Lecture 4

Chapter 18 Direct Current Circuit

You may ignore

* 18.4,18.5, 18.6, 18.7, and 18.8

This chapter is all

* The potential drop across a resistor is IR
* Power = I^2r
* Current is conserved
* The current is the same across resistors in series
* The potential drop is the same across resistors in parallel

Electric Circuits

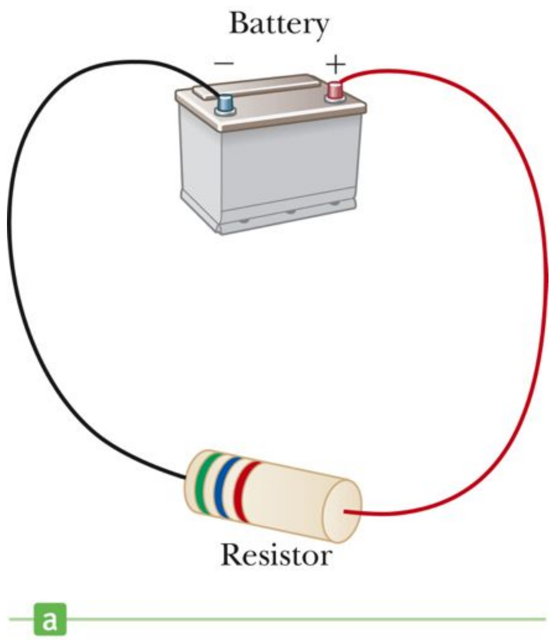
- Electric circuits control the flow of electricity and the energy associated with it.
- Circuits are used in many applications.
- Kirchhoff's Rules will simplify the analysis of simple circuits.
- Some circuits will be in steady state.
  - The currents are constant in magnitude and direction.
- In circuits containing resistors and capacitors, the current varies in time.

Sources of emf

- The source that maintains the current in a closed circuit is called a source of *emf.*
  - Any devices that increase the potential energy of charges circulating in circuits are sources of emf.
  - Examples include batteries and generators
- SI units are Volts
  - The emf is the work done per unit charge.

emf and Internal Resistance

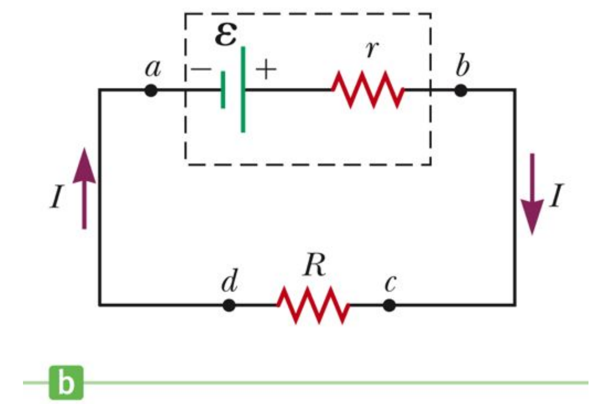
- A real battery has some internal resistance.
- Therefore, the terminal voltage is not equal to the emf.



More About Internal Resistance

- The schematic shows the internal resistance, r
- The terminal voltage is $\Delta V = V_b - V_a$
- $\Delta V = \varepsilon - Ir$
- For the entire circuit,

  $\varepsilon = IR + Ir$



Internal Resistance and emf, Cont.

- ε is equal to the terminal voltage when the current is zero.
  - Also called the *open-circuit voltage*
- R is called the *load resistance.*
- The current depends on both the resistance external to the battery and the internal resistance.

Internal Resistance and emf, Final

- When R >> r, r can be ignored.
  - Generally assumed in problems
- Power relationship
  - $I \varepsilon = I^2 R + I^2 r$
    - When R >> r, most of the power delivered by the battery is transferred to the load resistor.

Batteries and emf

- The current in a circuit depends on the resistance of the battery.
  - The battery cannot be considered a source of constant current.
- The terminal voltage of battery cannot be considered constant since the internal resistance may change.
- The battery *is* a source of constant emf.

Example

* A battery having an emf of 9 volts delivers 117 mA when connected to a 72 ohm load, determine the internal resistance of the battery.

Example

* Find the current in an 8 ohm resistor connected to a battery with an internal resistance of .15 ohms if the voltage across the battery is 9 volts. What is the emf of the battery?

The two ends of a 3.0-ohm resistor are connected to a 9.0-V battery. What is the total power delivered by the battery to the circuit?

a.3.0 W

b.27 W

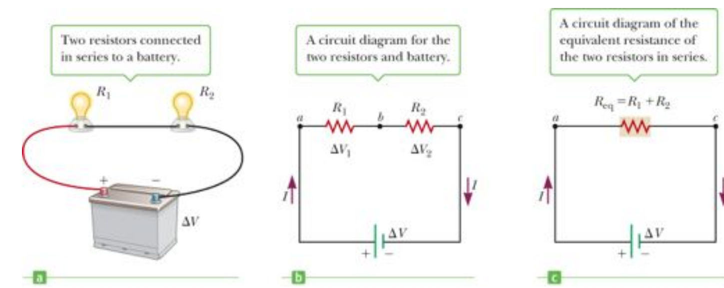
c.0.33 W

d.0.11 W

Resistors in Series

- When two or more resistors are connected end-to-end, they are said to be in *series.*
- The current is the same in all resistors because any charge that flows through one resistor flows through the other.
- The sum of the potential differences across the resistors is equal to the total potential difference across the combination.

Resistors in Series, Cont.



