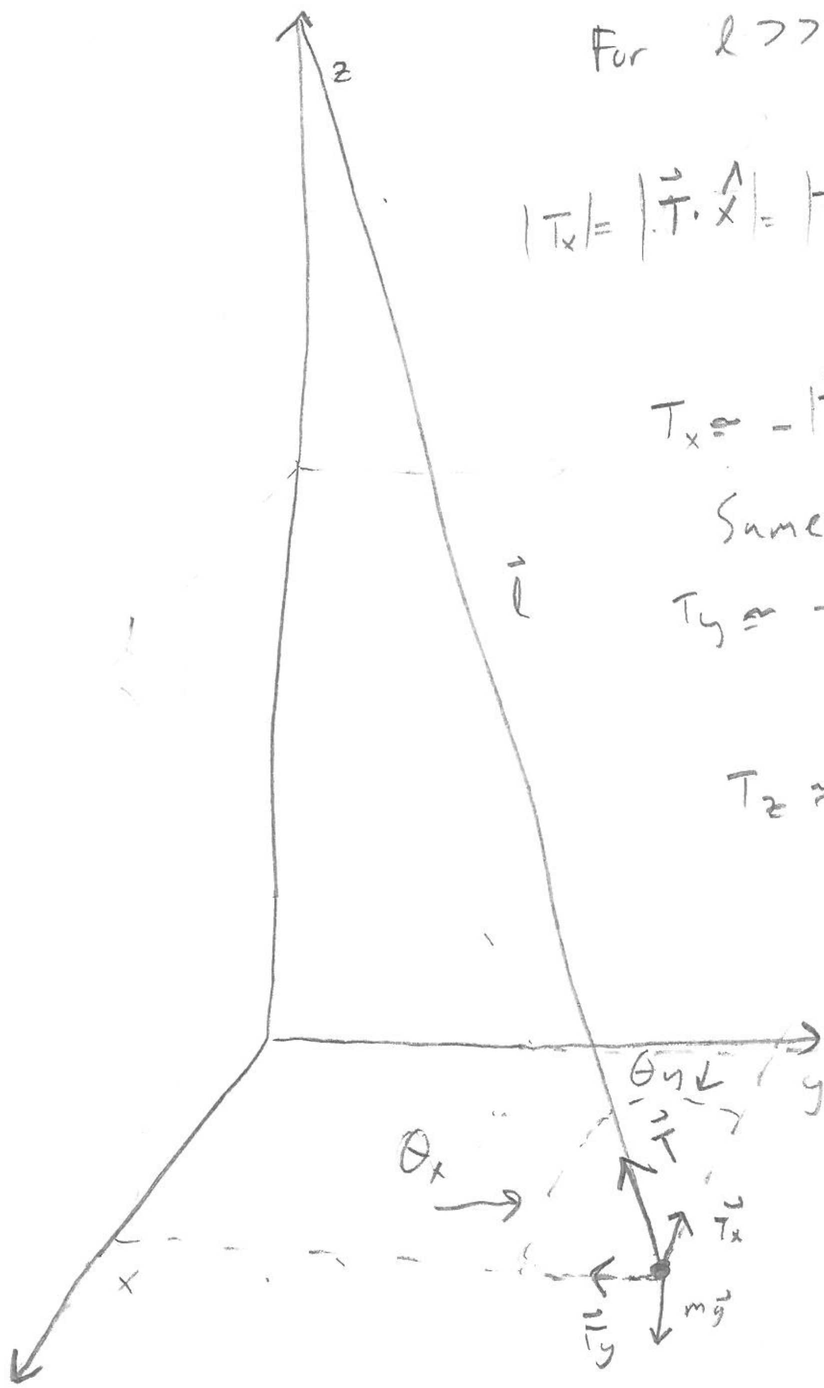
$$T_{Fall} \sim \sqrt{\frac{2h}{g}} \rightarrow T^3 = \frac{2h}{g} \sqrt{\frac{2h}{g}}$$

$$y(T_{Fall}) \approx \sqrt{\frac{8h^3}{g}} \left(\frac{1}{3} \omega \cos(\lambda) \right)$$

