

# Lecture 8

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## Chapter 23 Mirrors and Lenses

# You may skip

- \* 23.3 Convex mirrors
- \* Combinations of thin lenses

# Mirrors and Lenses

- The development of mirrors and lenses aided the progress of science.
- It led to the microscopes and telescopes.
  - Allowed the study of objects from microbes to distance planets
- Images can be formed by reflection from mirrors.
- Images can be formed by refraction through lenses.
- Will look at the formation of these images
- Will continue to use the ray approximation

Introduction

# Notation for Mirrors and Lenses

- The *object distance* is the distance from the object to the mirror or lens.
  - Denoted by  $p$
- The *image distance* is the distance from the image to the mirror or lens.
  - Images are formed at the point where rays actually intersect or appear to originate.
  - Denoted by  $q$
- The *lateral magnification* of the mirror or lens is the ratio of the image height to the object height.
  - Denoted by  $M$

# Types of Images

- Images are classified as real or virtual.
- **Real images** are formed at the point the rays of light actually intersect.
  - Real images can be displayed on screens.
- **Virtual images** are formed at the point the rays of light appear to originate.
  - The light appears to diverge from that point.
  - Virtual images cannot be displayed on screens.

Section 23.1



# More About Images

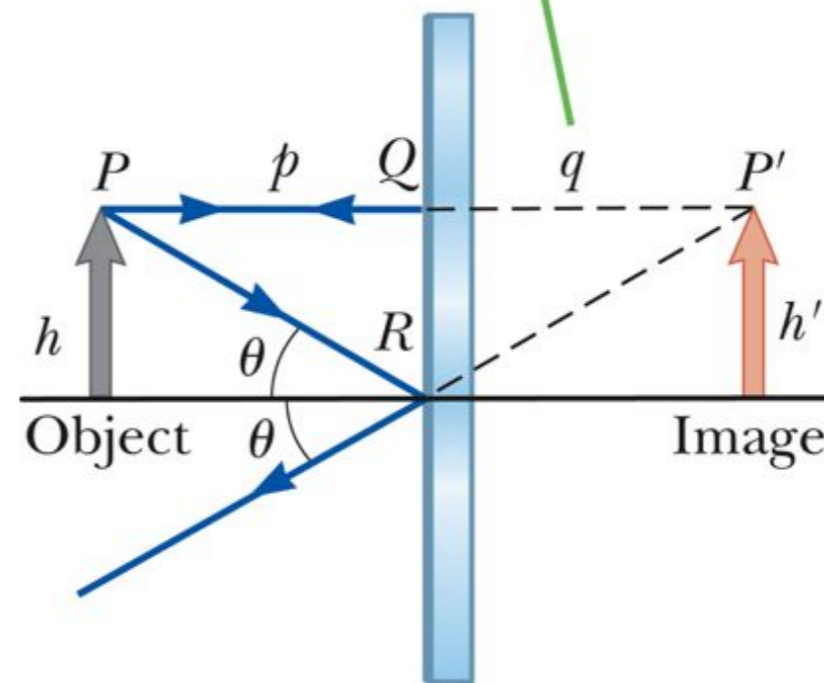
- To find where an image is formed, it is always necessary to follow at least two rays of light as they reflect from the mirror.

Section 23.1

# Flat Mirror

- Properties of the image can be determined by geometry.
- One ray starts at  $P$ , follows path  $PQ$  and reflects back on itself.
- A second ray follows path  $PR$  and reflects according to the Law of Reflection.

Because the triangles  $PQR$  and  $P'QR$  are identical,  $p = |q|$  and  $h = h'$ .



Section 23.1

# Magnification

- The lateral magnification is defined as

$$M = \frac{\text{image height}}{\text{object height}} = \frac{h'}{h}$$

- Magnification doesn't always mean enlargement.
  - The image can be smaller than the object.



## Properties of the Image Formed by a Flat Mirror

- The image is as far behind the mirror as the object is in front.
  - $p = |q|$
- The image is unmagnified.
  - The image height is the same as the object height.
  - $h' = h$  and  $M = 1$

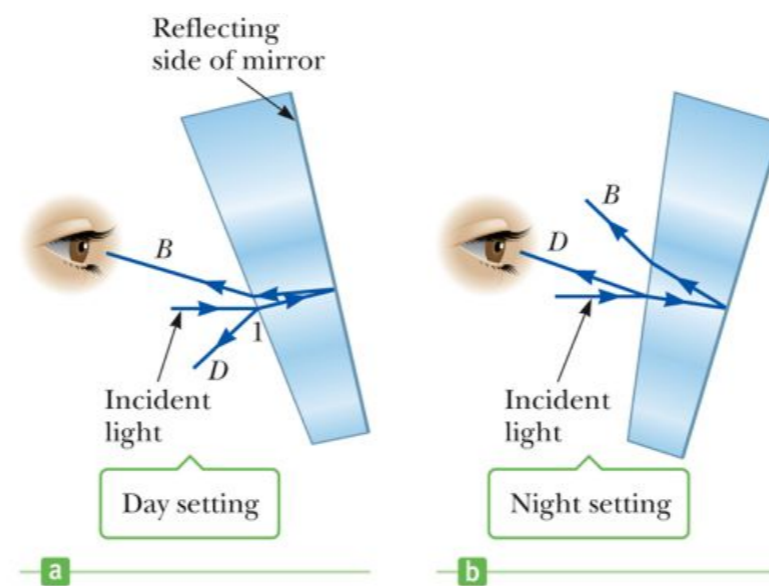
Section 23.1

## More Image Properties – Flat Mirror

- The image is virtual.
- The image is upright.
  - It has the same orientation as the object.
- There is an apparent left-right reversal in the image.

Section 23.1

## Application – Day and Night Settings on Auto Mirrors



- With the daytime setting, the bright beam of reflected light is directed into the driver's eyes.
- With the nighttime setting, the dim beam of reflected light is directed into the driver's eyes, while the bright beam goes elsewhere.

Section 23.1

# Hw 2

You stand two feet away from a plane mirror. How far is it from you to your image?

- a. 2.0 ft
- b. 3.0 ft
- c. 4.0 ft
- d. 5.0 ft

How large should a wall-mounted mirror be to view the upper half of one's height,  $h$ ?

- a.  $h$
- b.  $h/2$
- c.  $h/4$
- d. The answer is not given.



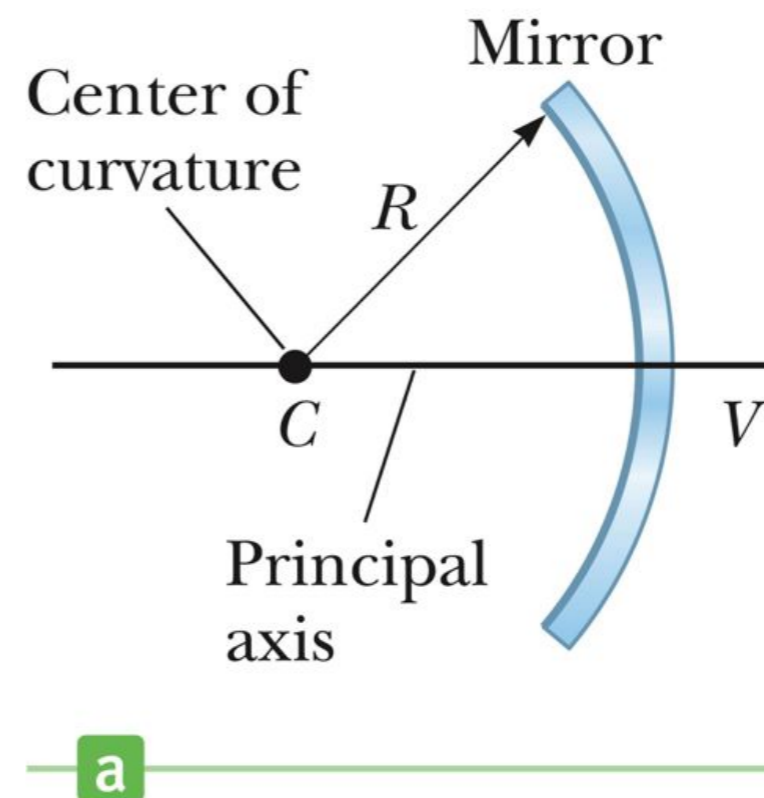
# Spherical Mirrors

- A *spherical mirror* has the shape of a segment of a sphere.
- A *concave spherical mirror* has the silvered surface of the mirror on the inner, or concave, side of the curve.
- A *convex spherical mirror* has the silvered surface of the mirror on the outer, or convex, side of the curve.

Section 23.2

## Concave Mirror, Notation

- The mirror has a *radius of curvature* of  $R$ .
- Its *center of curvature* is the point  $C$ .
- Point  $V$  is the center of the spherical segment.
- A line drawn from  $C$  to  $V$  is called the *principle axis* of the mirror.

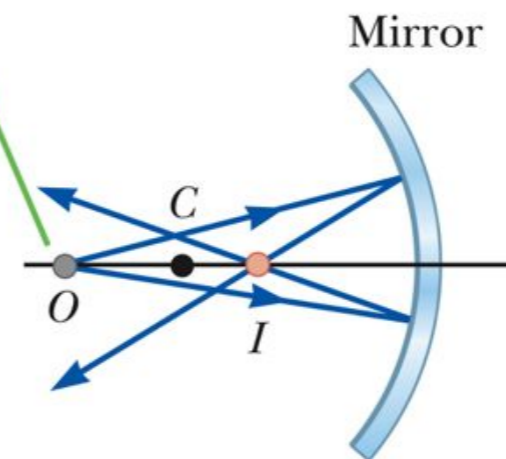


Section 23.2

# Concave Mirror, Image

- A point source of light is placed at  $O$ .
- Rays are drawn from  $O$ .
- After reflecting from the mirror, the rays converge at point  $I$ .
- Point  $I$  is called the Image point.
- Light actually passes through the point so the image is real.

If the rays diverge from  $O$  at small angles, they all reflect through the same image point at  $I$ .

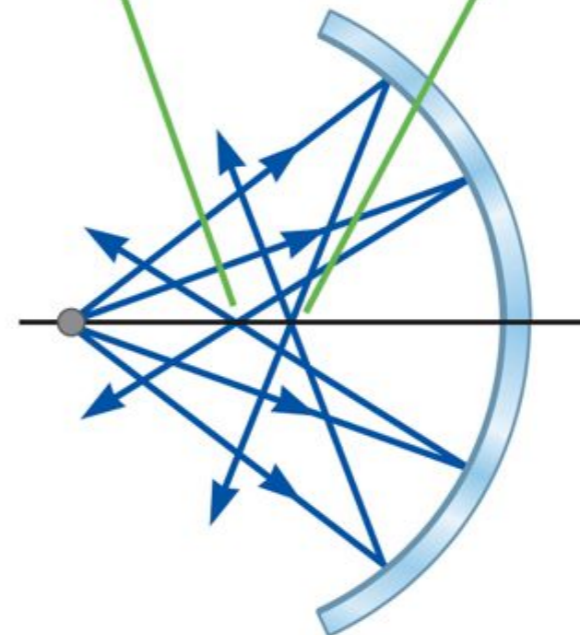


Section 23.2

# Spherical Aberration

- Rays are generally assumed to make small angles with the mirror.
- When the rays make large angles, they may converge to points other than the image point.
- This results in a blurred image.
- This effect is called **spherical aberration**.

