Lecture 9

Wave Optics

You may ignore

- Lloyd's Mirror
- Liquid Crystals

Wave Optics

- The wave nature of light is needed to explain various phenomena.
  - Interference
  - Diffraction
  - Polarization
- The particle nature of light was the basis for ray (geometric) optics.

Interference

- Light waves interfere with each other much like mechanical waves do.
- All interference associated with light waves arises when the electromagnetic fields that constitute the individual waves combine.

Conditions for Interference

- For sustained interference between two sources of light to be observed, there are two conditions which must be met.
  - The sources must be *coherent.*
    - The waves they emit must maintain a constant phase with respect to each other.
  - The waves must have identical wavelengths.

Producing Coherent Sources

- Light from a monochromatic source is allowed to pass through a narrow slit.
- The light from the single slit is allowed to fall on a screen containing two narrow slits.
- The first slit is needed to insure the light comes from a tiny region of the source which is coherent.
- Old method

Producing Coherent Sources, Cont.

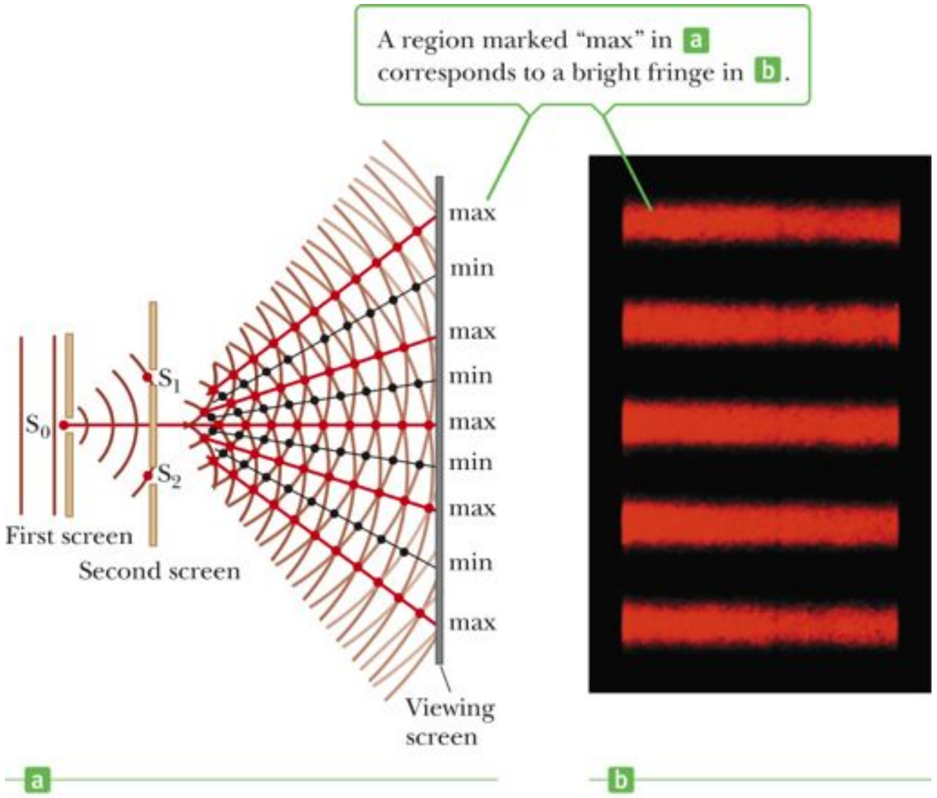
- Currently, it is much more common to use a laser as a coherent source.
- The laser produces an intense, coherent, monochromatic beam over a width of several millimeters.
- The laser light can be used to illuminate multiple slits directly.

Young' s Double Slit Experiment

- Thomas Young first demonstrated interference in light waves from two sources in 1801.
- Light is incident on a screen with a narrow slit, S_o
- The light waves emerging from this slit arrive at a second screen that contains two narrow, parallel slits, S_1 and S_2

Young's Double Slit Experiment, Diagram

- The narrow slits, S_1 and S_2 act as sources of waves.
- The waves emerging from the slits originate from the same wave front and therefore are always in phase.

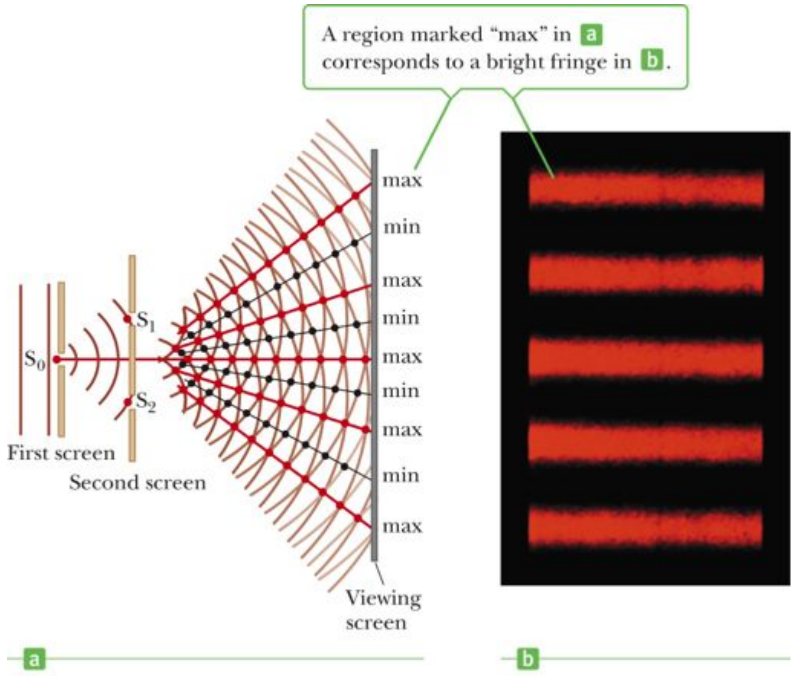


Resulting Interference Pattern

- The light from the two slits form a visible pattern on a screen.
- The pattern consists of a series of bright and dark parallel bands called **fringes.**
- *Constructive interference* occurs where a bright fringe appears.
- *Destructive interference* results in a dark fringe.

Fringe Pattern

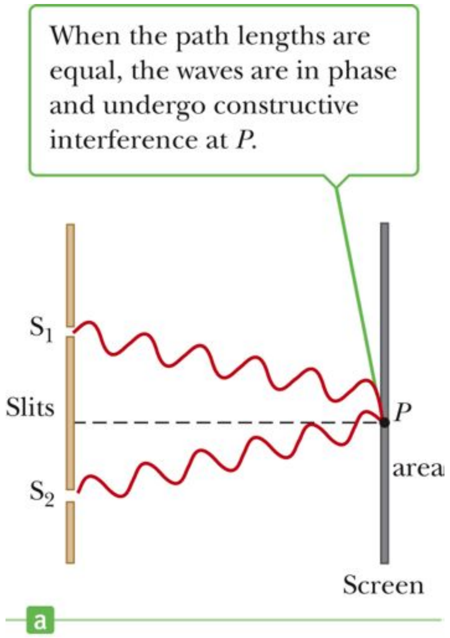
- The fringe pattern formed from a Young's Double Slit Experiment would look like this.
- The bright areas represent constructive interference.
- The dark areas represent destructive interference.



A region marked "max" in a corresponds to a bright fringe in b.

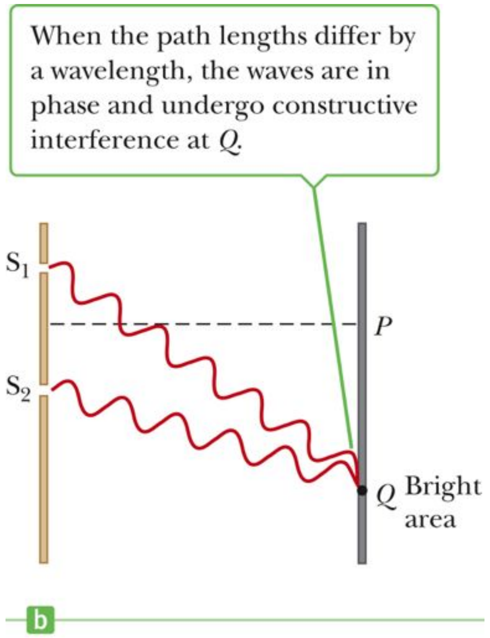
Interference Patterns

- Constructive interference occurs at the center point.
- The two waves travel the same distance.
  - Therefore, they arrive in phase.


When the path lengths are equal, the waves are in phase and undergo constructive interference at *P*.

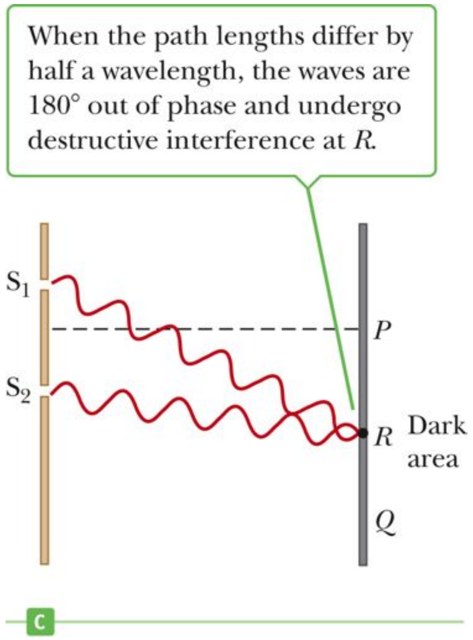
Interference Patterns, 2

- The upper wave has to travel farther than the lower wave.
- The upper wave travels one wavelength farther.
  - Therefore, the waves arrive in phase.
- A bright fringe occurs.