$z(0) = 100m$ $45°$ latitude 1.55 cm deflection

Foucalt Pendulum

$$\vec{a}_r = \vec{g} + \frac{\vec{T}}{m} - 2\vec{\omega} \times \vec{v}_r$$



For $l >> x\, y$ displacements

$$|T_x| = |\vec{T} \cdot \hat{x}| = |T| \cos\theta_x \approx T\theta_x \approx T\frac{x}{l}$$

$$T_x \approx -|\vec{T}|\frac{x}{L}$$

Same L

$$T_y \approx -|\vec{T}|\frac{y}{L}$$

$$T_z \approx |\vec{T}|\frac{z}{L} \approx |\vec{T}|$$

Then say $\dot{z} < \dot{x}$ and $\dot{y}$, $\dot{z} \approx 0$ $z \approx$ constant

$$\ddot{x} \approx -\frac{T}{m}\frac{x}{\ell} + 2\dot{y}\omega \sin\lambda$$

$$\ddot{y} \approx -\frac{T}{m}\frac{y}{\ell} - 2\dot{x}\omega \sin\lambda$$

ONLY CONSIDER OSCILLATIONS IN THE X Y PLANE

Let $\vec{T} \approx m\vec{g}$

$$\ddot{x} = -\overset{\alpha^2}{\left(\frac{g}{\ell}\right)} x + 2\dot{y}\underbrace{\omega \sin\lambda}_{\omega_z} = -\alpha^2 x + 2\dot{y}\omega_z$$

$$\ddot{y} = -\alpha^2 y - 2\dot{x}\omega_z$$

$$\ddot{x} + \alpha^2 x = 2\dot{y}\omega_z \quad \text{and} \quad \ddot{y} + \alpha^2 y = -2\dot{x}\omega_z$$

$$\eta = x + iy \quad \ddot{\eta} = \ddot{x} + i\ddot{y}$$

$$\begin{aligned} \ddot{x} + \alpha^2 x &= 2\dot{y}\omega_z \\ i\ddot{y} + \alpha^2 i y &= -2i\dot{x}\omega_z \end{aligned} \rightarrow \ddot{\eta} + \alpha^2\eta = 2\omega_z[\dot{y} - i\dot{x}]$$

$$\ddot{\eta} + \alpha^2\eta = 2i\omega_z\left[\frac{\dot{y}}{i} - \dot{x}\right]$$

$$\ddot{\eta} + \alpha^2 \eta = -2i\omega_z \left[i\dot{y} + \dot{x} \right]$$

$$\ddot{\eta} + \alpha^2 \eta + 2i\omega_z \dot{\eta} = 0$$

Now $\alpha^2 = \frac{g}{\ell}$, $\omega_z = \omega \sin \lambda$

$\eta \propto e^{\lambda \tau}$ $a = 1$ $b = 2i\omega_z$ $c = \alpha^2$

$$\lambda = -2i\omega_z \pm \sqrt{-4\omega^2 - 4\alpha^2} \left(\frac{1}{2} \right)$$

$$\lambda = -i\omega_z \pm \sqrt{(\omega_z^2 + \alpha^2)(-1)}$$

$$\eta(\tau) = A_{1,2} e^{-i\omega_z \tau} e^{\pm \sqrt{-\omega_z^2 - \alpha^2} \, \tau}$$

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if $\omega_z = 0$ $\eta(\tau) = A_{1,2} e^{\pm i \alpha \tau} = \underbrace{A_{1,2} e^{\pm \sqrt{\frac{g}{\ell}} \tau}}_{\text{Simple Pendulum}}$