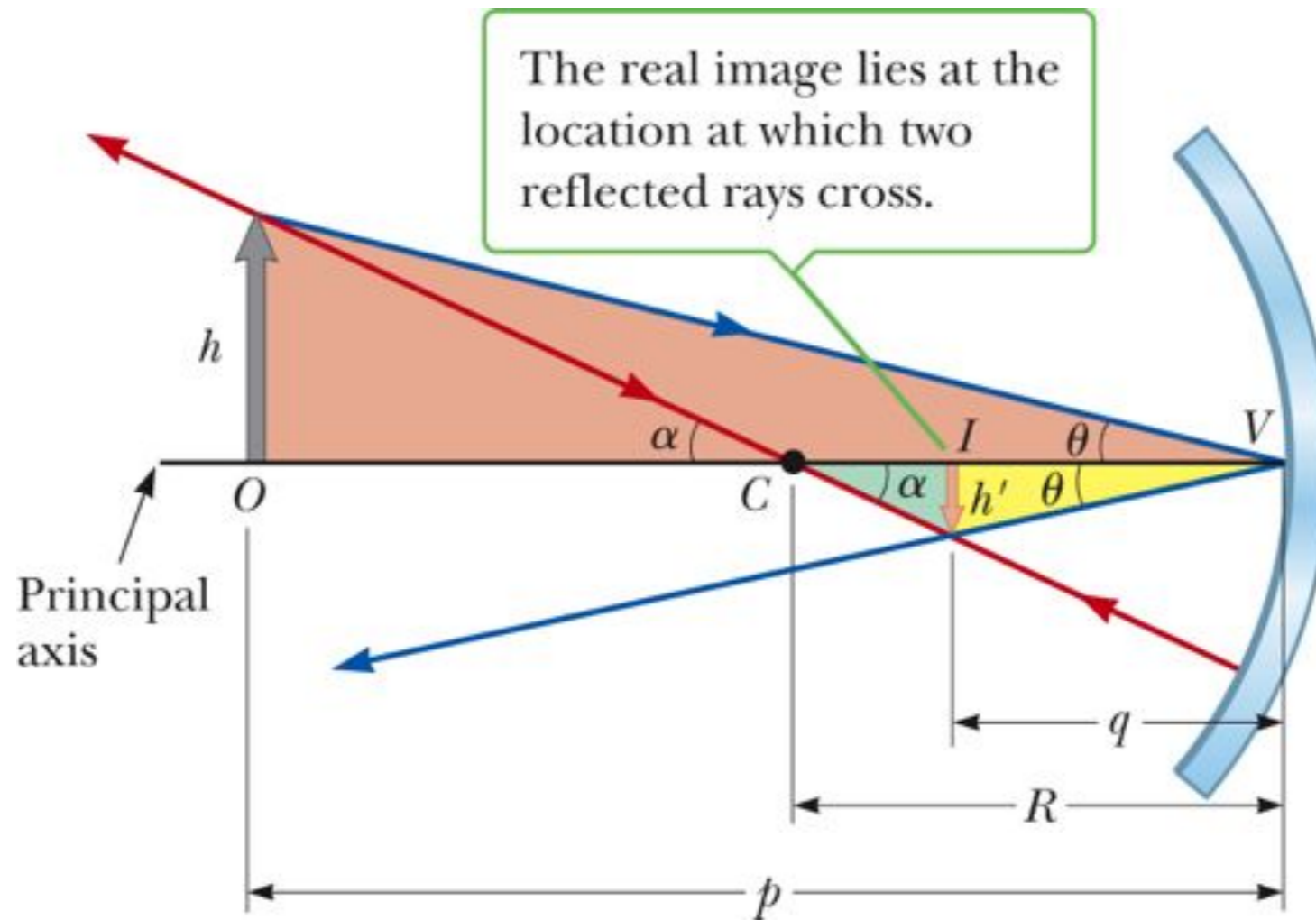
The reflected rays intersect at different points on the principal axis.



Section 23.2



# Image Formed by a Concave Mirror



Section 23.2



## Image Formed by a Concave Mirror, Equations

- Geometry can be used to determine the magnification of the image.

$$M = \frac{h'}{h} = -\frac{q}{p}$$

–  $h'$  is negative when the image is inverted with respect to the object.

- Geometry also shows the relationship between the image and object distances.

$$\frac{1}{p} + \frac{1}{q} = \frac{2}{R}$$

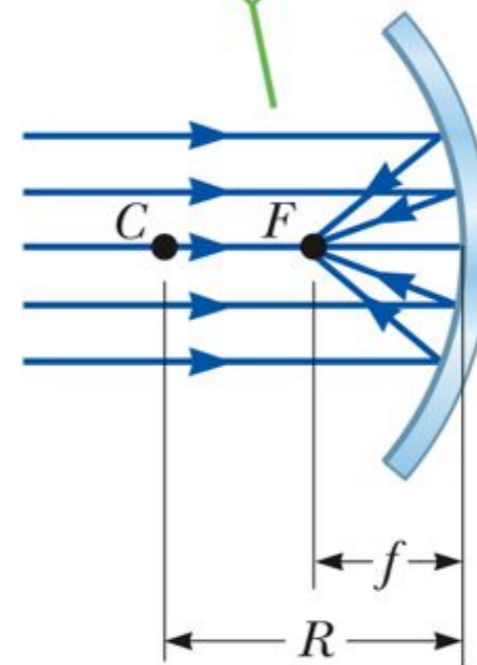
– This is called the **mirror equation**.

Section 23.2

# Focal Length

- If an object is very far away, then  $p = \infty$  and  $1/p = 0$ .
- Incoming rays are essentially parallel.
- In this special case, the image point is called the *focal point*.
- The distance from the mirror to the focal point is called the *focal length*.
  - The focal length is  $\frac{1}{2}$  the radius of curvature.

For  $p$  approaching infinity,  $q$  approaches  $R/2 = f$ , the focal length of the mirror.



a

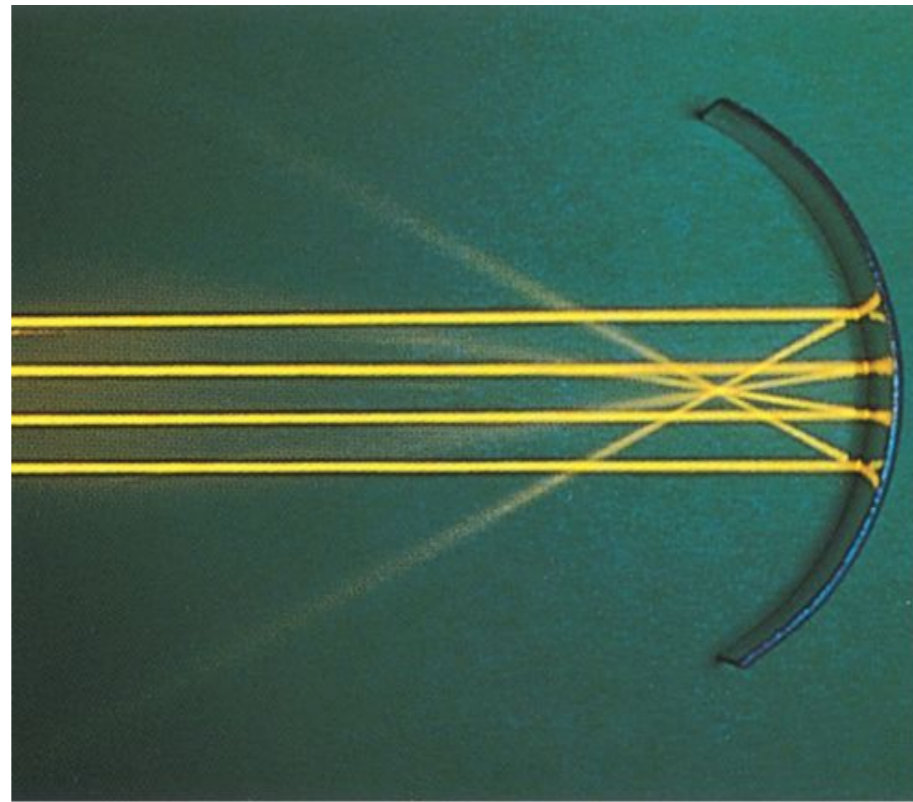
# Focal Point and Focal Length, Cont.

- The focal point is dependent solely on the curvature of the mirror, not by the location of the object.
- $f = R / 2$
- The mirror equation can be expressed as

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

Section 23.2

# Focal Length Shown by Parallel Rays



b

Section 23.2

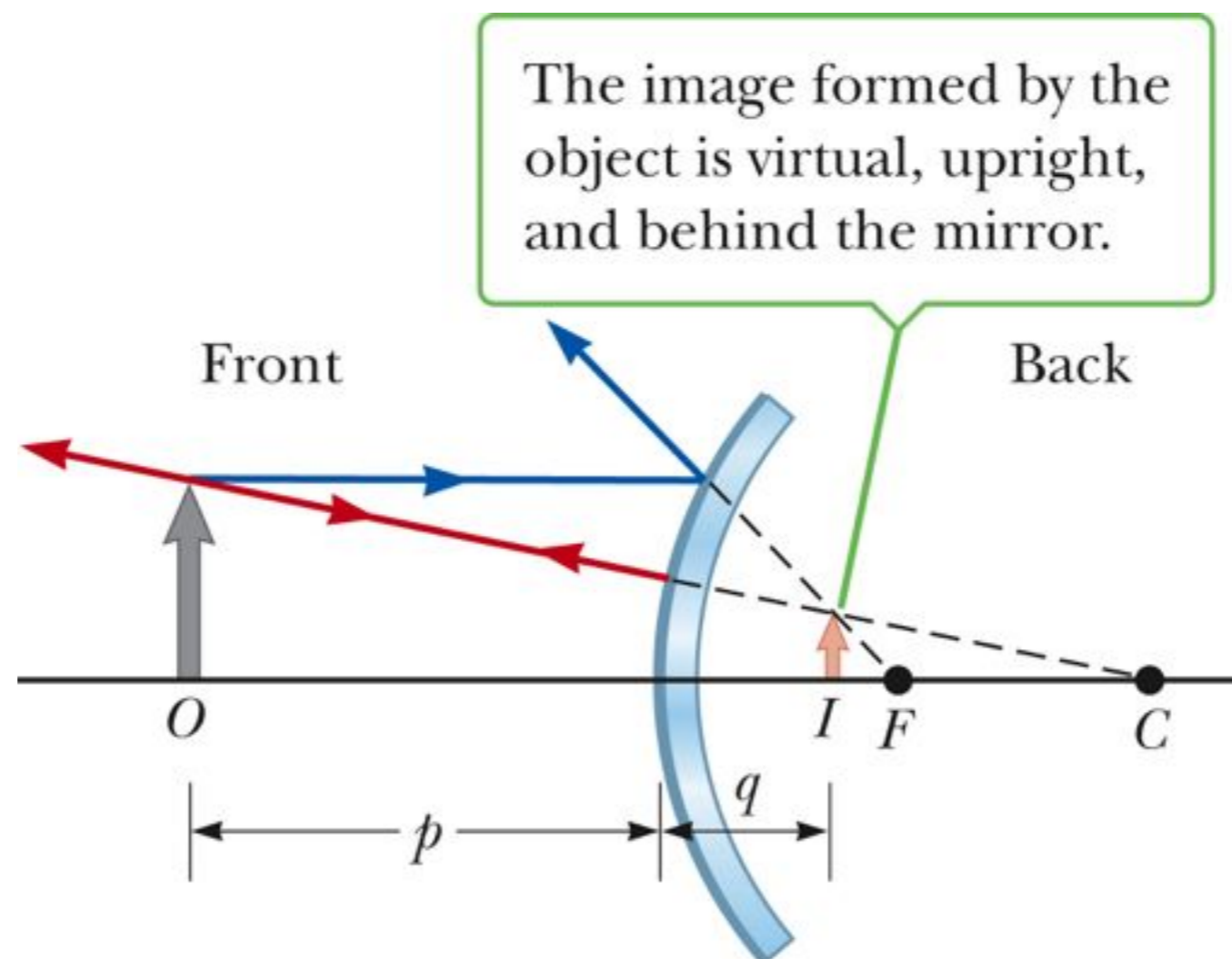
# Convex Mirrors

- A convex mirror is sometimes called a **diverging** mirror.
- The rays from any point on the object diverge after reflection as though they were coming from some point behind the mirror.
- The image is virtual because it lies behind the mirror at the point where the reflected rays appear to originate.
- In general, the image formed by a convex mirror is upright, virtual, and smaller than the object.