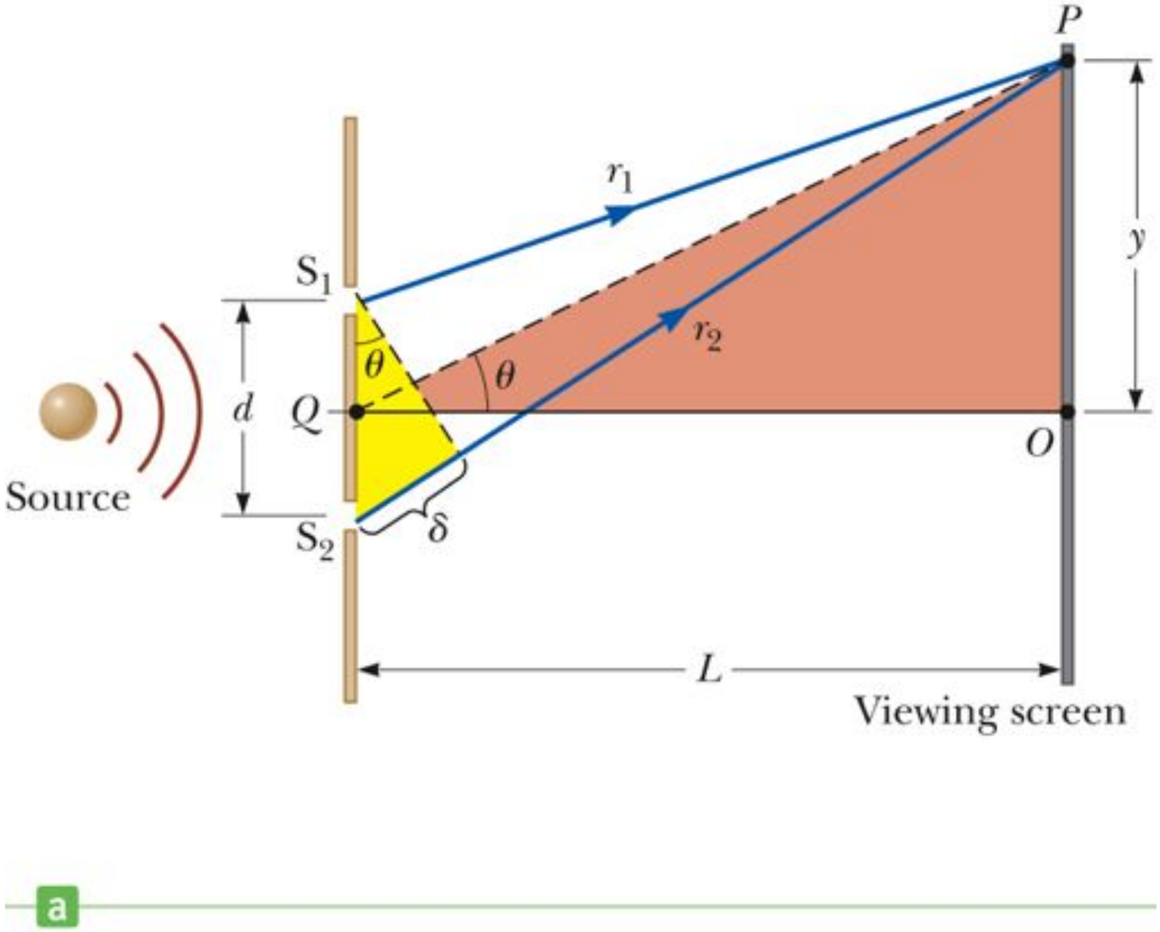

When the path lengths differ by a wavelength, the waves are in phase and undergo constructive interference at *Q*.

Interference Patterns, 3

- The upper wave travels one-half of a wavelength farther than the lower wave.
- The trough of the bottom wave overlaps the crest of the upper wave.
- This is destructive interference.
  - A dark fringe occurs.

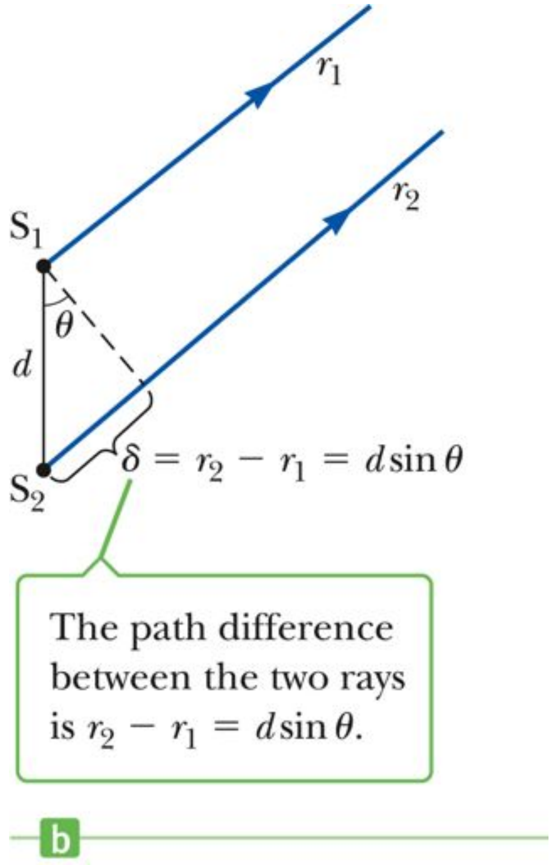

When the path lengths differ by half a wavelength, the waves are 180° out of phase and undergo destructive interference at *R*.

Geometry of Young's Double Slit Experiment



Interference Equations

- The path difference, δ, is found from the small triangle.
- $\delta = r_2 - r_1 = d \sin \theta$
  - This assumes the paths are parallel.
  - Not exactly parallel, but a very good approximation since L is much greater than d



The path difference between the two rays is $r_2 - r_1 = d \sin \theta$.

Interference Equations, 2

- For a bright fringe, produced by constructive interference, the path difference must be either zero or some integral multiple of the wavelength.
- $\delta = d \sin \theta_{\text{bright}} = m\lambda$
  - m = 0, ±1, ±2, …
  - m is called the *order number.*
    - When m = 0, it is the zeroth order maximum.
    - When m = ±1, it is called the first order maximum.

Interference Equations, 2

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Interference Equations, 3

- When destructive interference occurs, a dark fringe is observed.
- This needs a path difference of an odd half wavelength.
- $\delta = d \sin \theta_{\text{dark}} = (m + \frac{1}{2})\,\lambda$
  - $m = 0, \pm 1, \pm 2, \ldots$

Interference Equations, 4

- The positions of the fringes can be measured vertically from the zeroth order maximum.
- $y = L \tan\theta \approx L \sin\theta$
- Assumptions
  - $L >> d$
  - $d >> \lambda$
- Approximation
  - θ is small and therefore the approximation $\tan\theta \approx \sin\theta$ can be used.
    - The approximation is true to three-digit precision only for angles less than about 4°

Interference Equations, Final

- For bright fringes

$$y_{bright} = \frac{\lambda L}{d} m \quad m = 0, \pm 1, \pm 2$$

- For dark fringes

$$y_{dark} = \frac{\lambda L}{d}\left(m + \frac{1}{2}\right) \quad m = 0, \pm 1, \pm 2$$

Hw 3 & 5 & 13

A Young’s double slit has a slit separation of 2.50 x 10-^{5} m on which a monochromatic light beam is directed. The resultant bright fringes on a screen 1.00 m
the double slit are separated by 2.30 x 10-^{2} m. What is the wavelength of this beam? (1 nm = 10-^{9} m)
a.373 nm
b.454 nm
c.575 nm
d.667 nm

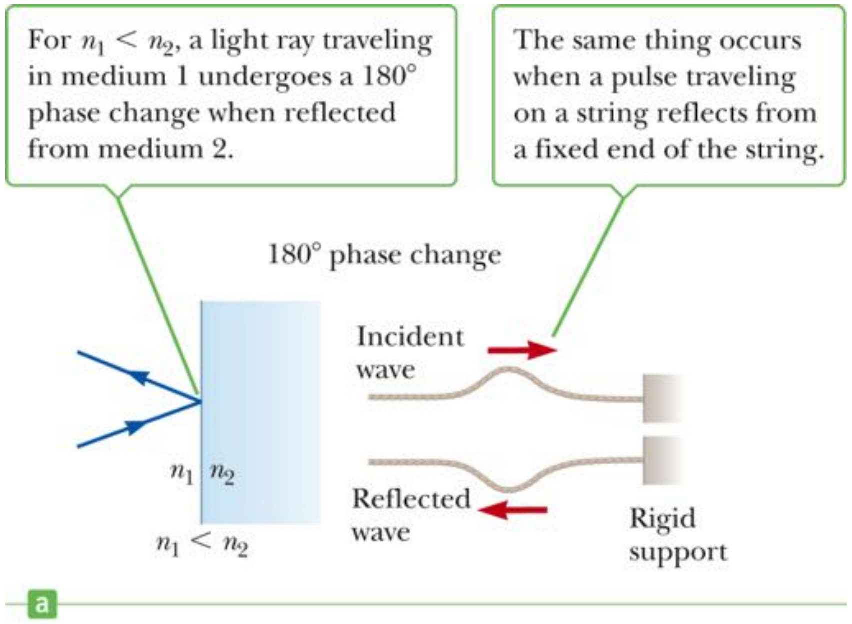
In a Young’s double-slit interference apparatus, by what factor is the distance between adjacent light and dark fringes changed when the separation between slits is doubled?
a.1/4
b.1/2
c.1
d.2

In a Young’s double-slit interference apparatus, the distance from the slits to the screen is doubled. The distance between adjacent light and dark fringes changes by a factor of:
a.1/4.
b.1/2.
c.1.
d.2.

A light source simultaneously emits light of two wavelengths, 480 nm and 560 nm, respectively. The source is used in a double-slit interference experiment where the s
spacing is a 0.040 mm, and the distance between double slits and the screen is 1.2 m. What is the separation between the second-order bright fringes of the two wavelengths as they appear on the screen? (1 nm = 10-^{9} m)
a.0.16 cm
b.0.32 cm
c.0.48 cm
d.0.64 cm

Phase Change

Phase Changes Due To Reflection



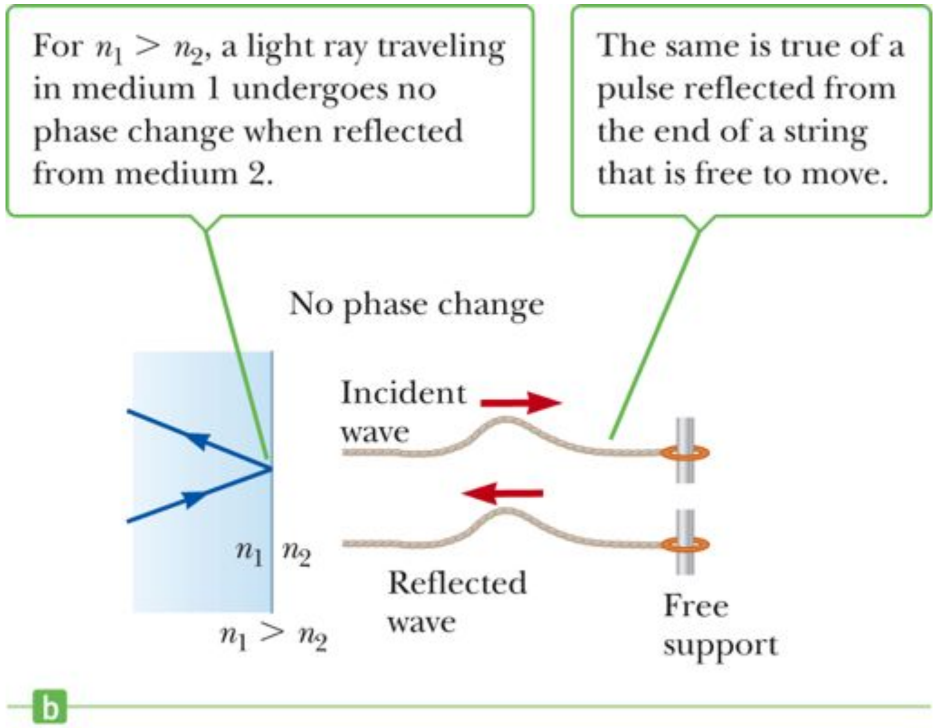
For $n_1 < n_2$, a light ray traveling in medium 1 undergoes a 180° phase change when reflected from medium 2.

The same thing occurs when a pulse traveling on a string reflects from a fixed end of the string.