$\sqrt{\frac{g}{\ell}}$? $\sqrt{\frac{10}{10{,}1000}} = 1, .1 = \omega_{\text{Pendulum}} \gg\gg \omega_{z_{\text{earth}}}$

So $\alpha >> \omega_z$

and $e^{-i\omega_z T}\left[Ae^{i\alpha T} + Be^{-i\alpha T}\right] = \eta(T)$

$$\eta(T) = e^{-i\omega_z T}\,\eta'(T) = e^{-i\omega_z T}\left[x'(T) + iy'(T)\right]$$

$$\eta(T) = \left[x'(T) + iy'(T)\right]\left[\cos(\omega_z T) - i\sin(\omega_z T)\right]$$

$$x(T) + iy(T) = \left[x'\cos - ix'\sin + iy'\cos + y'\sin\right]$$

or $x = x'\cos(\omega_z T) + y'\sin(\omega_z)T$

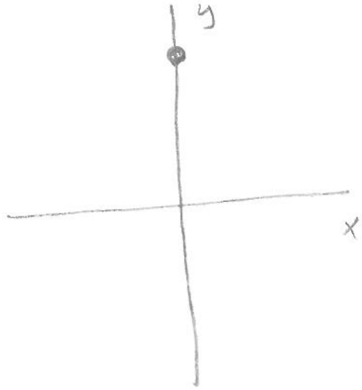
$y = -x'\sin(\omega_z T) + y'\cos(\omega_z T)$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \underbrace{\begin{pmatrix} \cos(\omega_z T) & \sin(\omega_z T) \\ -\sin(\omega_z T) & \cos(\omega_z T) \end{pmatrix}}_{\text{rotation matrix}} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

$\theta = \omega_z T$ Period ω_z Example

Also $q(\tau) \approx e^{-i\omega_z\tau}\left(Ae^{i\alpha\tau} + Be^{-i\alpha\tau}\right)$

pendulum



$x_0 = 0 \quad \dot{x}_0 = 0$

$y_0 = A \quad \dot{y}_0 = 0$

but $A = A_1 + iA_2 \qquad B = B_1 + iB_2$

$$q(\tau) = (A_1 + iA_2)\left[e^{i(\alpha-\omega)\tau}\right] + [B_1 + iB_2]\,e^{-i(\alpha+\omega)\tau}$$

$$q(\tau) = (A_1 + iA_2)\left[\cos[(\alpha-\omega)\tau] + i\sin[(\alpha-\omega)\tau]\right] + (B_1 + iB_2)\left[\cos[(\alpha+\omega)\tau] - i\sin[(\alpha+\omega)\tau]\right]$$

$$x \to \text{Real} \quad \overset{1}{A_1\cos(\alpha-\omega)\tau} - \overset{2}{A_2\sin(\alpha-\omega)\tau} + \overset{3}{B_1\cos(\alpha+\omega)\tau} + \overset{4}{B_2\sin(\alpha+\omega)\tau}$$

$$y \to \text{Im} \quad \overset{5}{A_2\cos(\alpha-\omega)\tau} + \overset{6}{\underline{A_1}\sin(\alpha-\omega)\tau} - \overset{7}{B_1\sin(\alpha+\omega)\tau} + \overset{8}{B_2\cos(\alpha+\omega)\tau}$$

$x(0) = 0 \quad A_1 + B_1 = 0 \quad \underline{\underline{A_1 = -B_1}}$

$\dot{x}(0) = 0 \quad -A_2(\alpha-\omega) + B_2(\alpha+\omega) = 0 \qquad A_2 = B_2\dfrac{(\alpha+\omega)}{(\alpha-\omega)} \quad \alpha \sim \sqrt{\dfrac{g}{L}} \sim \sqrt{\dfrac{10}{100}} \sim \dfrac{1}{\sqrt{10}} \gg \omega$