Section 23.3



# Image Formed by a Convex Mirror



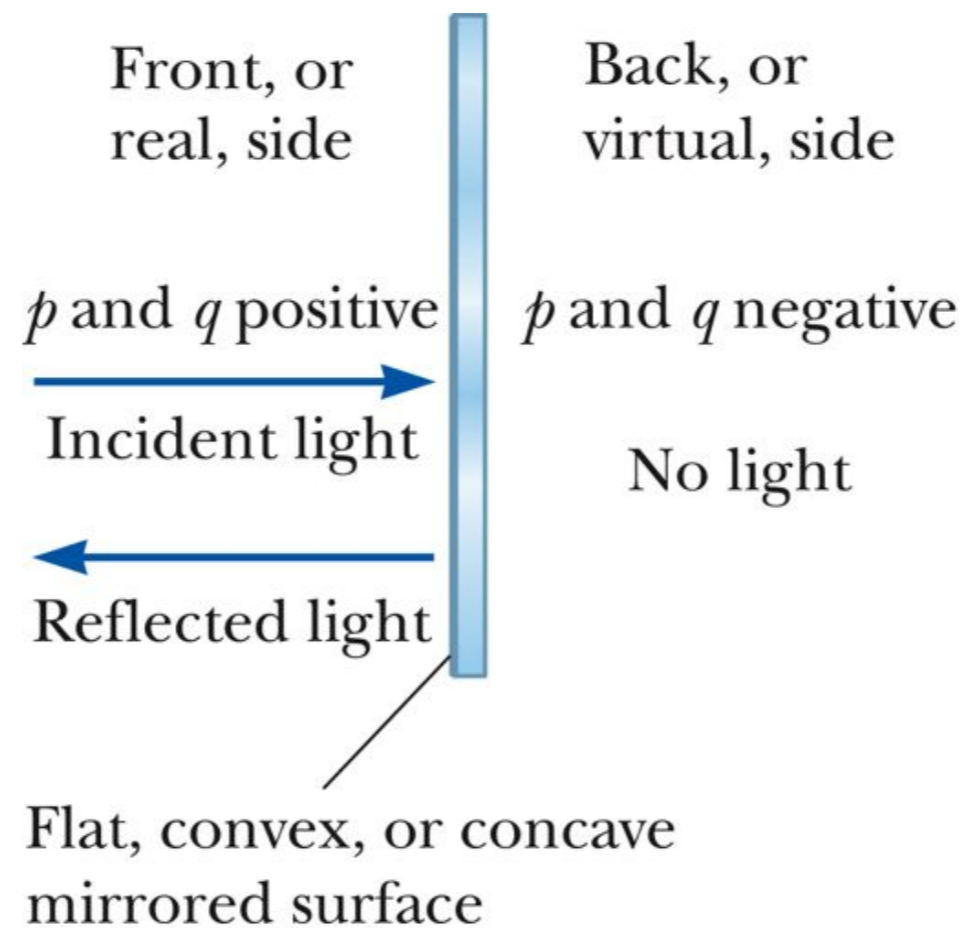
Section 23.3

# Convex Mirror, Equations

- The equations for convex mirrors are the same as for concave mirrors.
  - Need to use sign conventions
- A positive sign is used where the light is
  - In front (the front side) of the mirror
- A negative sign is used behind the mirror.
  - The back side
  - Where virtual images are formed

Section 23.3

# Diagram for Signs



Section 23.3

# Sign Conventions for Mirrors

**Table 23.1** Sign Conventions for Mirrors

Quantity	Symbol	In Front	In Back	Upright Image	Inverted Image
Object location	$p$	+	-		
Image location	$q$	+	-		
Focal length	$f$	+	-		
Image height	$h'$			+	-
Magnification	$M$			+	-

# Ray Diagrams

- A *ray diagram* can be used to determine the position and size of an image.
- They are graphical constructions which tell the overall nature of the image.
- They can also be used to check the parameters calculated from the mirror and magnification equations.



# Drawing A Ray Diagram

- To make the ray diagram, you need to know
  - The position of the object
  - The position of the center of curvature
- Three rays are drawn
  - They all start from the same position on the object
- The intersection of any two of the rays at a point locates the image.
  - The third ray serves as a check of the construction.

# The Rays in a Ray Diagram

- Ray 1 is drawn parallel to the principle axis and is reflected back through the focal point, F.
- Ray 2 is drawn through the focal point and is reflected parallel to the principle axis.
- Ray 3 is drawn through the center of curvature and is reflected back on itself.

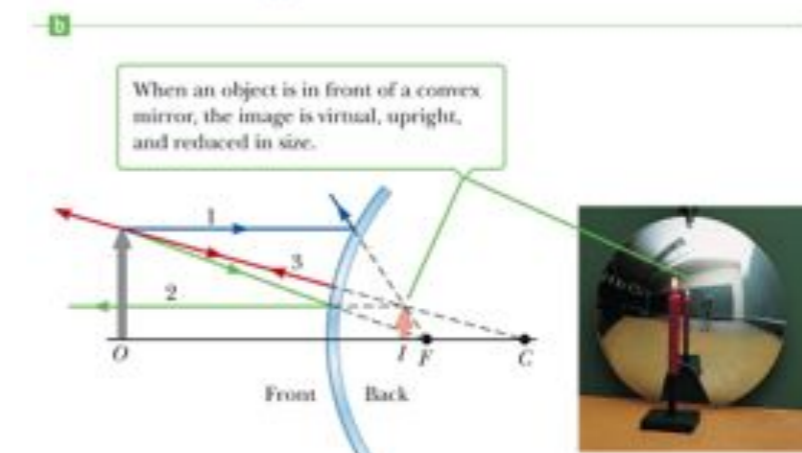
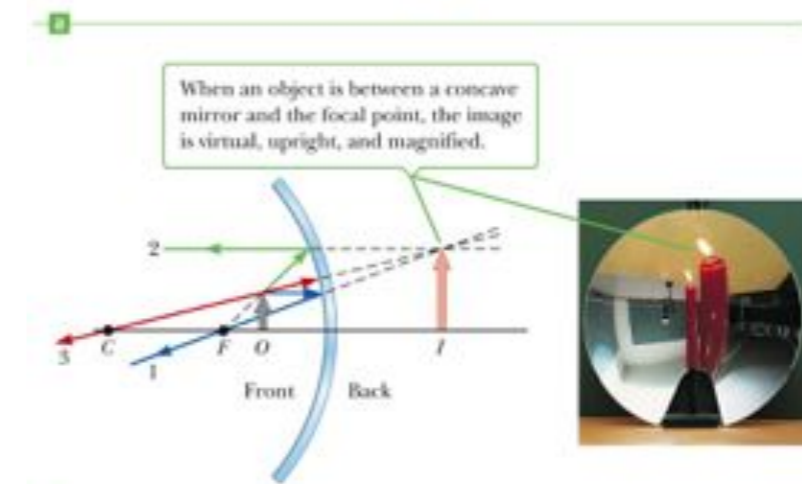
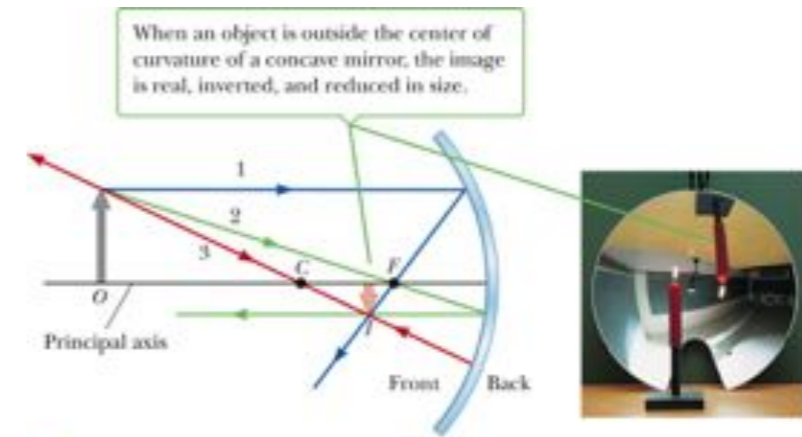
## Notes About the Rays

- The rays actually go in all directions from the object.
- The three rays were chosen for their ease of construction.
- The image point obtained by the ray diagram must agree with the value of  $q$  calculated from the mirror equation.

Section 23.3

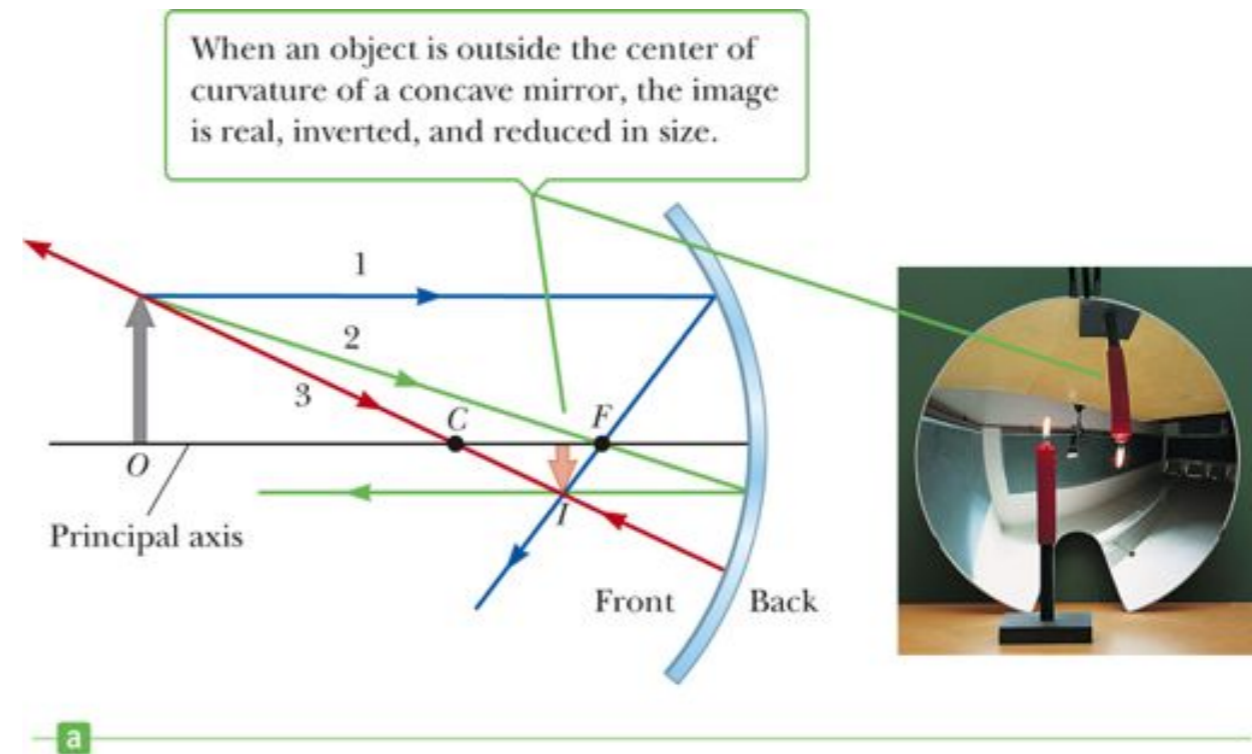
# Ray Diagram Examples

- Note the changes in the image as the object moves through the focal point.