- An electromagnetic wave undergoes a phase change of 180° upon reflection from a medium of higher index of refraction than the one in which it was traveling.
  - Analogous to a reflected pulse on a string

Section 24.3

Phase Changes Due To Reflection, Cont.



- There is no phase change when the wave is reflected from a boundary leading to a medium of lower index of refraction.
  - Analogous to a pulse in a string reflecting from a free support

Interference in Thin Films

- Interference effects are commonly observed in thin films.
  - Examples are soap bubbles and oil on water
- The interference is due to the interaction of the waves reflected from both surfaces of the film.

Section 24.4

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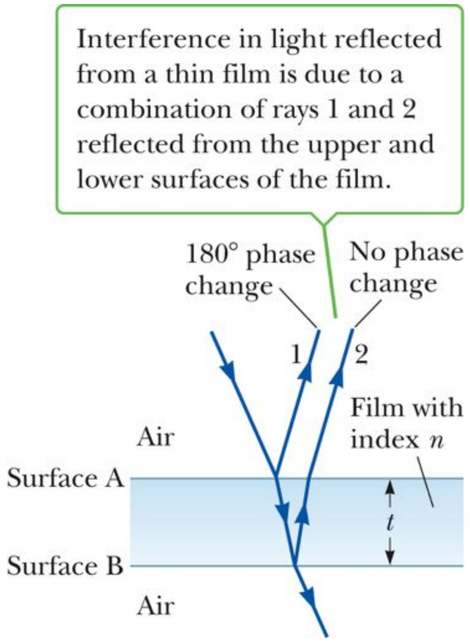
Interference in Thin Films, 2

- Facts to remember
  - An electromagnetic wave traveling from a medium of index of refraction n_1 toward a medium of index of refraction n_2 undergoes a 180° phase change on reflection when $n_2 > n_1$
    - There is no phase change in the reflected wave if $n_2 < n_1$
  - The wavelength of light λ_n in a medium with index of refraction n is $\lambda_n = \lambda/n$ where λ is the wavelength of light in vacuum.

Interference in Thin Films, 3

- Ray 1 undergoes a phase change of 180° with respect to the incident ray.
- Ray 2, which is reflected from the lower surface, undergoes no phase change with respect to the incident wave.



Interference in light reflected from a thin film is due to a combination of rays 1 and 2 reflected from the upper and lower surfaces of the film.

Interference in Thin Films, 4

- Ray 2 also travels an additional distance of 2t before the waves recombine.
- For constructive interference
  - $2nt = (m + \frac{1}{2})\lambda \quad m = 0, 1, 2 \ldots$
    - This takes into account both the difference in optical path length for the two rays and the 180° phase change
- For destructive interference
  - $2nt = m\lambda \quad m = 0, 1, 2 \ldots$

Interference in Thin Films, 5

- Two factors influence interference.
  - Possible phase reversals on reflection
  - Differences in travel distance
- The conditions are valid if the medium above the top surface is the same as the medium below the bottom surface.
- If the thin film is between two different media, one of lower index than the film and one of higher index, the conditions for constructive and destructive interference are *reversed.*

Interference in Thin Films, Final

- Be sure to include two effects when analyzing the interference pattern from a thin film.
  - Path length
  - Phase change

Problem Solving Strategy with Thin Films, 1

- Identify the thin film causing the interference.
- Determine the indices of refraction in the film and the media on either side of it.
- Determine the number of phase reversals: zero, one or two.

Problem Solving with Thin Films,

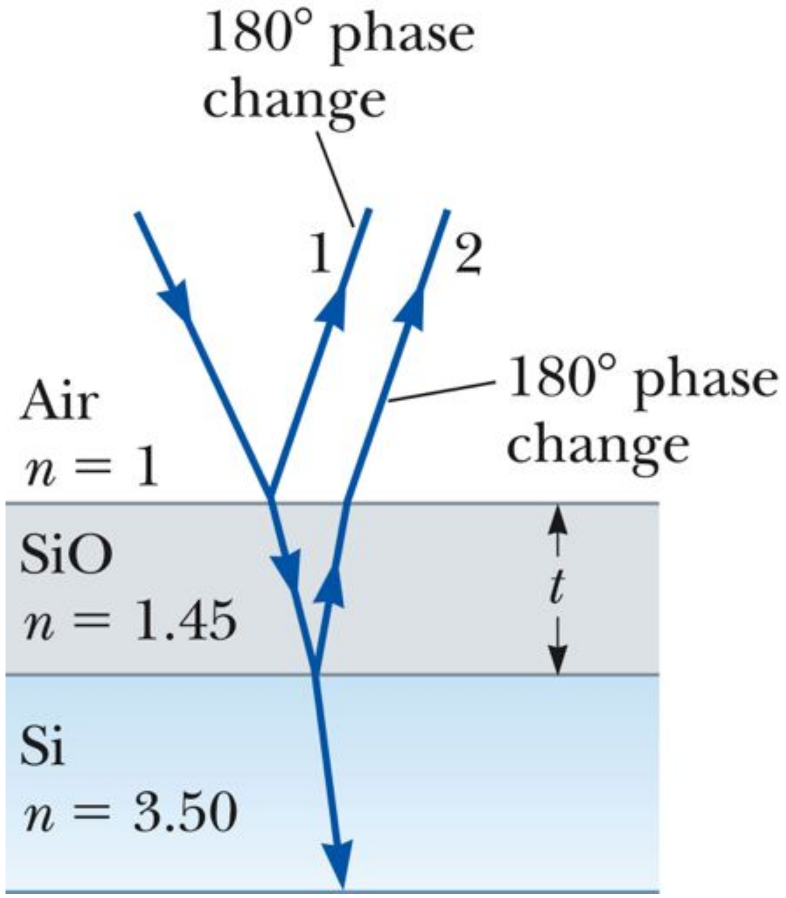
- The interference is constructive if the path differen is an integral multiple of λ and destructive if the pa difference is an odd half multiple of λ.
  - The conditions are reversed if one of the waves underg a phase change on reflection.
- Substitute values in the appropriate equation.
- Solve and check.

Problem Solving with Thin Films, 3

Equation $m = 0, 1, 2, \ldots$	1 phase reversal	0 or 2 phase reversals
$2nt = (m + \frac{1}{2})\lambda$	constructive	destructive
$2nt = m\lambda$	destructive	constructive

Interference in Thin Films, Example

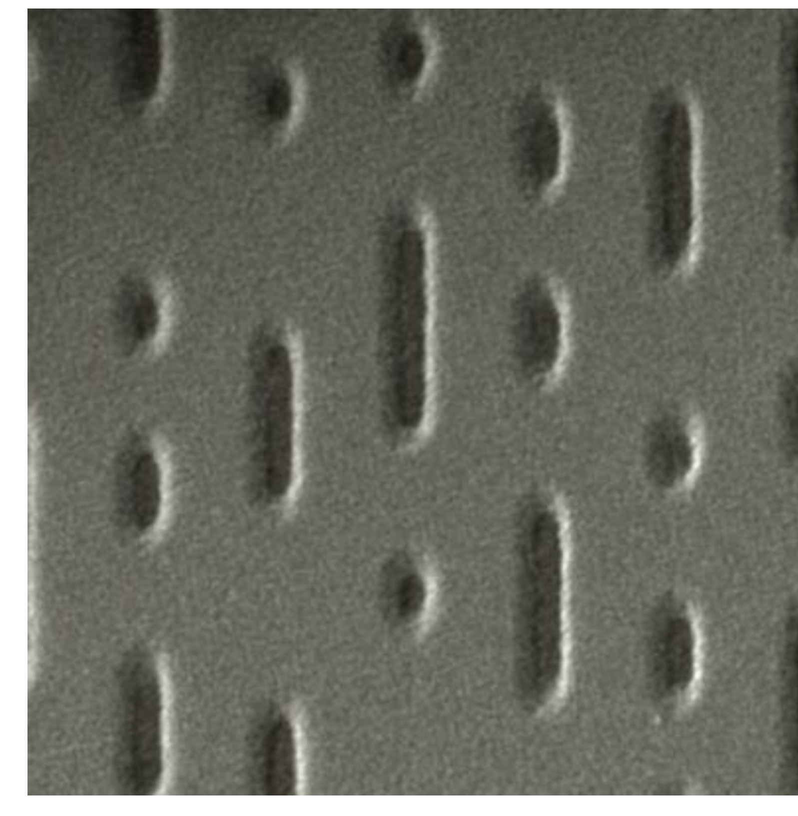
- An example of different indices of refraction
- A coating on a solar cell
- There are two phase changes

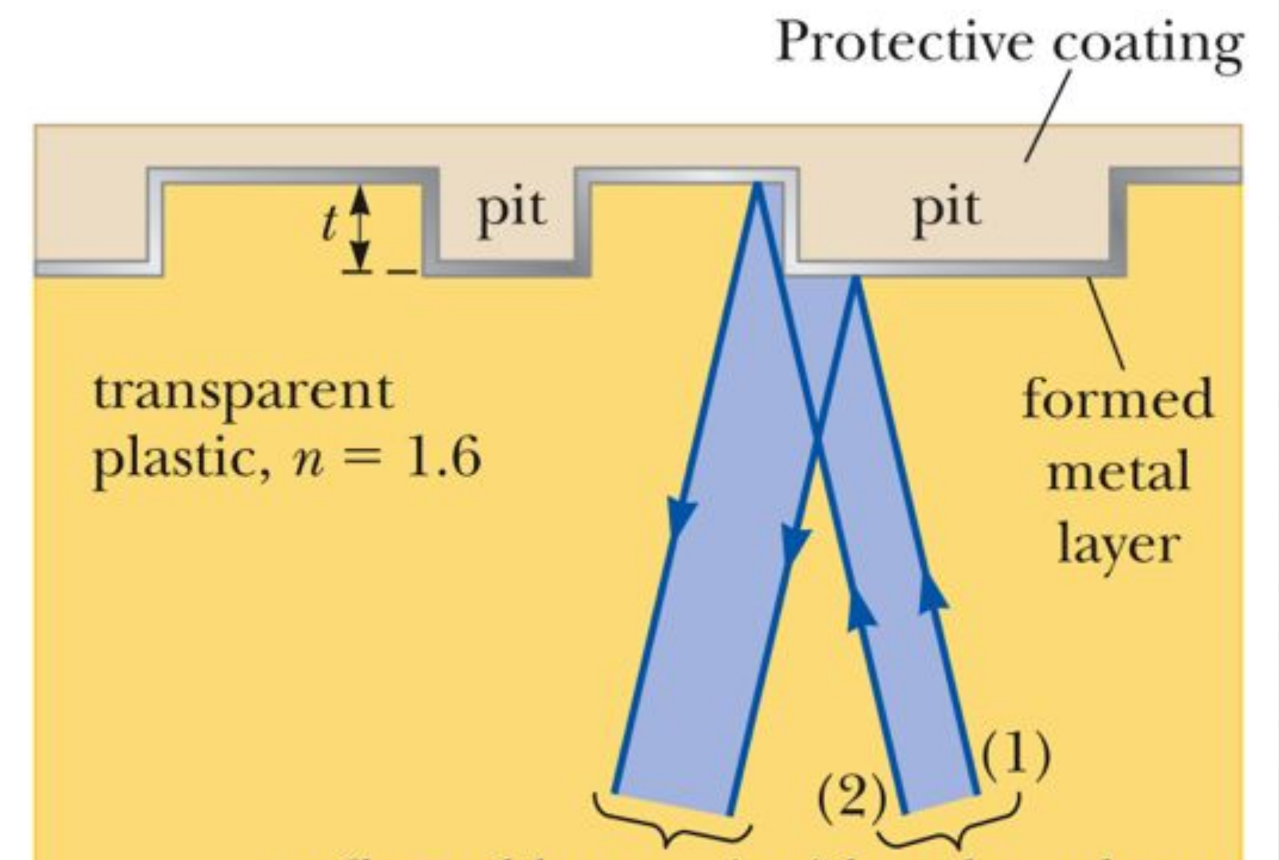


Section 24.4

CD’s and DVD’s

- Data is stored digitally.
  - A series of ones and zeros read by laser light reflected from the disk
- Strong reflections correspond to constructive interference.
  - These reflections are chosen to represent zeros.
- Weak reflections correspond to destructive interference.
  - These reflections are chosen to represent ones.