$\underline{\underline{A_2 \approx B_2}}$

$y(0) = A \qquad A_2 + B_2 = A \qquad A_2 \approx \frac{A}{2}$

$\dot{y}(0) = 0 \qquad (\alpha - \omega) A_1 - (\alpha + \omega) B_1 = 0$

$(\alpha - \omega) A_1 + (\alpha + \omega) A_1 = 0$

$A_1 \sim 0 \quad \text{So} \quad B_1 \sim 0$

$X(T) = -\frac{A}{2} \sin(\alpha - \omega)T + \frac{A}{2} \sin(\alpha + \omega)T$

$y(T) = \frac{A}{2} \cos(\alpha - \omega)T + \frac{A}{2} \cos(\alpha + \omega)T$

$X(T) = \frac{A}{2} \left[S\alpha C\omega + S\omega C\alpha - (S\alpha C\omega - S\omega C\alpha) \right]$

$X(T) = A \sin(\omega_2 T) \cos(\alpha T)$

Similarly

$y(T) = \frac{A}{2} \left[C\alpha C\omega - \sin\alpha \sin\omega + C\alpha C\omega + S\alpha S\omega \right]$

$y(T) = A \cos(\alpha_2 T) \cos(\omega_2 T)$

$\vec{r} = X(T)\hat{i} + y(T)\hat{j} = A \cos\left(\frac{\sqrt{g}}{\sqrt{L}} T\right) \left[\sin(\omega_2 T)\hat{i} + \cos(\omega_2 T)\hat{j} \right]$

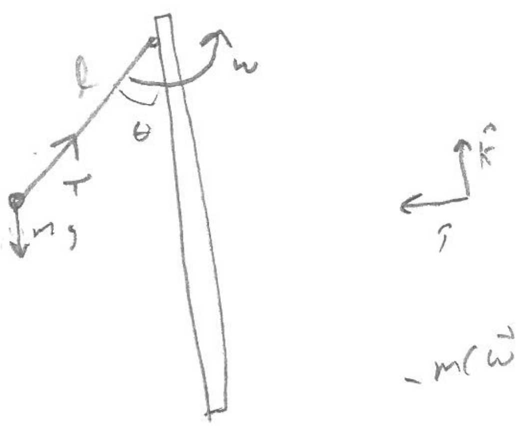
$\vec{r} = A\cos\sqrt{\frac{g}{l}}t\ \hat{n}$ $\quad \uparrow \downarrow \quad$ in $\omega = \sqrt{\frac{g}{l}} \rightarrow T_p = \frac{2\pi}{\omega} \rightarrow$ check

$\hat{n} = \sin(\omega_z t)\hat{i} + \cos(\omega_z t)\hat{j}$

$t = 0 \quad \hat{n} = \hat{j} \quad T_\omega = \frac{2\pi}{\omega_z} \qquad \uparrow \ \rightarrow \ \downarrow \ \leftarrow$

Northern hemisphere, opposite for $\lambda < 0$ Southern

Last Problem then Ch. #11



$-m(\vec{\omega}\times\vec{\omega}\times\vec{r}) = -m(\omega\hat{k}\times\omega\hat{k}\times[l\sin\theta\hat{i} - l\cos\theta\hat{k}])$

$= -m(\omega\hat{k}\times\omega l\sin\theta\hat{j})$

$= m\omega^2 l\hat{i} \rightarrow$ out

$\vec{T} + m\vec{g} - m\vec{\omega}\times\vec{\omega}\times\vec{r} = 0$

$-mg\hat{k} + m\omega^2 l\sin\theta\hat{i} + T\cos\theta\hat{k} - T\sin\theta\hat{i} = 0$

$T\cos\theta = mg, \quad T\sin\theta = m\omega^2 l\sin\theta$

$mg\tan\theta = m\omega^2 l\sin\theta \qquad T = m\omega^2 l, \quad \theta = \cos^{-1}\left(\frac{g}{\omega^2 l}\right)$