# Ray Diagram for Concave Mirror, $p > R$

- The object is outside the center of curvature of the mirror.
- The image is real.
- The image is inverted.
- The image is smaller than the object.

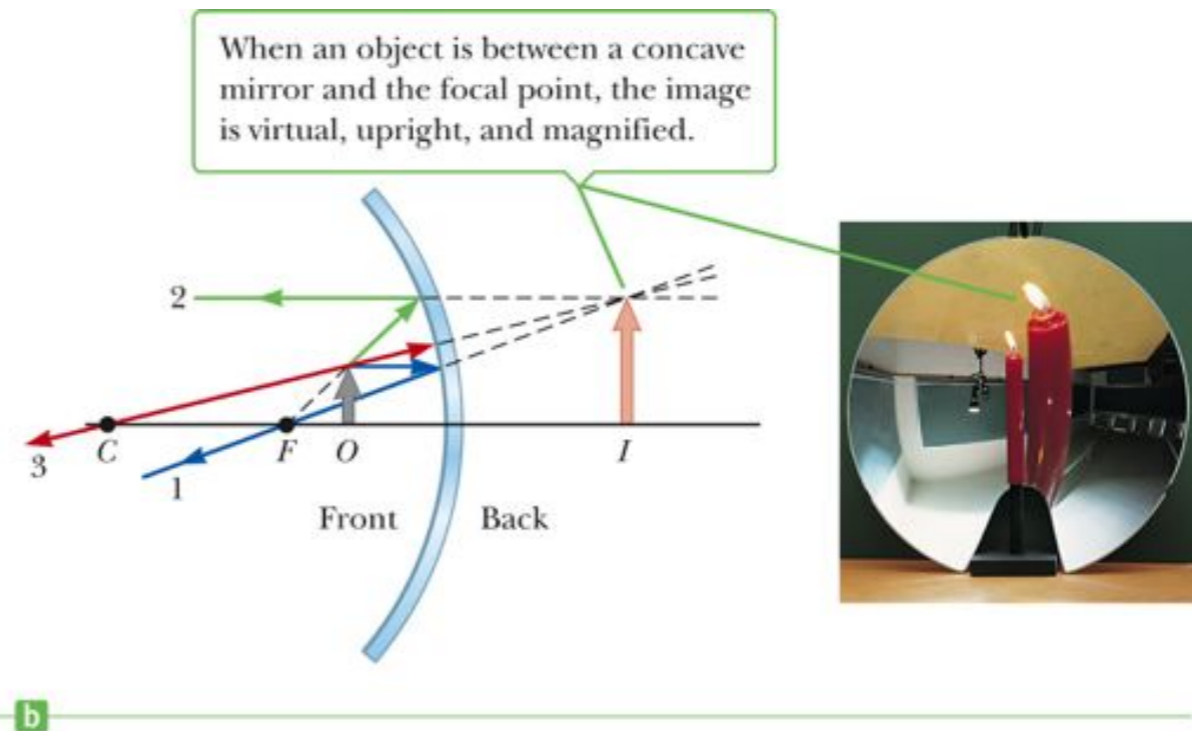


Section 23.3



# Ray Diagram for a Concave Mirror, $p < f$

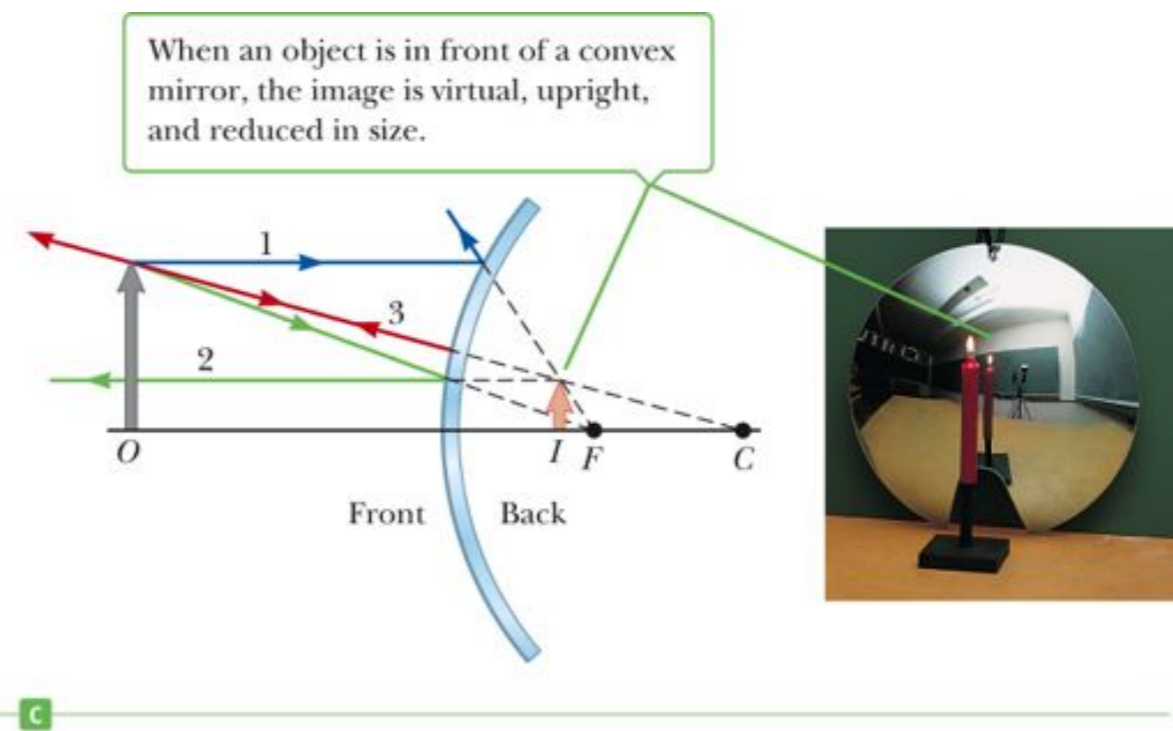
- The object is between the mirror and the focal point.
- The image is virtual.
- The image is upright.
- The image is larger than the object.



Section 23.3

# Ray Diagram for a Convex Mirror

- The object is in front of a convex mirror.
- The image is virtual.
- The image is upright.
- The image is smaller than the object.



Section 23.3

# Notes on Images

- With a concave mirror, the image may be either real or virtual.
  - When the object is outside the focal point, the image is real.
  - When the object is at the focal point, the image is infinitely far away.
  - When the object is between the mirror and the focal point, the image is virtual.
- With a convex mirror, the image is always virtual and upright.
  - As the object distance increases, the virtual image gets smaller.

Hw 9, 13, & 17

The real image of an object is located 45.0 cm away from a concave mirror, which has a focal length of 10.0 cm. How far is the object from the mirror?

- a. 40.0 cm
- b. 35.0 cm
- c. 22.5 cm
- d. 12.9 cm

A concave mirror forms a real image at 25.0 cm from the mirror surface along the principal axis. If the corresponding object is at a 10.0-cm distance, what is the mirror's focal length?

- a. 1.43 cm
- b. 16.7 cm
- c. 12.4 cm
- d. 7.14 cm



Which best describes the image of a concave mirror when the object is located somewhere between the focal point and twice the focal point distance from the mirror?

- a.virtual, upright and magnification greater than one
- b.real, inverted and magnification less than one
- c.virtual, upright and magnification less than one
- d.real, inverted and magnification greater than one

Which of the following best describes the image of a concave mirror when the object's distance from the mirror is less than the focal point distance?

- a.virtual, upright and magnification greater than one
- b.real, inverted and magnification less than one
- c.virtual, upright and magnification less than one
- d.real, inverted and magnification greater than one

Which of the following best describes the image of a convex mirror when the object's distance from the mirror is less than the absolute value of the focal point distance?

- a.virtual, upright and magnification greater than one
- b.real, inverted and magnification less than one
- c.virtual, upright and magnification less than one
- d.real, inverted and magnification greater than one

A convex mirror with focal length of 20 cm forms an image 12 cm behind the surface. Where is the object located as measured from the surface?

- a. 7.5 cm
- b. 15 cm
- c. 22 cm
- d. 30 cm

An object is held at a distance of 12 cm from a convex mirror creating an image that is  $\frac{1}{3}$  the object size. What is the focal length of the mirror?

- a. -6.0 cm
- b. -3.0 cm
- c. -9.0 cm
- d. -18 cm

# Images Formed by Refraction

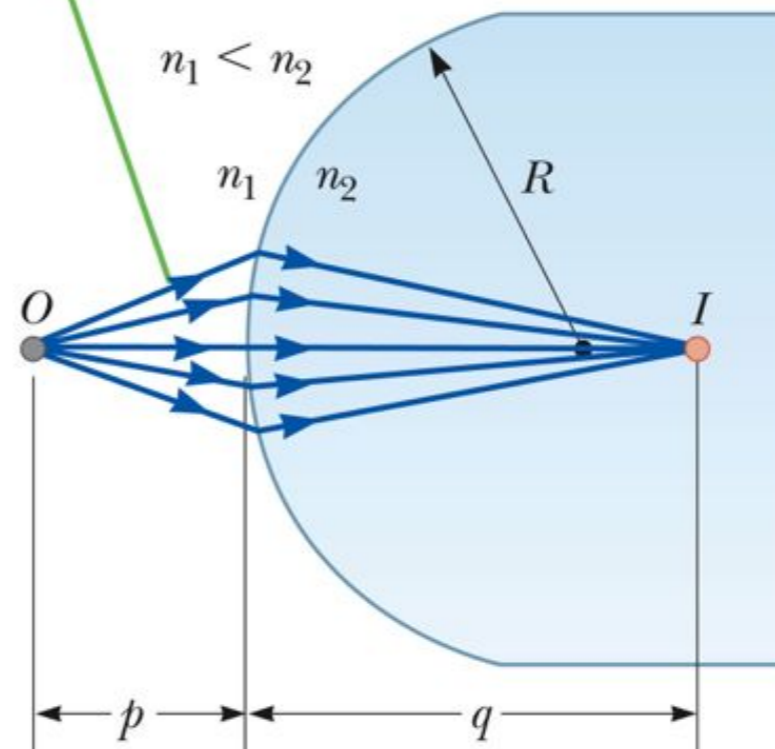
- Rays originate from the object point,  $O$ , and pass through the image point,  $I$
- When  $n_2 > n_1$ ,

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$$

$$M = \frac{h'}{h} = -\frac{n_1 q}{n_2 p}$$

- Real images are formed on the side opposite from the object.

Rays making small angles with the principal axis diverge from a point object at  $O$  and pass through the image point  $I$ .