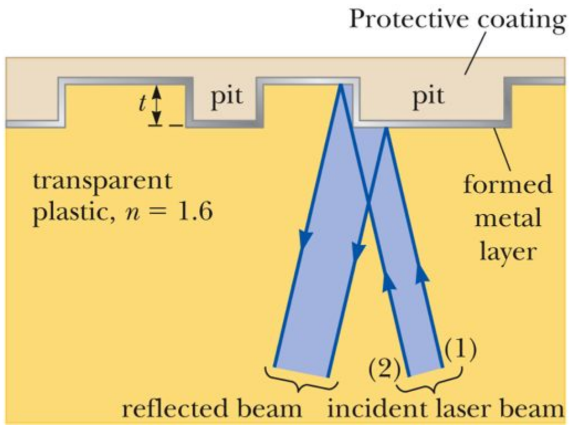


CD’s and Thin Film Interference

- A CD has multiple tracks.
  - The tracks consist of a sequence of pits of varying length formed in a reflecting information layer.
- The pits appear as bumps to the laser beam.
  - The laser beam shines on the metallic layer through a clear plastic coating.

Reading a CD

- As the disk rotates, the laser reflects off the sequence of bumps and lower areas into a photodector.
  - The photodector converts the fluctuating reflected light intensity into an electrical string of zeros and ones.
- The pit depth is made equal to one-quarter of the wavelength of the light.



Reading a CD, Cont.

- When the laser beam hits a rising or falling bump edge, part of the beam reflects from the top of the bump and part from the lower adjacent area.
  - This ensures destructive interference and very low intensity when the reflected beams combine at the detector.
- The bump edges are read as ones.
- The flat bump tops and intervening flat plains are read as zeros.

DVD’s

- DVD’s use shorter wavelength lasers.
  - The track separation, pit depth and minimum pit length are all smaller.
  - Therefore, the DVD can store about 30 times more information than a CD.

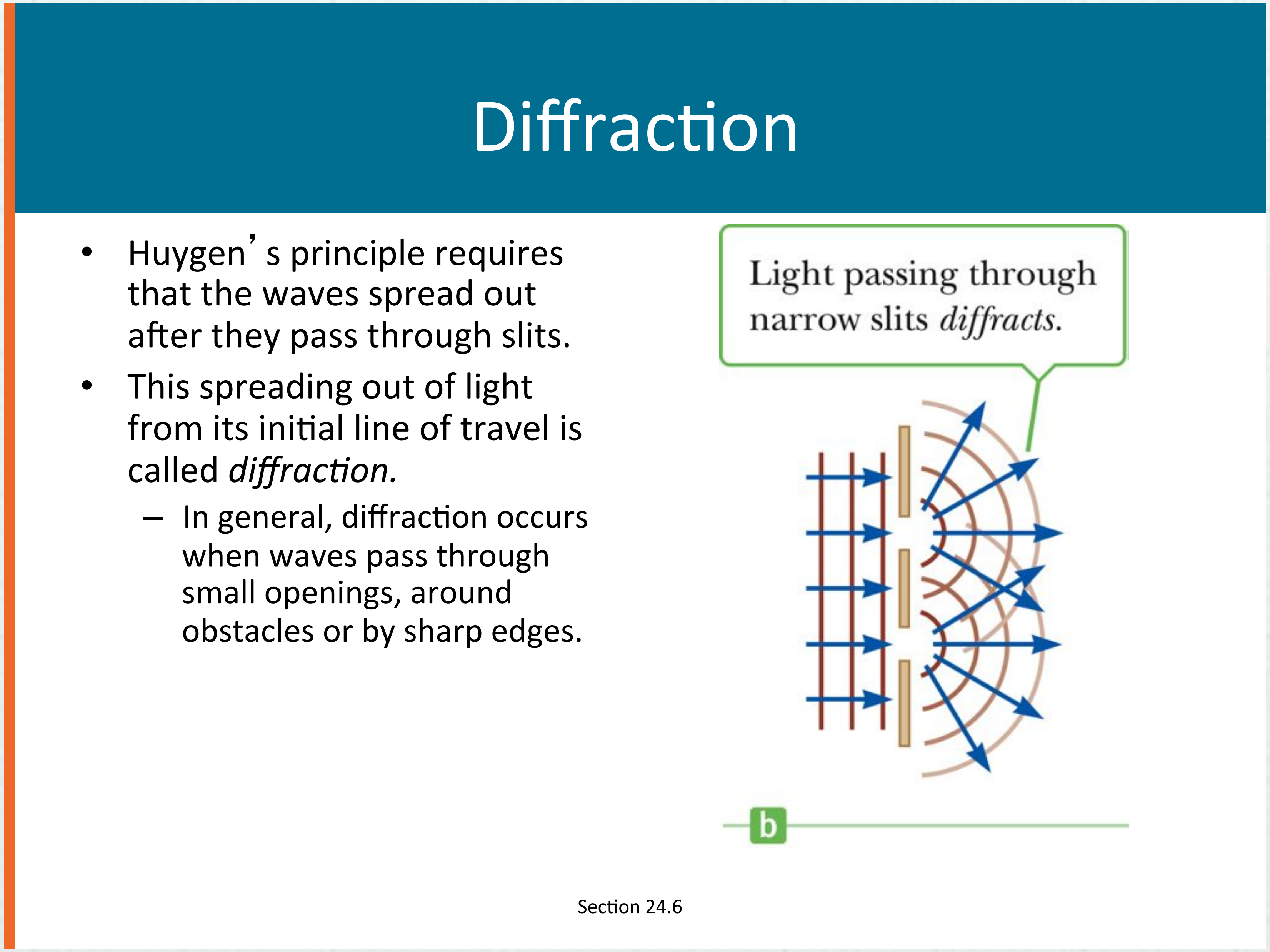
Hw 16 & 23

What is the minimum thickness of a glycerin film (*n* = 1.47) on which light of wavelength 600 nm shines that results in constructive interference of the reflected light? Assume the film is surrounded front and back by air.
a.75 nm
b.102 nm
c.150 nm
d.204 nm

A silicon monoxide thin film (*n* = 1.45) of thickness 90.0 nm is applied to a camera lens made of glass (*n* = 1.55). This will result in a destructive interference for reflected light of what wavelength?
a.720 nm
b.558 nm
c.522 nm
d.450 nm

Two closely spaced parallel glass plates are separated by 750 nm. What wavelength light source in the visible region (390 nm to 710 nm) will experience maximum transmission through the two plates?
a.500 nm
b.429 nm
c.600 nm
d.684 nm

Diffraction

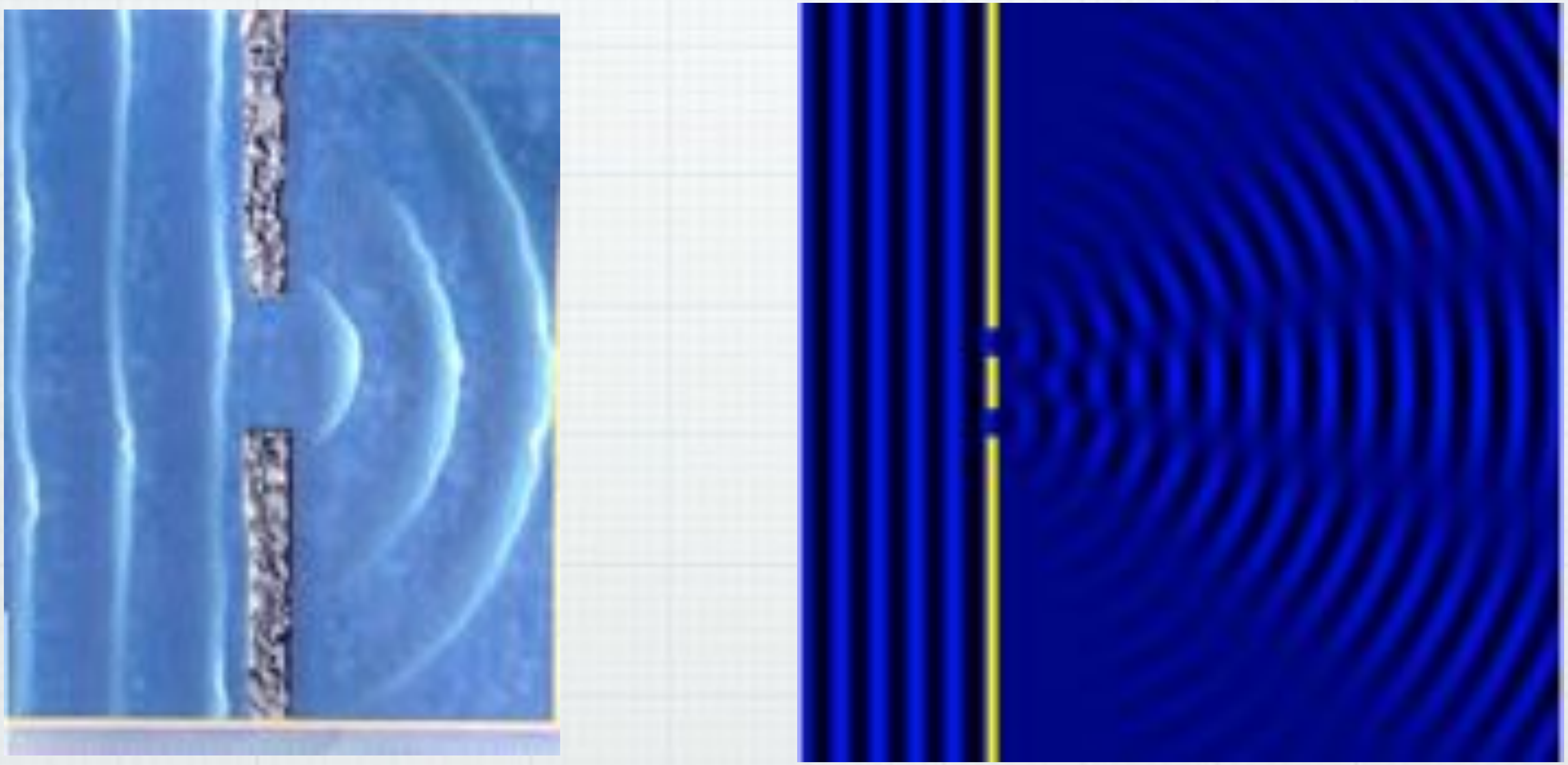


Diffraction

- Huygen's principle requires that the waves spread out after they pass through slits.
- This spreading out of light from its initial line of travel is called *diffraction.*
  - In general, diffraction occurs when waves pass through small openings, around obstacles or by sharp edges.