# Sign Conventions for Refracting Surfaces

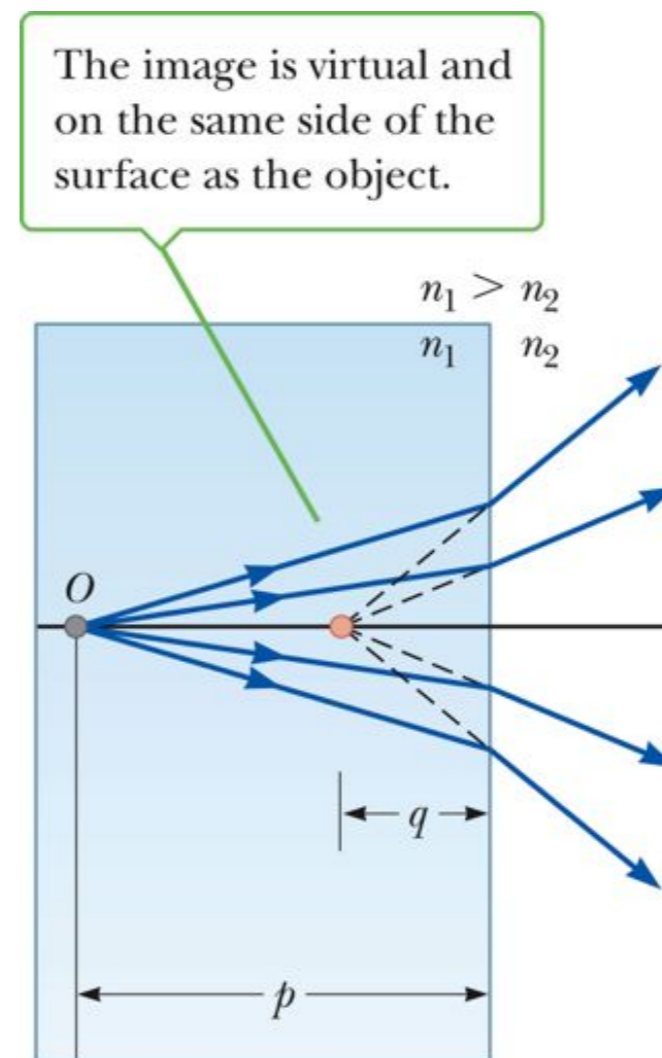
**Table 23.2** Sign Conventions for Refracting Surfaces

Quantity	Symbol	In Front	In Back	Upright Image	Inverted Image
Object location	$p$	+	-		
Image location	$q$	-	+		
Radius	$R$	-	+		
Image height	$h'$			+	-



# Flat Refracting Surface

- The image formed by a flat refracting surface is on the *same* side of the surface as the object.
  - The image is *virtual*.
  - The image forms between the object and the surface.
  - The rays bend away from the normal since  $n_1 > n_2$



Section 23.4



# Hw 21 & 24

Ron fills a beaker with glycerin ( $n = 1.473$ ) to a depth of 5.0 cm. If he looks straight down through the glycerin surface, he will perceive the liquid to be what apparent depth?

- a. 7.4 cm
- b. 5.0 cm
- c. 3.4 cm
- d. 1.0 cm

A solid glass sphere with a radius of 5.00 cm and index of refraction of 1.52 has a small coin embedded 3.00 cm from the front surface of the sphere. For the viewer looking at the coin through the glass, at what distance from the front surface of the glass does the coin's image appear to be located?

- a. 2.48 cm
- b. 3.20 cm
- c. 5.00 cm
- d. 6.85 cm

A glass block, for which  $n = 1.52$ , has a blemish located 3.2 cm from one surface. At what distance from that surface does the image of the blemish appear to the outside observer?

- a. 1.6 cm
- b. 2.1 cm
- c. 4.9 cm
- d. 6.4 cm

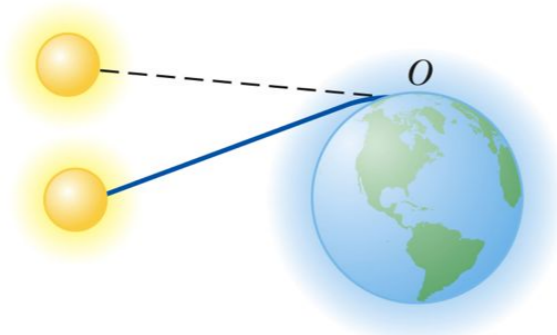
## Atmospheric Refraction

- There are many interesting results of refraction in the atmosphere.
  - Sun's position
  - Mirages

Section 23.5

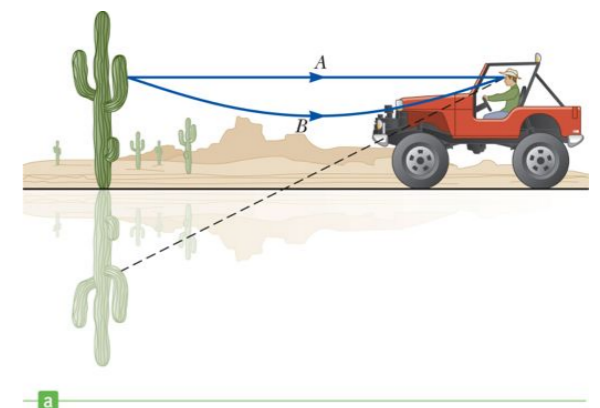
### Atmospheric Refraction and Sun's Position

- Light rays from the sun are bent as they pass into the atmosphere.
- It is a gradual bend because the light passes through layers of the atmosphere.
  - Each layer has a slightly different index of refraction.
- The Sun is seen to be above the horizon even after it has



### Atmospheric Refraction and Mirages

- A mirage can be observed when the air above the ground is warmer than the air at higher elevations.
- The rays in path B are directed toward the ground and then bent by refraction.
- The observer sees both an upright and an inverted image.



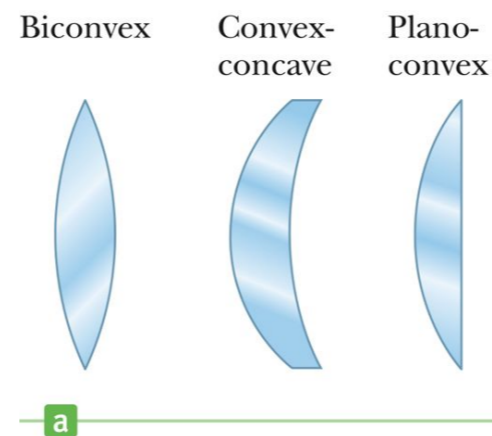
# Thin Lenses

- A thin lens consists of a piece of glass or plastic, ground so that each of its two refracting surfaces is a segment of either a sphere or a plane.
- Lenses are commonly used to form images by refraction in optical instruments.

Section 23.6



# Thin Lens Shapes

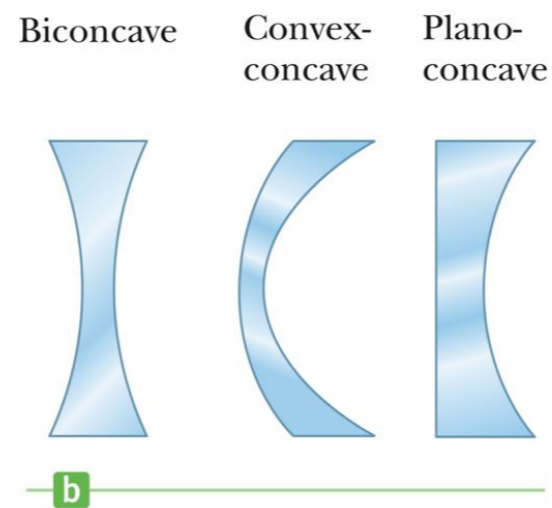


- These are examples of *converging* lenses.
- They have positive focal lengths.
- They are thickest in the middle.

Section 23.6



# More Thin Lens Shapes



- These are examples of *diverging* lenses.
- They have negative focal lengths.
- They are thickest at the edges.

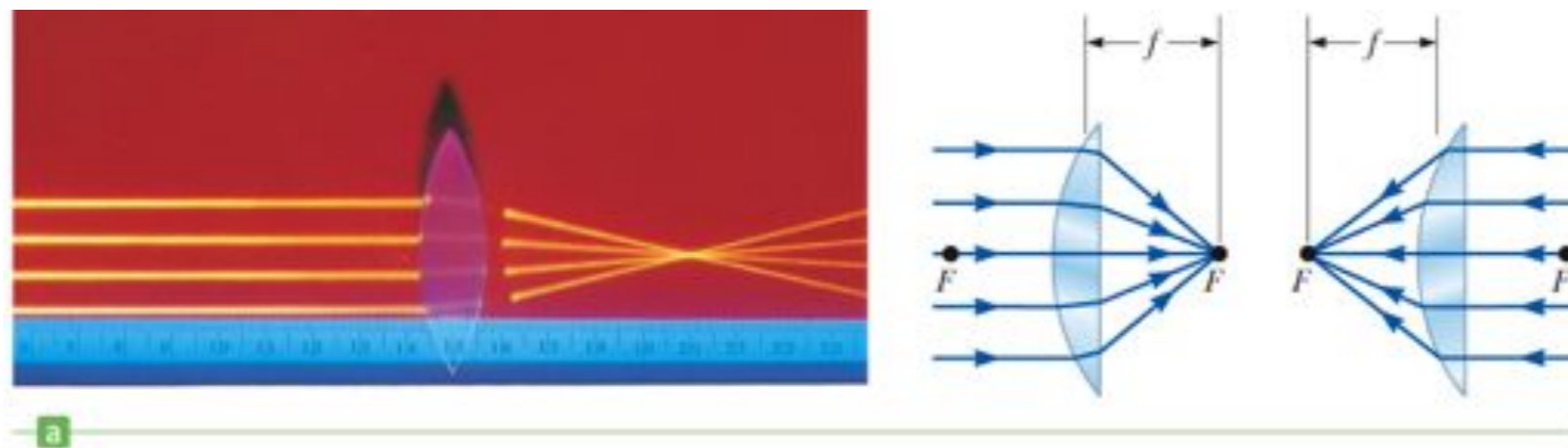
Section 23.6

# Focal Length of Lenses

- The focal length,  $f$ , is the image distance that corresponds to an infinite object distance.
  - This is the same as for mirrors.
- A thin lens has two focal points, corresponding to parallel rays from the left and from the right.
  - A thin lens is one in which the distance between the surface of the lens and the center of the lens is negligible.

Section 23.6

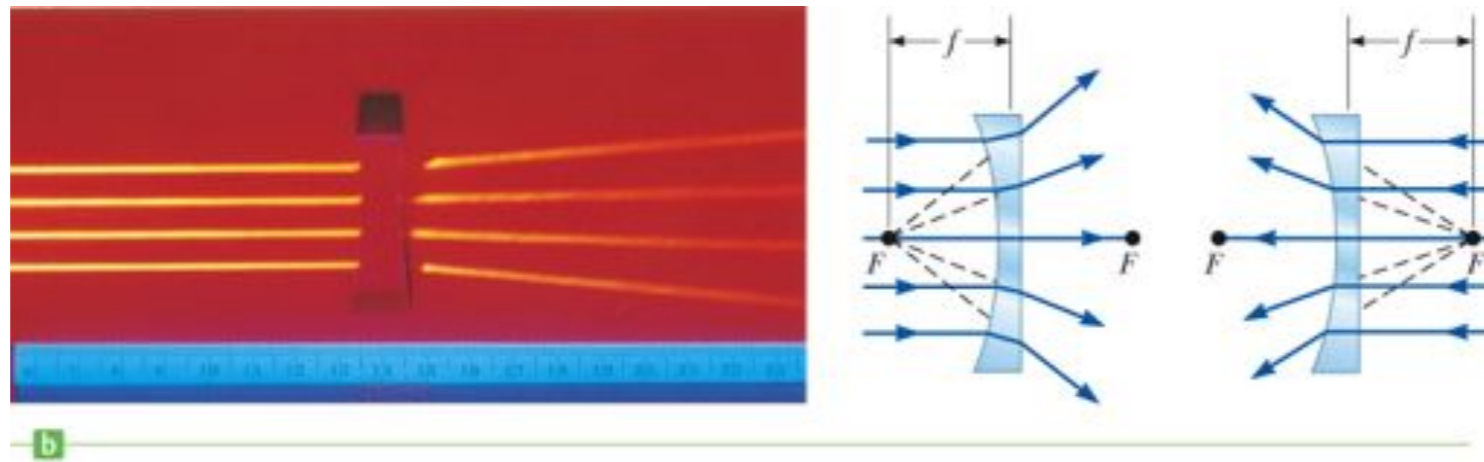
# Focal Length of a Converging Lens



- The parallel rays pass through the lens and converge at the focal point.
- The parallel rays can come from the left or right of the lens.

Section 23.6

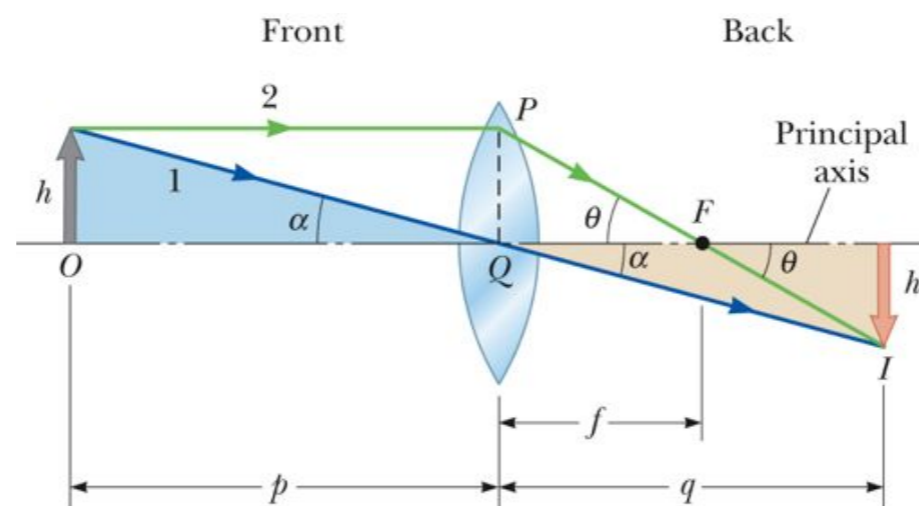
# Focal Length of a Diverging Lens



- The parallel rays diverge after passing through the diverging lens.
- The focal point is the point where the rays appear to have originated.

Section 23.6

# Lens Equations



- The geometric derivation of the equations is very similar to that of mirrors.

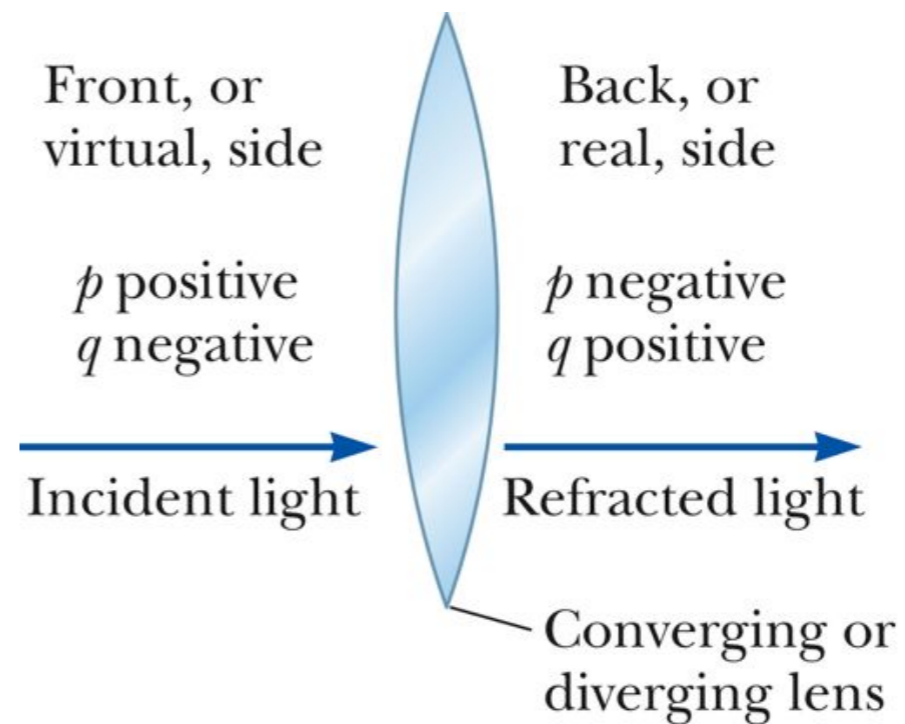
$$M = \frac{h'}{h} = -\frac{q}{p} \quad \text{and} \quad \frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

Section 23.6



# Lens Equations and Signs

- The equations can be used for both converging and diverging lenses.
  - A converging lens has a positive focal length.
  - A diverging lens has a negative focal length.
- See other sign conventions in the diagram.



Section 23.6

# Sign Conventions, Table

**Table 23.3** Sign Conventions for Thin Lenses

Quantity	Symbol	In Front	In Back	Convergent	Divergent
Object location	$p$	+	-		
Image location	$q$	-	+		
Lens radii	$R_1, R_2$	-	+		
Focal length	$f$			+	-

Section 23.6

# Focal Length for a Lens

- The focal length of a lens is related to the curvature of its front and back surfaces and the index of refraction of the material.

$$\frac{1}{f} = (n - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$

- This is called the *lens-maker's equation*.

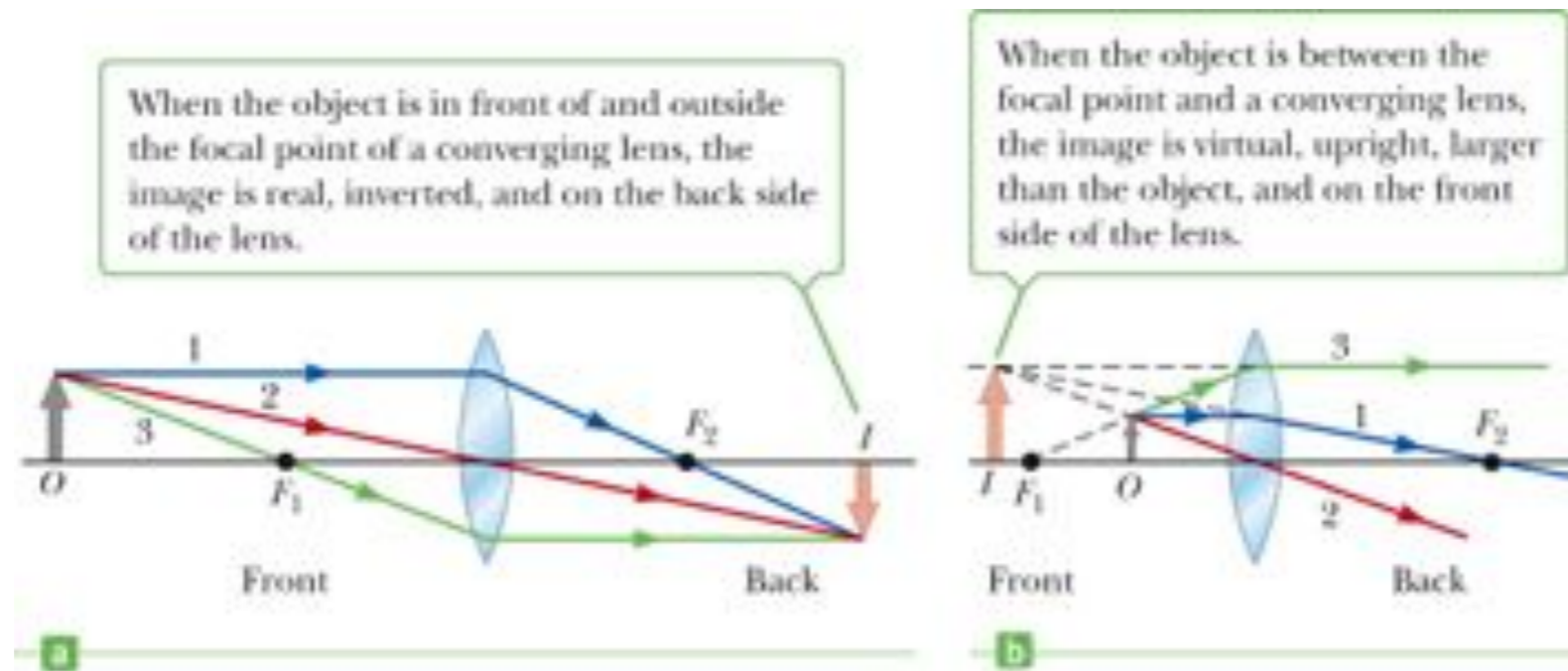
Section 23.6

# Ray Diagrams for Thin Lenses

- Ray diagrams are essential for understanding the overall image formation.
- Three rays are drawn.
  - The first ray is drawn parallel to the first principle axis and then passes through (or appears to come from) one of the focal lengths.
  - The second ray is drawn through the center of the lens and continues in a straight line.
  - The third ray is drawn from the other focal point and emerges from the lens parallel to the principle axis.
- There are an infinite number of rays, these are convenient

Section 23.6

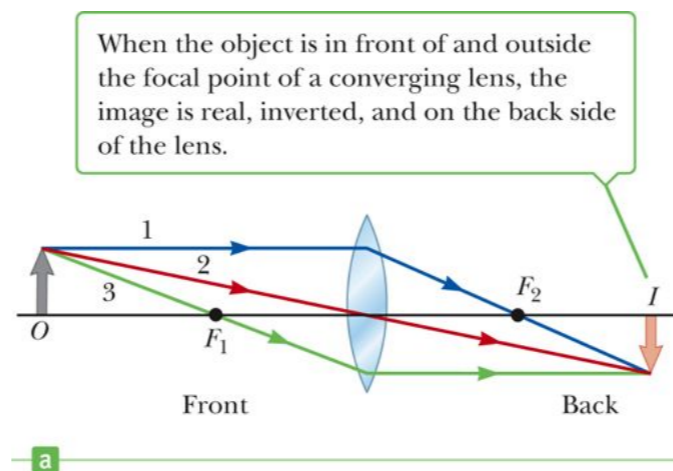
# Ray Diagram Examples



- Note the changes in the image as the object moves through the focal point.

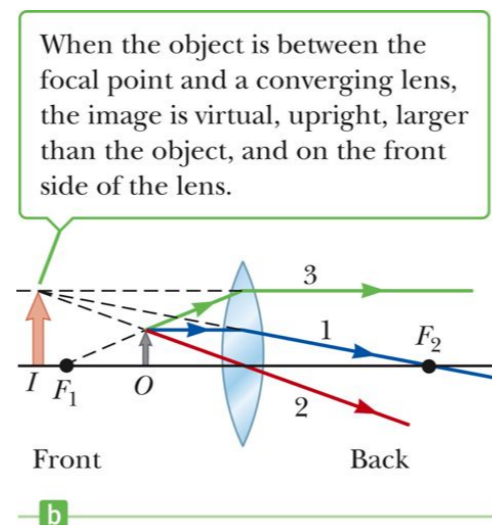


# Ray Diagram for Converging Lens, $p > f$



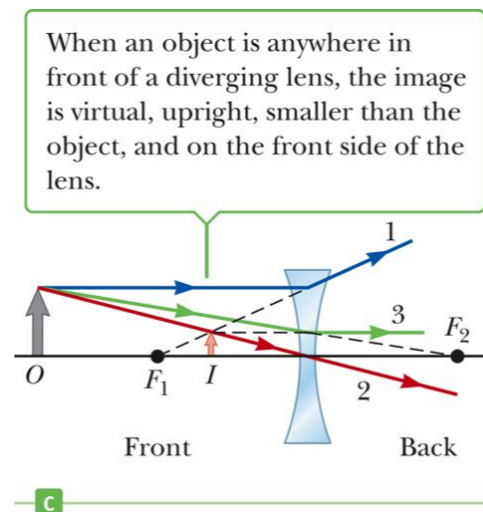
- The image is real.
- The image is inverted.
- The image is on the back side of the lens.