Light passing through narrow slits *diffracts.*

Section 24.6

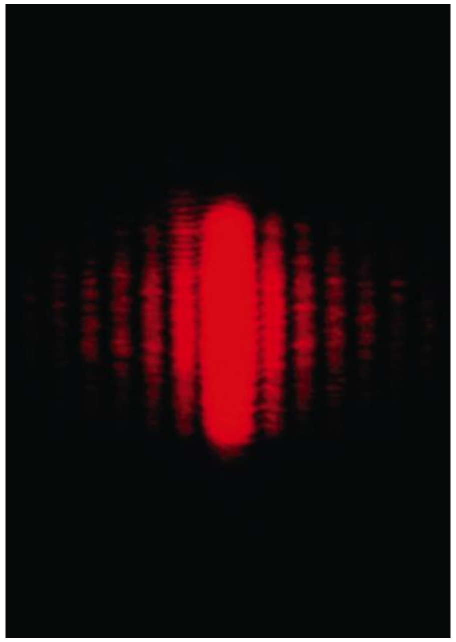


Diffraction, 2

- A single slit placed between a distant light source and a screen produces a diffraction pattern.
  - It will have a broad, intense central band.
  - The central band will be flanked by a series of narrower, less intense secondary bands.
    - Called secondary maxima
  - The central band will also be flanked by a series of dark bands.
    - Called minima

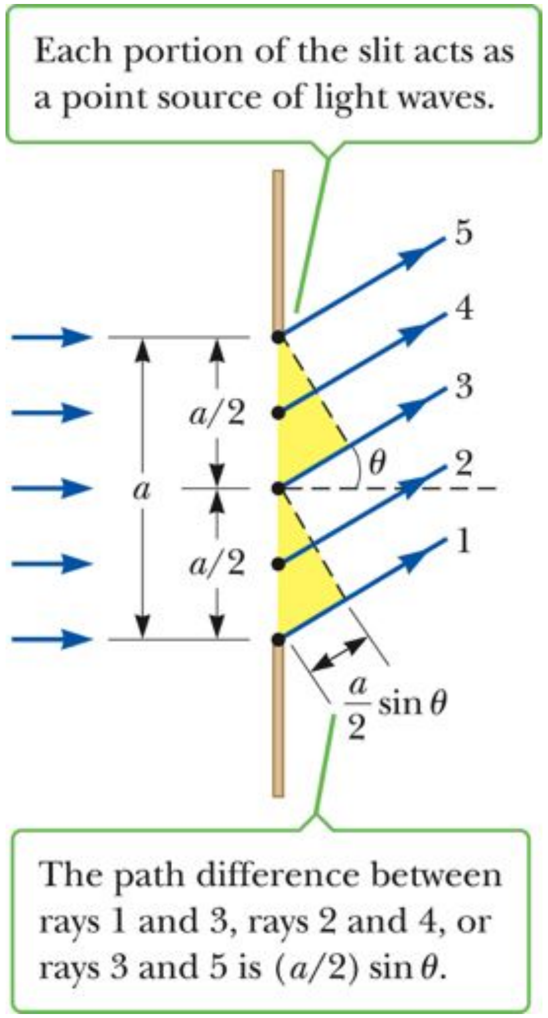
Diffraction, 3

- The results of the single slit cannot be explained by geometric optics.
  - Geometric optics would say that light rays traveling in straight lines should cast a sharp image of the slit on the screen.



Single Slit Diffraction

- According to Huygen’s principle, each portion of the slit acts as a source of waves.
- The light from one portion of the slit can interfere with light from another portion.
- The resultant intensity on the screen depends on the direction θ



Each portion of the slit acts as a point source of light waves.

The path difference between rays 1 and 3, rays 2 and 4, or rays 3 and 5 is $(a/2)\sin\theta$.

Single Slit Diffraction, 2

- All the waves that originate at the slit are in phase.
- Wave 1 travels farther than wave 3 by an amount equal to the path difference (a/2) sin θ
  - a is the width of the slit
- If this path difference is exactly half of a wavelength, the two waves cancel each other and destructive interference results.

Single Slit Diffraction, 3

- In general, *destructive interference* occurs for a single slit of width *a* when $\sin \theta_{dark} = m\lambda / a$
  - $m = \pm 1, \pm 2, \pm 3, \ldots$
- Doesn’t give any information about the variations in intensity along the screen

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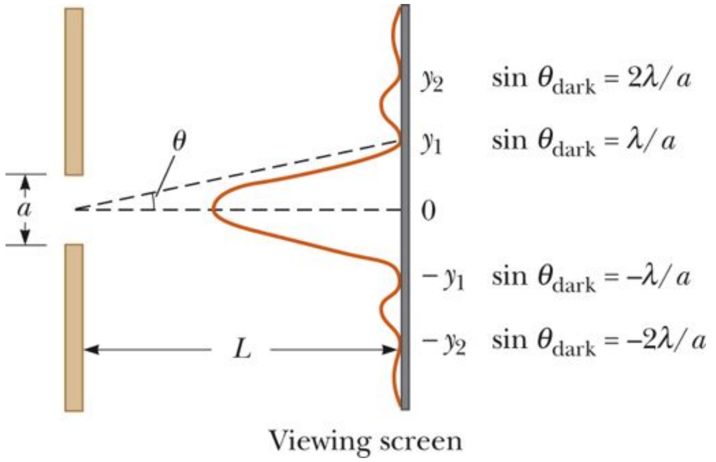
Single Slit Diffraction, 4

- The general features of the intensity distribution are shown.
- A broad central bright fringe is flanked by much weaker bright fringes alternating with dark fringes.
- The points of constructive interference lie approximately halfway between the dark fringes.

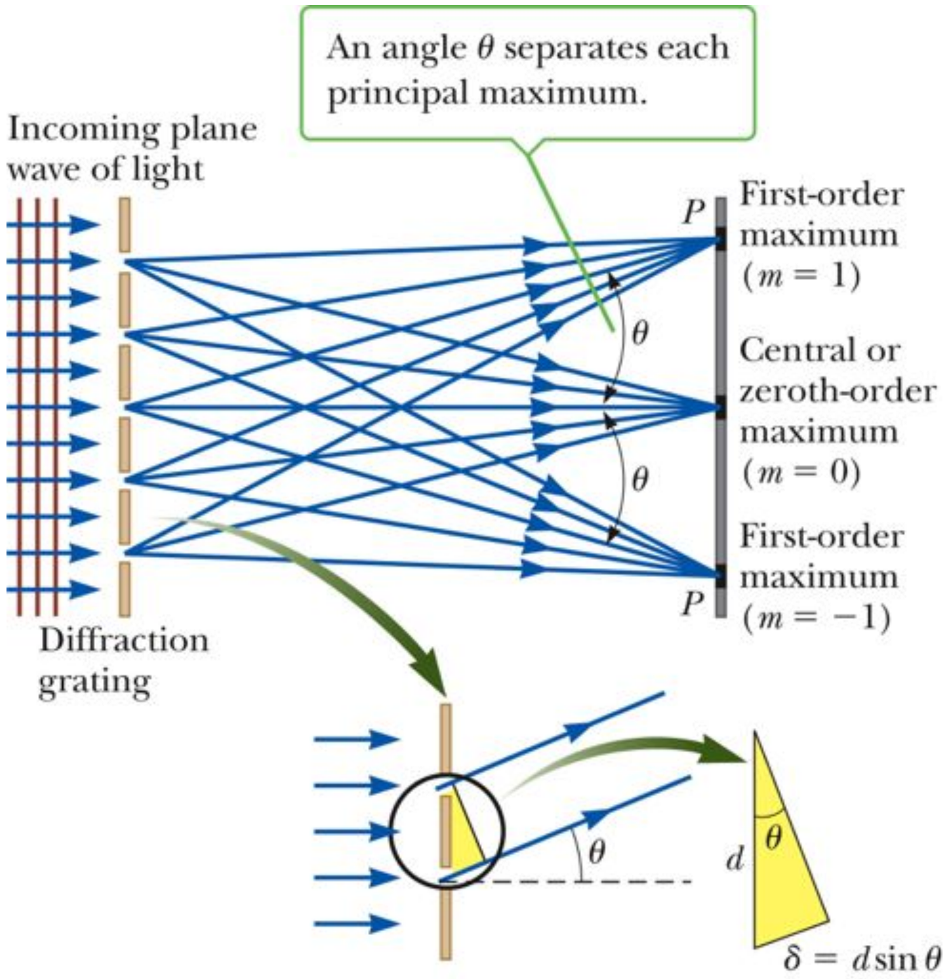


Diffraction Grating

- The diffracting grating consists of many equally spaced parallel slits.
  - A typical grating contains several thousand lines per centimeter.
- The intensity of the pattern on the screen is the result of the combined effects of interference and diffraction.

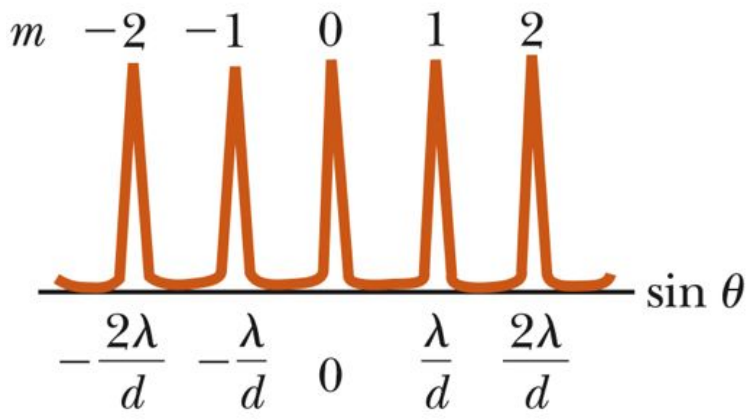
Diffraction Grating, Cont.

- The condition for *maxima* is
  - $d \sin \theta_{bright} = m \lambda$
    - $m = 0, \pm 1, \pm 2, \ldots$
- The integer m is the *order number* of the diffraction pattern.
- If the incident radiation contains several wavelengths, each wavelength deviates through a specific angle.



Diffraction Grating, Final

- All the wavelengths are focused at m = 0
  - This is called the zeroth order maximum
- The first order maximum corresponds to m = 1
- Note the sharpness of the principle maxima and the broad range of the dark area.
  - This is in contrast to the broad, bright fringes characteristic of the two-slit interference pattern.