# Ray Diagram for Converging Lens, $p < f$

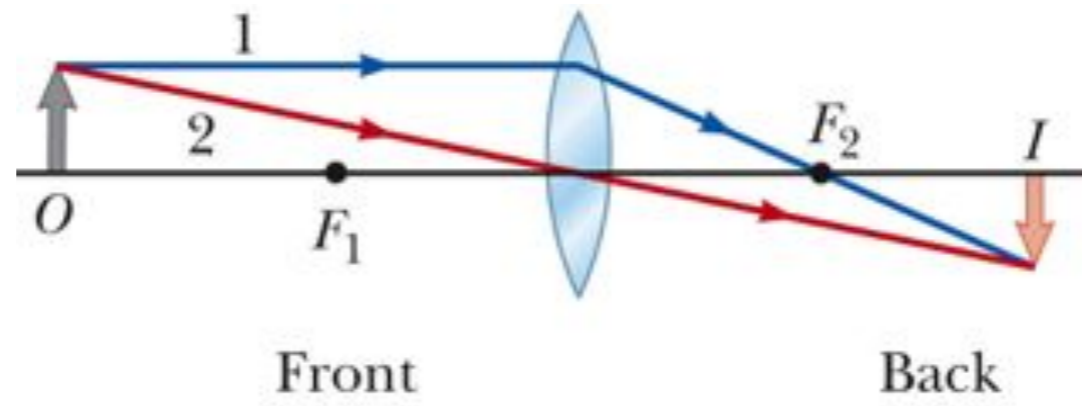


- The image is virtual.
- The image is upright.
- The image is on the front side of the lens.

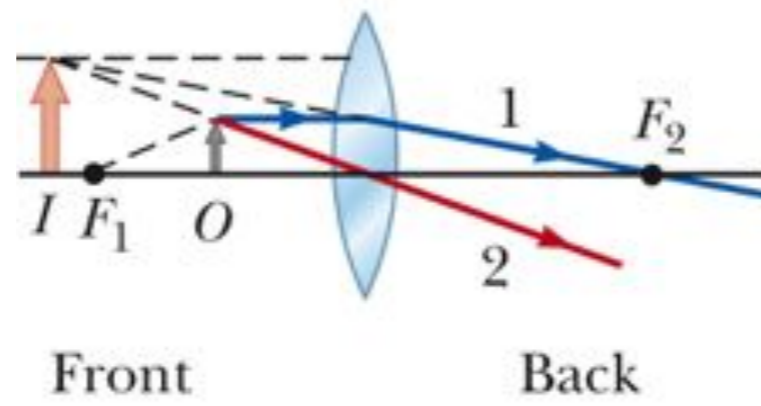
# Ray Diagram for Diverging Lens



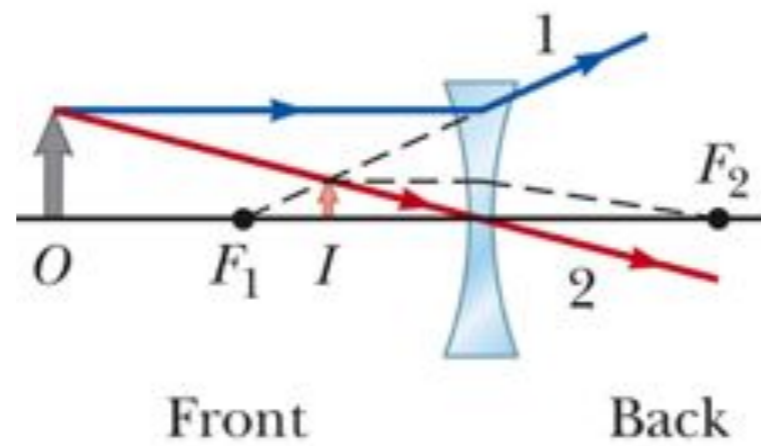
- The image is virtual.
- The image is upright.
- The image is on the front side of the lens.



a



b



c

## Thin Lens, Final Notes

- The point of intersection of any two of these rays can be used to locate the image.
  - The third ray serves as a check.
- The values of  $p$  and  $q$  are positive where the light is.
  - For real objects, the light originates with the object in front of the lens.
    - So  $p$  is positive
  - If the image is in back of the lens,  $q$  is positive.



Hw 29 & 31

Sally places an object 6.0 cm from a thin convex lens along its axis. The lens has a focal length of 9.0 cm. What are the respective values of the image distance and magnification?

- a. -18 cm and 3.0
- b. 18 cm and 3.0
- c. 3.0 cm and -0.50
- d. -18 cm and -3.0

Ansel places an object 30 cm from a thin convex lens along the axis. If a real image forms at a distance of 10 cm from the lens, what is the focal length of the lens?

- a. 30 cm
- b. 15 cm
- c. 10 cm
- d. 7.5 cm

Ellen places an object 40.0 cm from a concave lens. If a virtual image appears 10.0 cm from the lens on the same side as the object, what is the focal length of the lens?

- a. -50.0 cm
- b. -13.3 cm
- c. -10.0 cm
- d. -8.00 cm

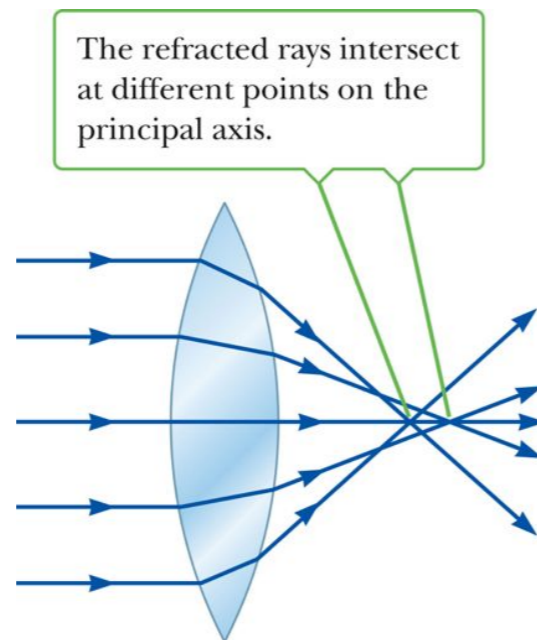
# Lens and Mirror Aberrations

- One of the basic problems of systems containing mirrors and lenses is the imperfect quality of the images.
  - Largely the result of defects in shape and form
- Two common types of aberrations exist
  - Spherical aberration
  - Chromatic aberration

Section 23.7

## Spherical Aberration

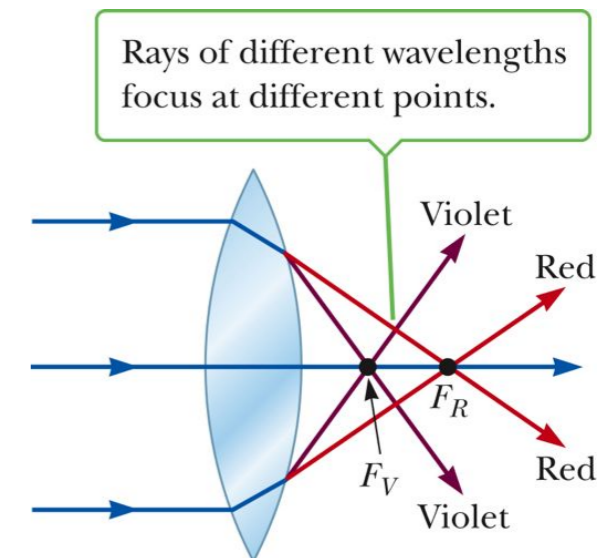
- Results from the focal points of light rays far from the principle axis are different from the focal points of rays passing near the axis.
- For a mirror, parabolic shapes can be used to correct for spherical aberration.



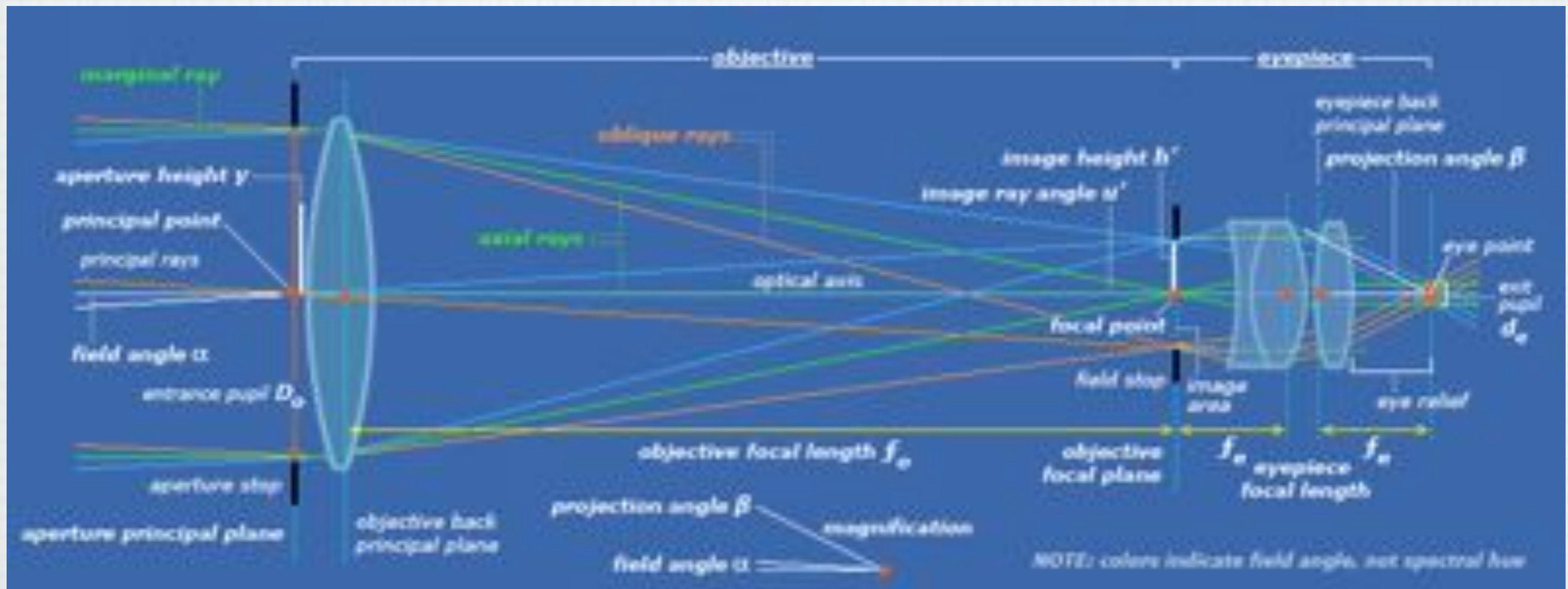
Section 23.7

## Chromatic Aberration

- Different wavelengths of light refracted by a lens focus at different points.
  - Violet rays are refracted more than red rays.
  - The focal length for red light is greater than the focal length for violet light.
- Chromatic aberration can be minimized by the use of a combination of converging and diverging lenses.