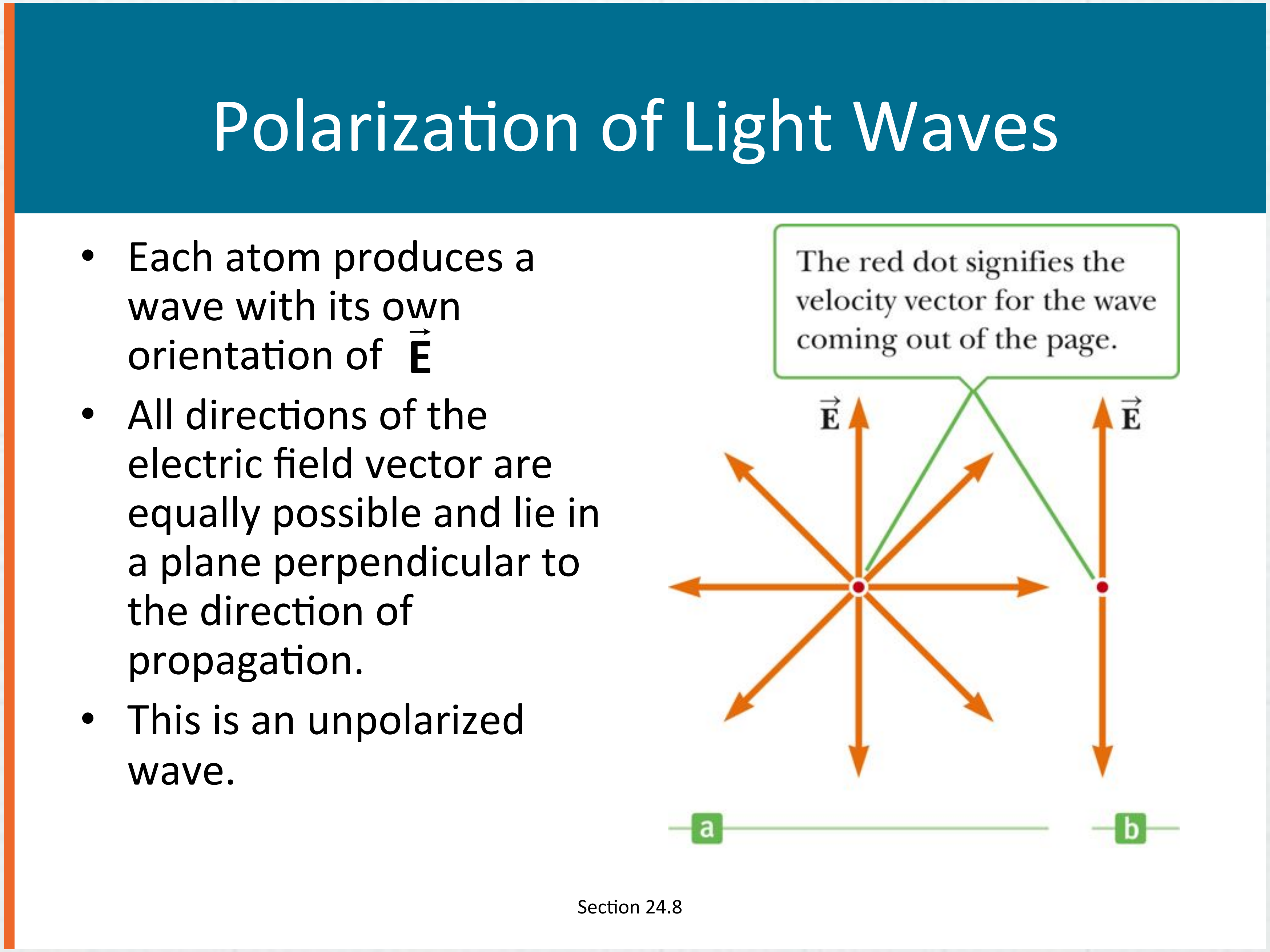
Hw 32 & 48

Light of wavelength 610 nm is incident on a slit of width 0.20 mm, and a diffraction pattern is produced on a screen that is 1.5 m from the slit. What is the distance of the second dark fringe from the center of the bright fringe? (1 nm = 10^{-9} m)
a.0.68 cm
b.0.92 cm
c.1.2 cm
d.1.4 cm

A multiple slit diffraction grating has a slit separation of 2.00×10^{-6} m. Find the wavelength of the monochromatic light that will have its second order bright fringe diffracted through an angle of 38.0°. (1 nm = 10^{-9} m)
a.120 nm
b.500 nm
c.616 nm

A wavelength of 573 nm yields a first order maximum at 35° with a grating. At what angle will the second order maximum appear for this wavelength?
a.17.5°
b.–35°
c.70°
d.No second order maximum exists in this case.

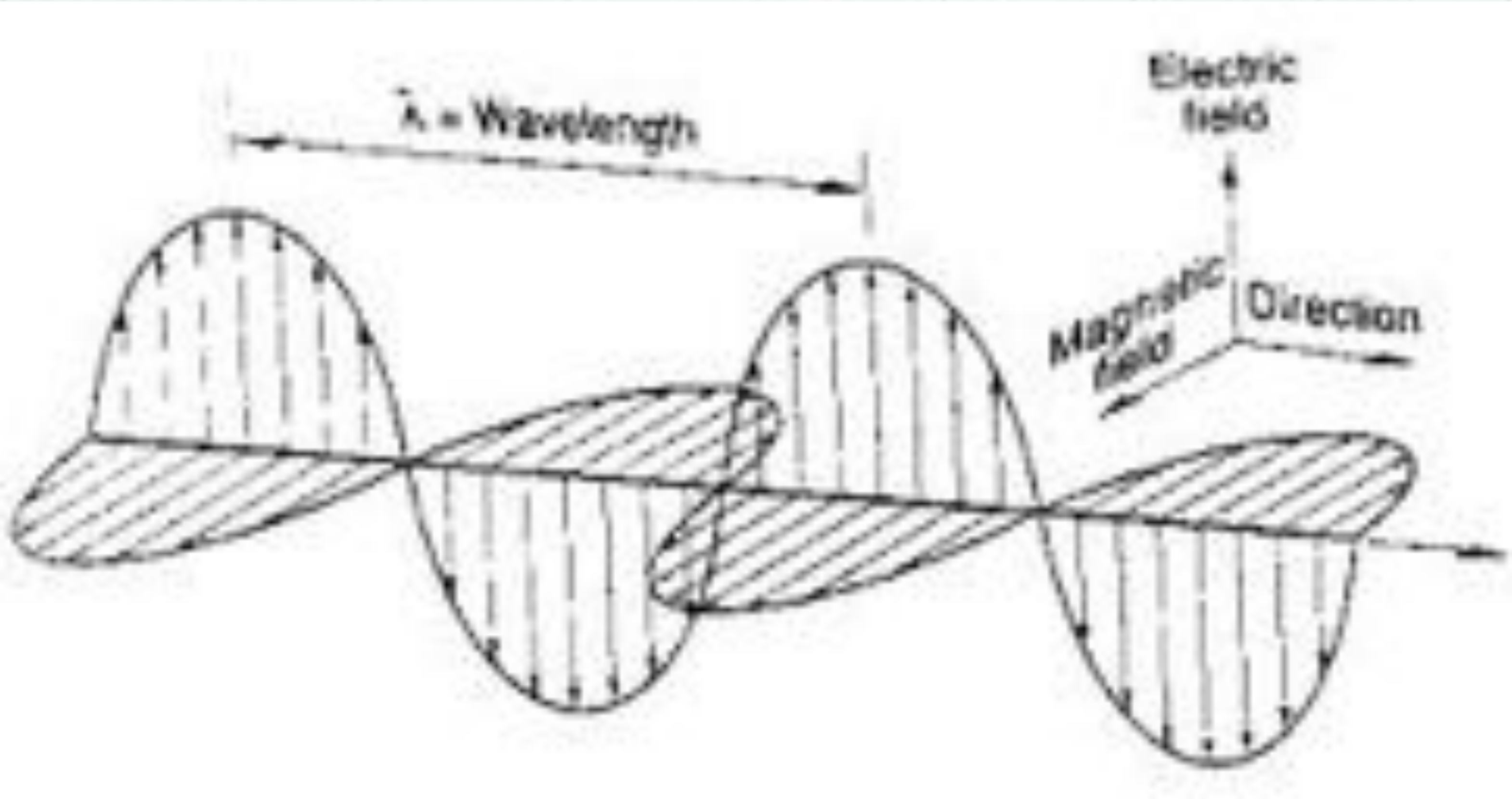
Polarization of Light



Polarization of Light Waves

- Each atom produces a wave with its own orientation of $\vec{E}$
- All directions of the electric field vector are equally possible and lie in a plane perpendicular to the direction of propagation.
- This is an unpolarized wave.

The red dot signifies the velocity vector for the wave coming out of the page.



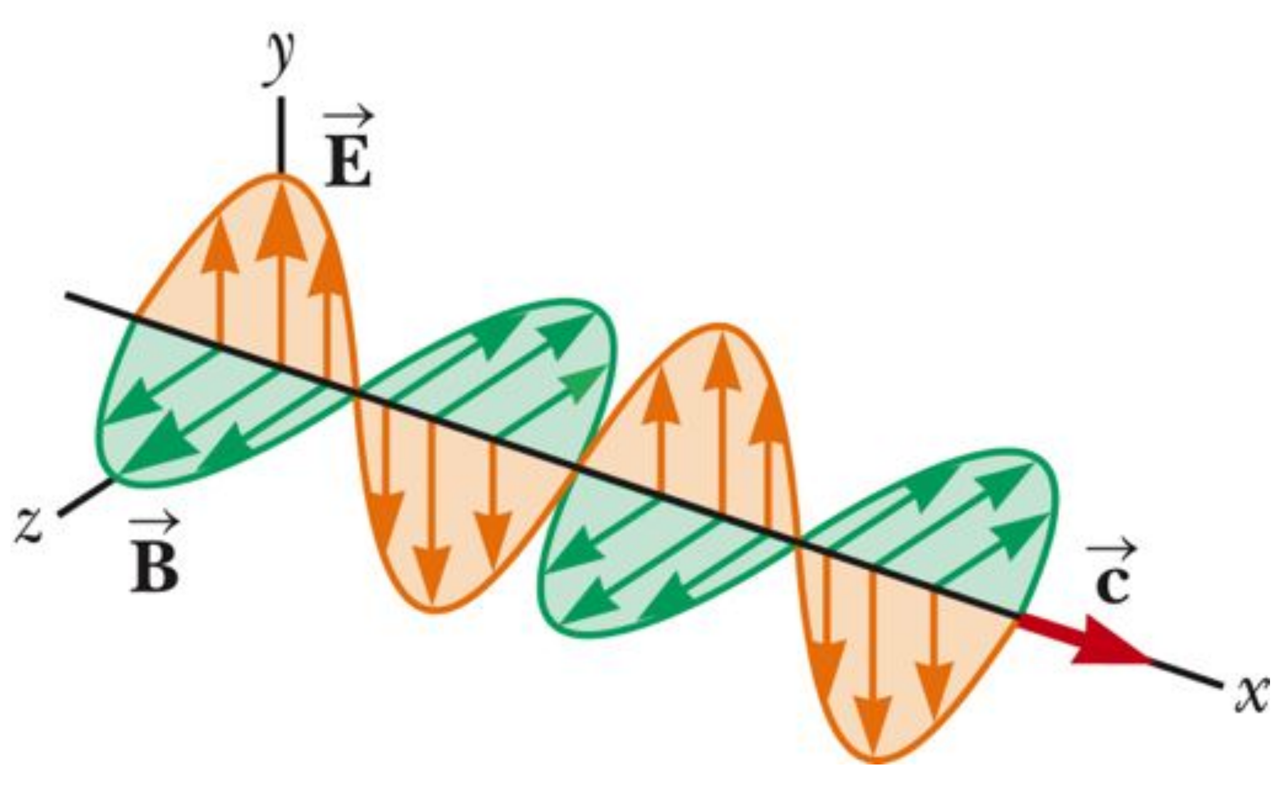
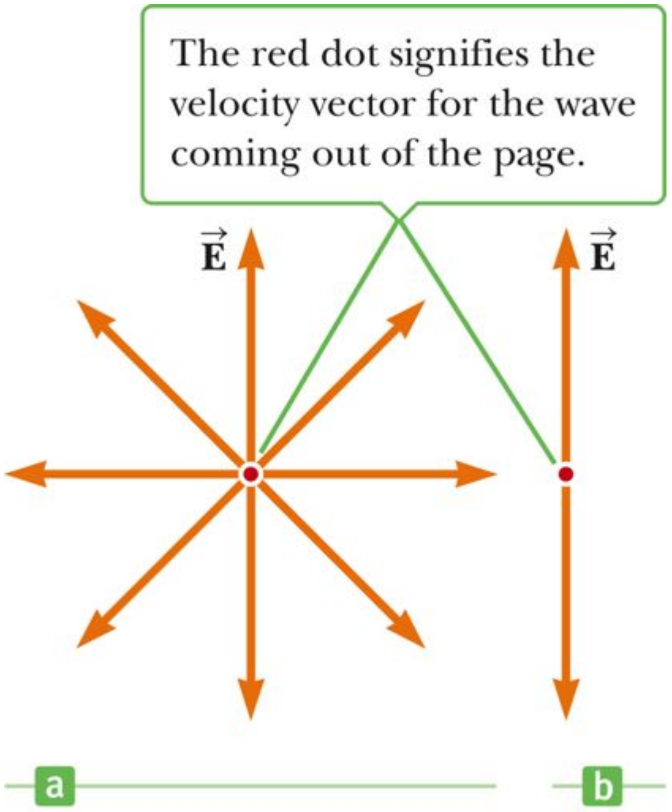


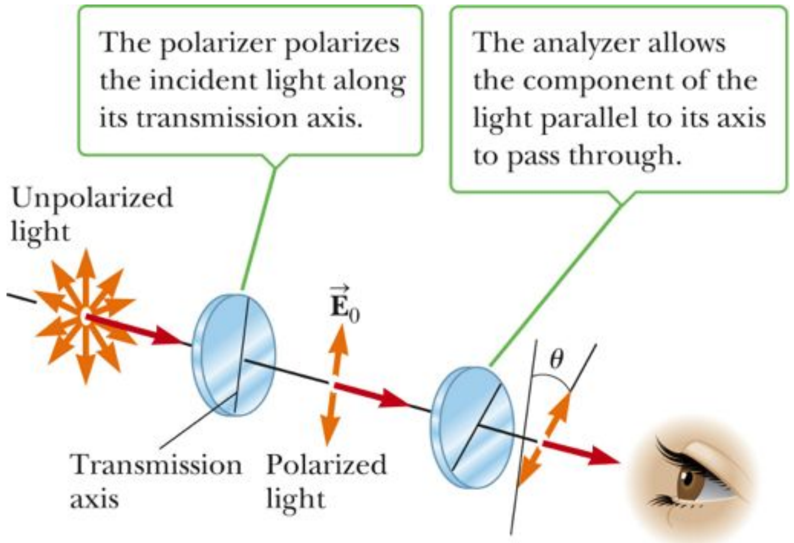
Fig. 24-24, p. 843

Polarization of Light, Cont.

- A wave is said to be *linearly polarized* if the resultant electric field vibrates in the same direction at all times at a particular point.
- Polarization can be obtained from an unpolarized beam by
  - Selective absorption
  - Reflection
  - Scattering


The red dot signifies the velocity vector for the wave coming out of the page.

Polarization by Selective Absorption


The polarizer polarizes the incident light along its transmission axis.

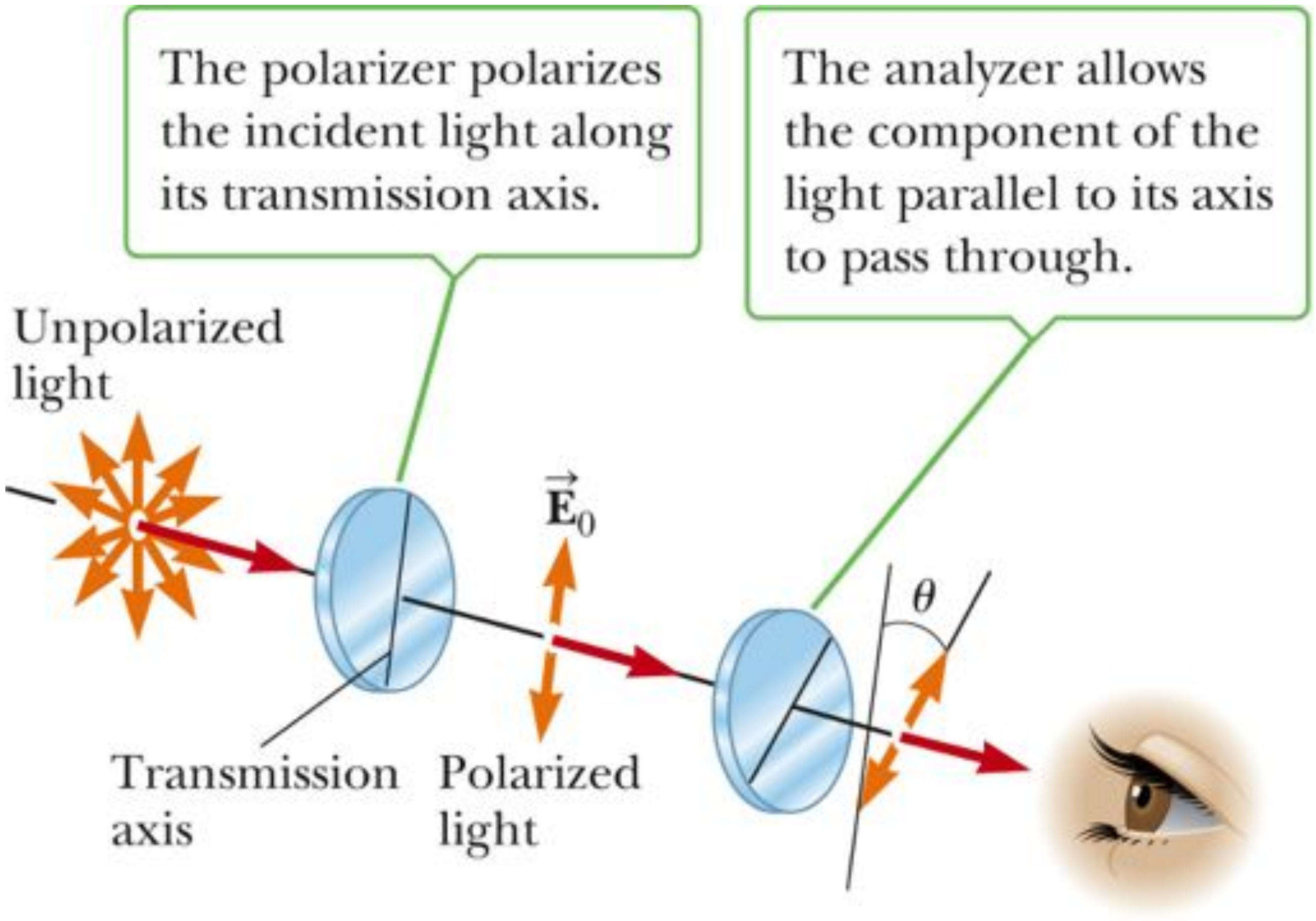
The analyzer allows the component of the light parallel to its axis to pass through.

- The most common technique for polarizing light
- Uses a material that transmits waves whose electric field vectors in the plane are parallel to a certain direction and absorbs waves whose electric field vectors are perpendicular to that direction

Section 24.8

Selective Absorption, Cont.

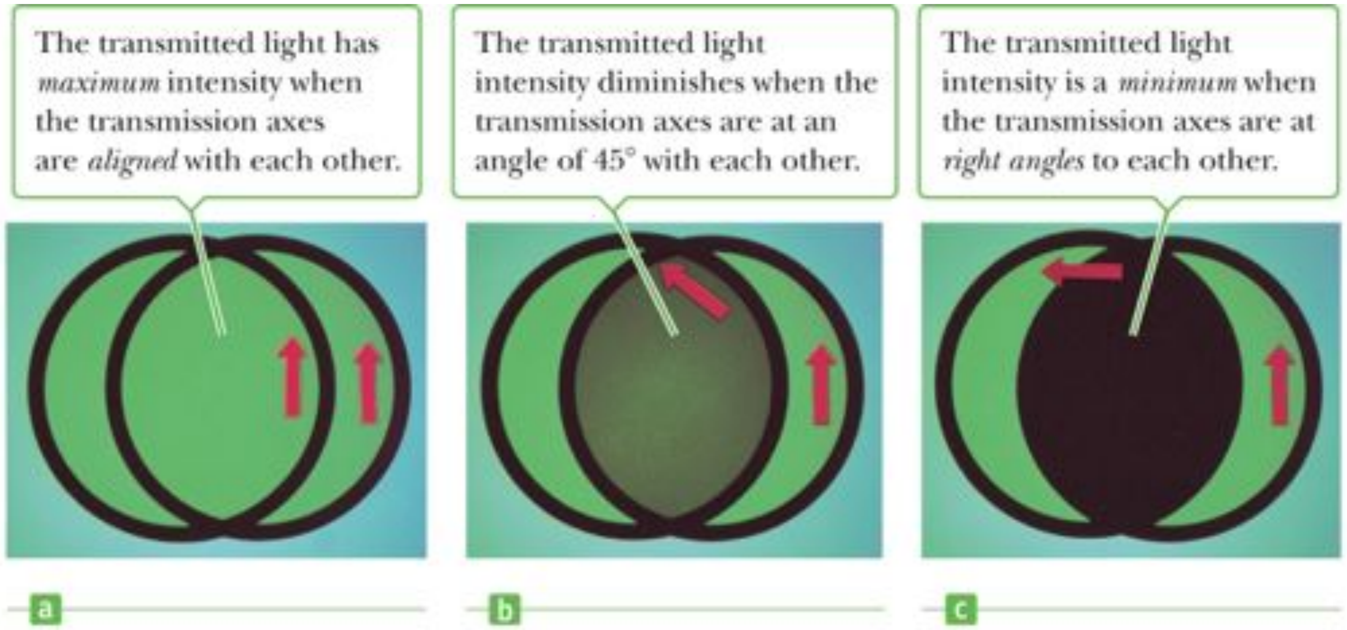
- E. H. Land discovered a material that polarizes light through selective absorption.
  - He called the material **Polaroid.**
  - The molecules readily absorb light whose electric field vector is parallel to their lengths and transmit light whose electric field vector is perpendicular to their lengths.