# Key Concepts

Because the triangles  $PQR$  and  $P'QR$  are identical,  $p = |q|$  and  $h = h'$ .

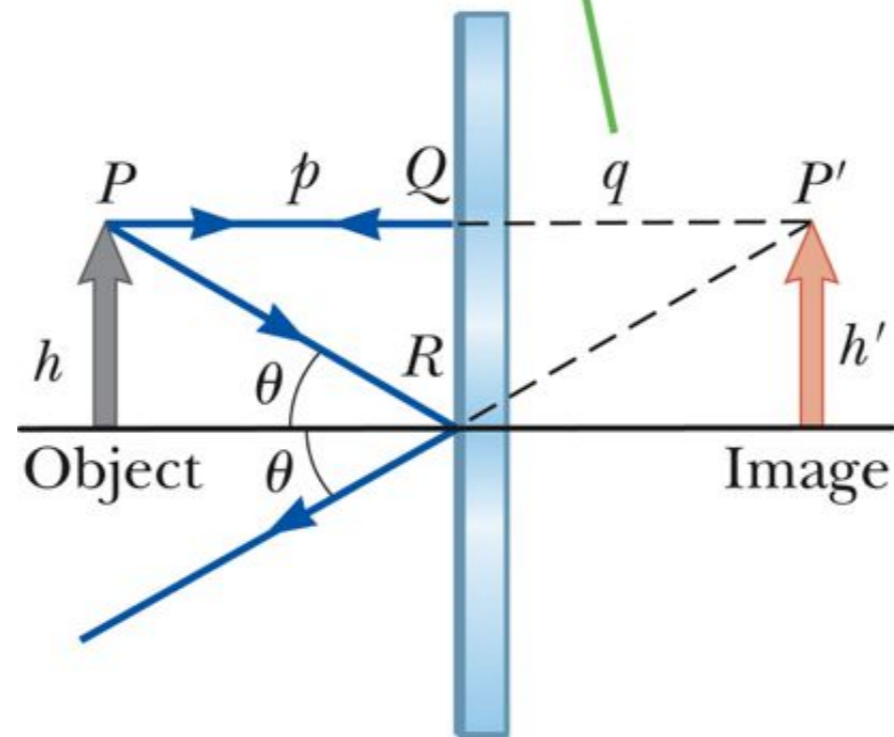
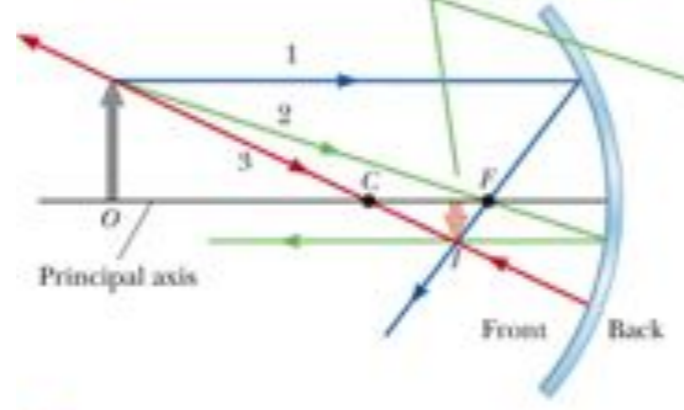


Fig. 23-2, p. 791

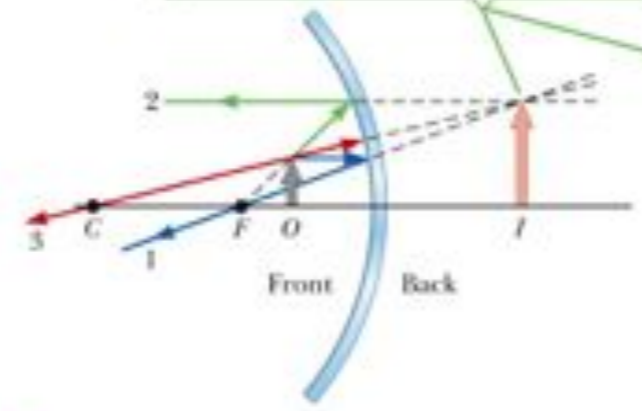


When an object is outside the center of curvature of a concave mirror, the image is real, inverted, and reduced in size.



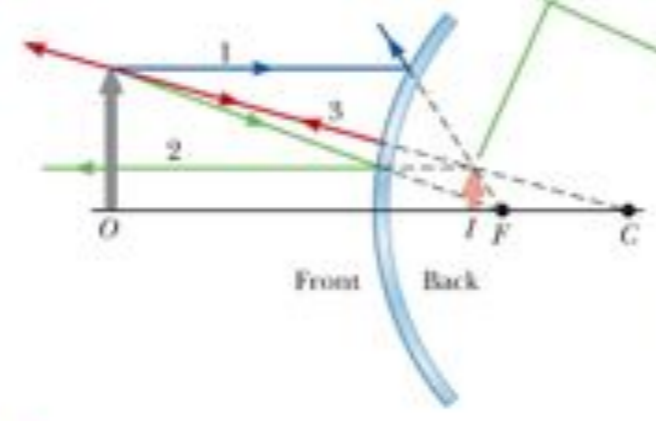
a

When an object is between a concave mirror and the focal point, the image is virtual, upright, and magnified.



b

When an object is in front of a convex mirror, the image is virtual, upright, and reduced in size.



c

Fig. 23-13, p. 797

Rays making small angles with the principal axis diverge from a point object at  $O$  and pass through the image point  $I$ .

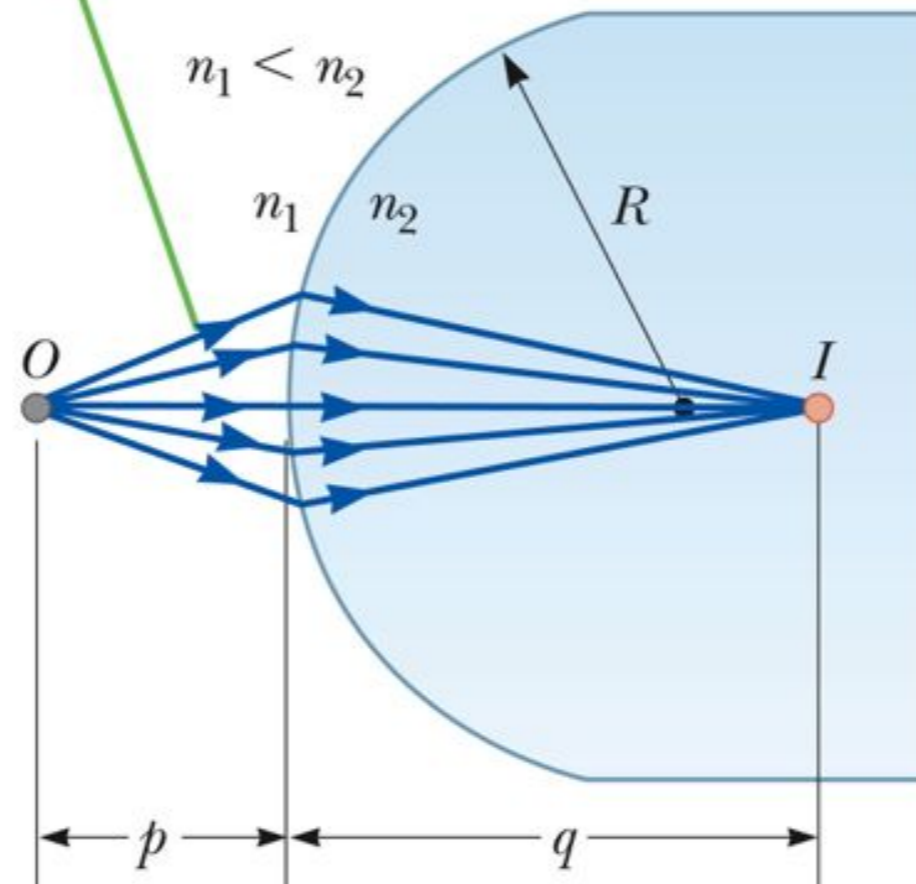
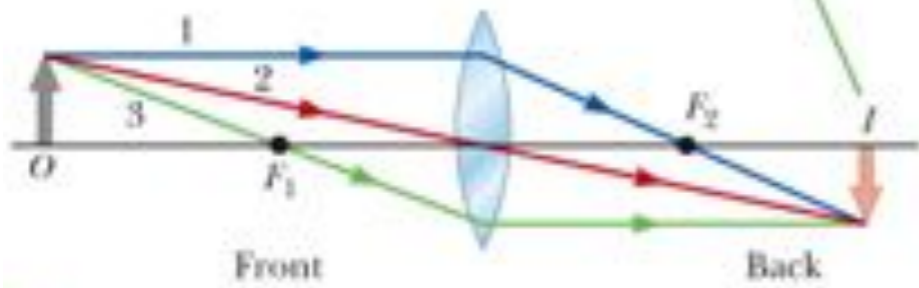


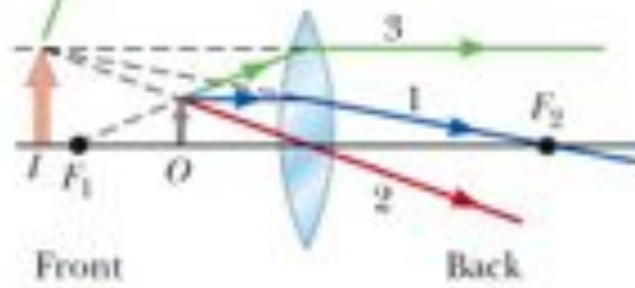
Fig. 23-15, p. 801

When the object is in front of and outside the focal point of a converging lens, the image is real, inverted, and on the back side of the lens.



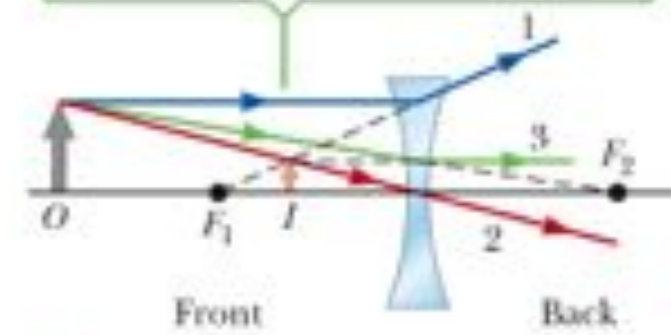
a

When the object is between the focal point and a converging lens, the image is virtual, upright, larger than the object, and on the front side of the lens.



b

When an object is anywhere in front of a diverging lens, the image is virtual, upright, smaller than the object, and on the front side of the lens.



c

Fig. 23-25, p. 807



# Key Equations

$$f = \frac{R}{2} \quad \text{Focal length in terms of radius of curvature}$$

$p$  = object distance,  $q$  = image distance,  $h$  = object height  $h'$  = image height

$$M = \frac{h'}{h} = -\frac{q}{p} \quad \text{Magnification of a mirror or thin lens}$$

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \quad \text{Focal length of a mirror or thin lens}$$

$$M = \frac{h'}{h} = -\frac{n_1 q}{n_2 p} \quad \text{Magnification of a refracting surface}$$

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R} \quad \text{relation between object and image distance for a refracting surface}$$

$$\frac{1}{f} = (n - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \quad \text{Lens-maker's equation}$$