The polarizer polarizes the incident light along its transmission axis.

The analyzer allows the component of the light parallel to its axis to pass through.

Fig. 24-26, p. 843



The transmitted light has *maximum* intensity when the transmission axes are *aligned* with each other.

The transmitted light intensity diminishes when the transmission axes are at an angle of 45° with each other.

The transmitted light intensity is a *minimum* when the transmission axes are at *right angles* to each other.

Fig. 24-27, p. 844

Selective Absorption, Final

- The intensity of the polarized beam transmitted through the second polarizing sheet (the analyzer) varies as
  - $I = I_o \cos^2 \theta$
    - I_o is the intensity of the polarized wave incident on the analyzer.
    - This is known as **Malus' Law** and applies to any two polarizing materials whose transmission axes are at an angle of θ to each other.

Polarization by Reflection

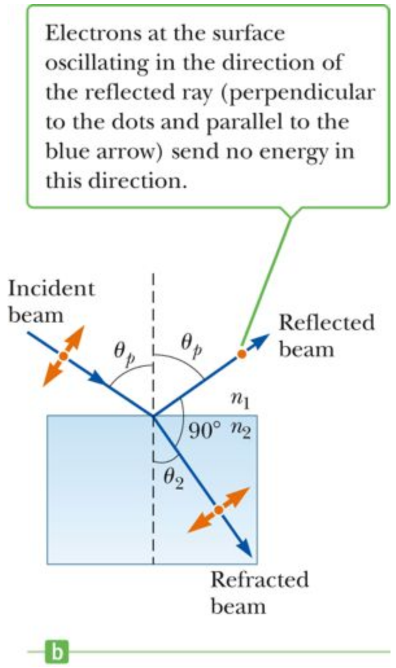
- When an unpolarized light beam is reflected from a surface, the reflected light is
  - Completely polarized
  - Partially polarized
  - Unpolarized
- It depends on the angle of incidence.
  - If the angle is 0° or 90°, the reflected beam is unpolarized.
  - For angles between this, there is some degree of polarization.
  - For one particular angle, the beam is completely polarized.

Polarization by Reflection, Cont.

- The angle of incidence for which the reflected beam is completely polarized is called the *polarizing angle*, θ_p.
- Brewster's Law relates the polarizing angle to the index of refraction for the material.

$$n = \frac{\sin\theta_p}{\cos\theta_p} = \tan\theta_p$$

- θ_p may also be called Brewster's Angle.


Electrons at the surface oscillating in the direction of the reflected ray (perpendicular to the dots and parallel to the blue arrow) send no energy in this direction.

Polarization by Reflection, Cont.

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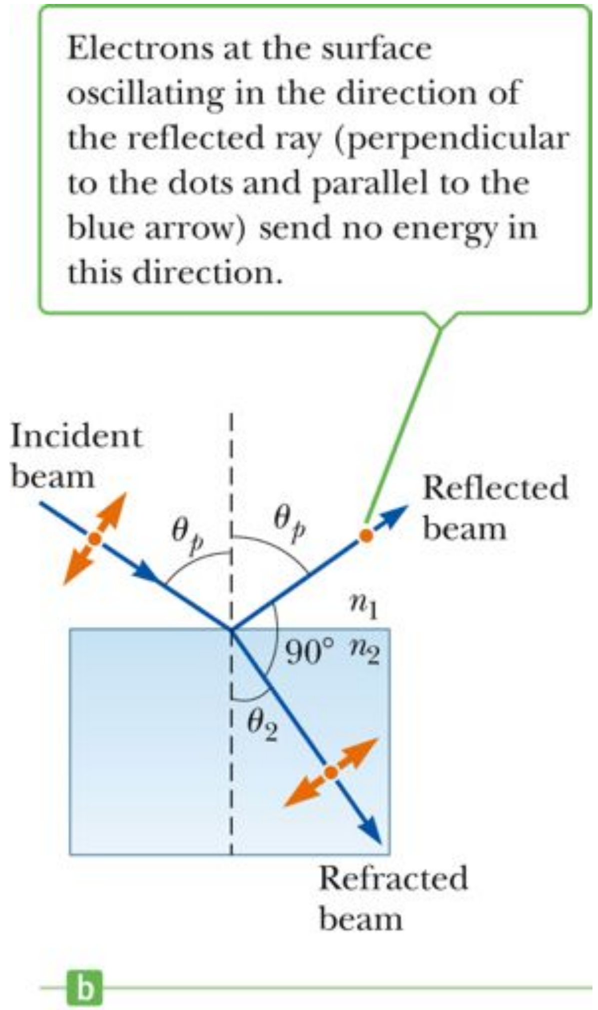


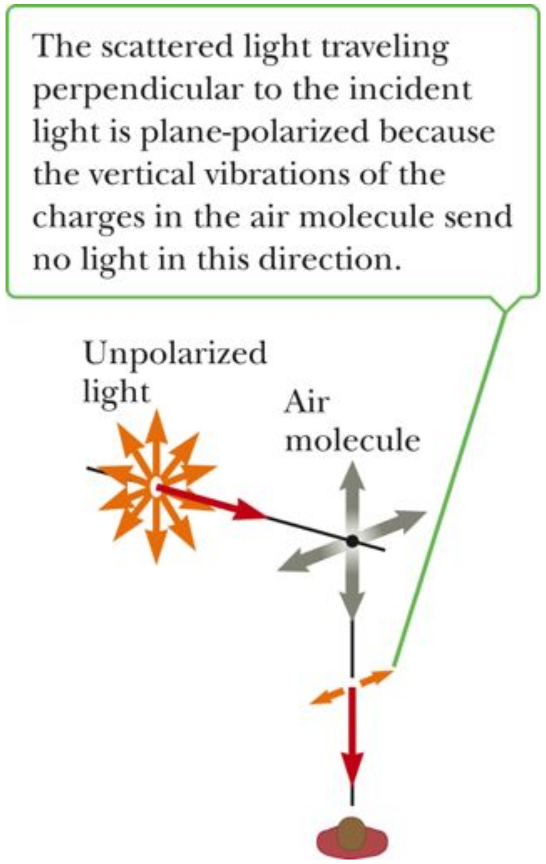
Electrons at the surface oscillating in the direction of the reflected ray (perpendicular to the dots and parallel to the blue arrow) send no energy in this direction.

Polarization by Scattering

- When light is incident on a system of particles, the electrons in the medium can absorb and reradiate part of the light.
  - This process is called **scattering.**
- An example of scattering is the sunlight reaching an observer on the earth becoming polarized.

Polarization by Scattering, Cont.

- The horizontal part of the electric field vector in the incident wave causes the charges to vibrate horizontally.
- The vertical part of the vector simultaneously causes them to vibrate vertically.
- Horizontally and vertically polarized waves are emitted.



The scattered light traveling perpendicular to the incident light is plane-polarized because the vertical vibrations of the charges in the air molecule send no light in this direction.

Hw 52 & 53 & 59

How far above the horizon is the moon when its image reflected in calm water is completely polarized? (n_{water} = 1.333)
a.53.12°
b.18.44°
c.22.20°
d.36.88°

Unpolarized light is passed through polarizer 1. The light then goes though polarizer 2 with its plane of polarization at 45.0° to that of polarizer 1. What fraction of the intensity of the original light gets though the second polarizer?
a.0.707
b.0.500
c.0.250
d.0.125

If the polarizing angle for diamond is 67.5°, what is the index of refraction of this material?
a.2.00
b.2.20
c.2.41
d.2.65

Key Concepts

- Interference
- Diffraction
- Polarization

Key Equations

$$dsin(\theta)_{bright} = m\lambda \rightarrow y_{bright} = \frac{\lambda L}{d} m$$

$$dsin(\theta)_{dark} = (m + 1/2)\lambda \rightarrow y_{bright} = \frac{\lambda L}{d}(m + 1/2)$$

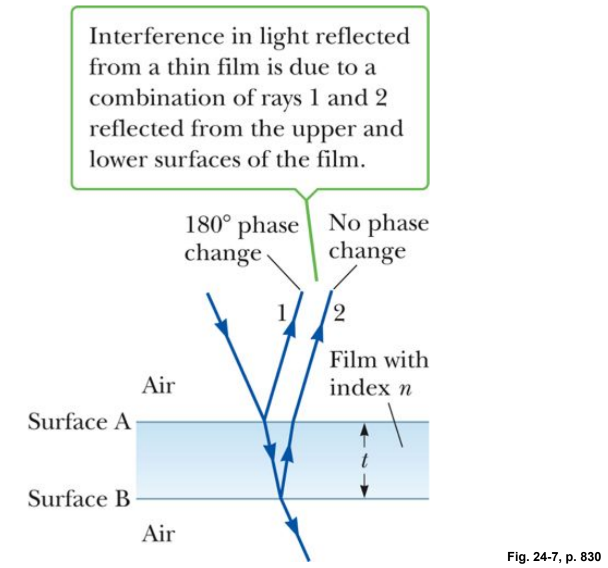

Interference in light reflected from a thin film is due to a combination of rays 1 and 2 reflected from the upper and lower surfaces of the film.

Fig. 24-7, p. 830

$2nt = (m + 1/2)\lambda$ 1 Con 0/2 Des

$2nt = (m)\lambda$ 1 Des 0/2 Con

$$sin(\theta_{dark}) = m\frac{\lambda}{a}$$

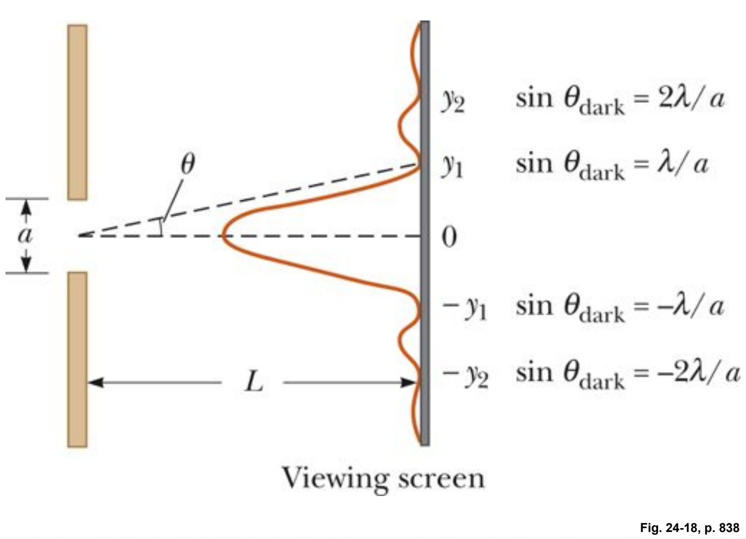


Fig. 24-18, p. 838

$dsin(\theta_{bright}) = m\lambda$ Diffraction grating

$I = I_0 cos^2(\theta)$ Malus Law $\qquad n = tan(\theta_p)$