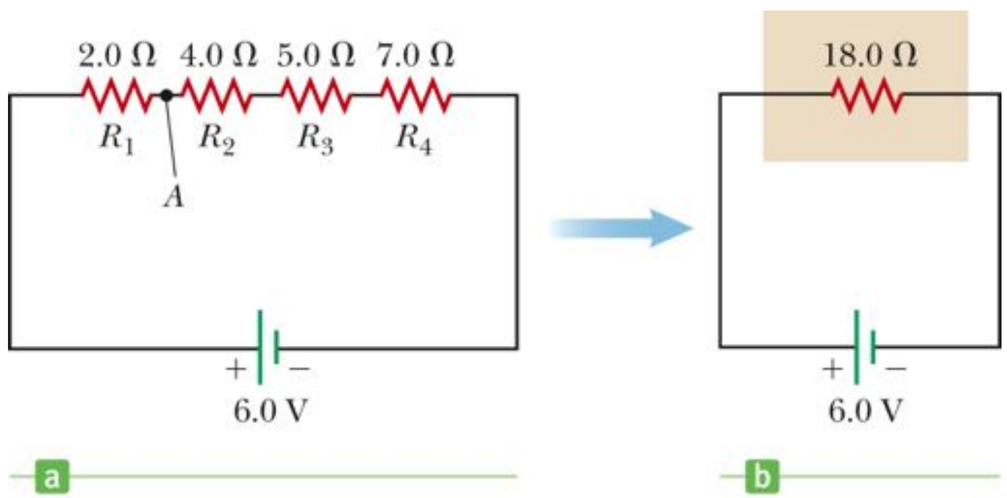
- Potentials add
  - $\Delta V = IR_1 + IR_2 = I(R_1+R_2)$
  - Consequence of Conservation of Energy
- The equivalent resistance has the effect on the circuit as the original combination of resistors.

Section 18.2

Equivalent Resistance – Series

- $R_{eq} = R_1 + R_2 + R_3 + \ldots$
- The equivalent resistance of a series combination of resistors is the algebraic sum of the individual resistances and is always greater than any of the individual resistors.
- If one element in the series circuit fails, the circuit would no longer be complete and none of the elements would work.

Equivalent Resistance – Series
An Example

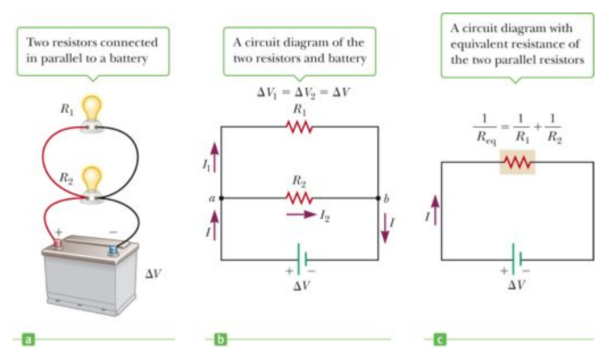


- Four resistors are replaced with their equivalent resistance.

Resistors in Parallel

- The potential difference across each resistor is the same because each is connected directly across the battery terminals.
- The current, I, that enters a point must be equal to the total current leaving that point.
  - $I = I_1 + I_2$
  - The currents are generally not the same.
  - Consequence of Conservation of Charge

Equivalent Resistance – Parallel An Example



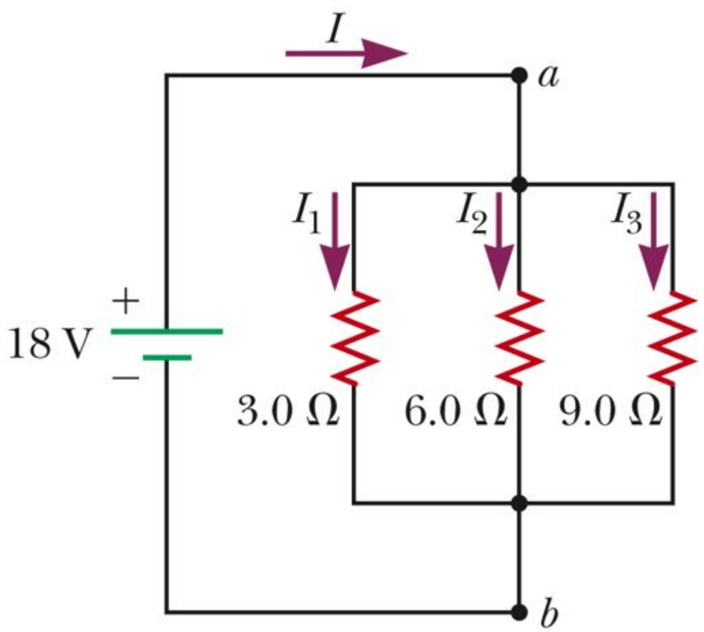
- Equivalent resistance replaces the two original resistances.
- *Household circuits* are wired so the electrical devices are connected in parallel.
  - Circuit breakers may be used in series with other circuit elements for safety purposes.

Equivalent Resistance – Parallel

- Equivalent Resistance

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

- The inverse of the equivalent resistance of two or more resistors connected in parallel is the algebraic sum of the inverses of the individual resistance.
  - The equivalent is always less than the smallest resistor in the group.



Problem-Solving Strategy, 1

- Combine all resistors in series.
  - They carry the same current.
  - The potential differences across them are not the same.
  - The resistors add directly to give the equivalent resistance of the series combination: $R_{eq} = R_1 + R_2 + \ldots$
  - Draw the simplified circuit diagram.

Problem-Solving Strategy, 2

- Combine all resistors in parallel.
  - The potential differences across them are the same.
  - The currents through them are not the same.
  - The equivalent resistance of a parallel combination is found through reciprocal addition:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

  - Remember to invert the answer after summing the reciprocals.
  - Draw the simplified circuit diagram.

Problem-Solving Strategy, 3

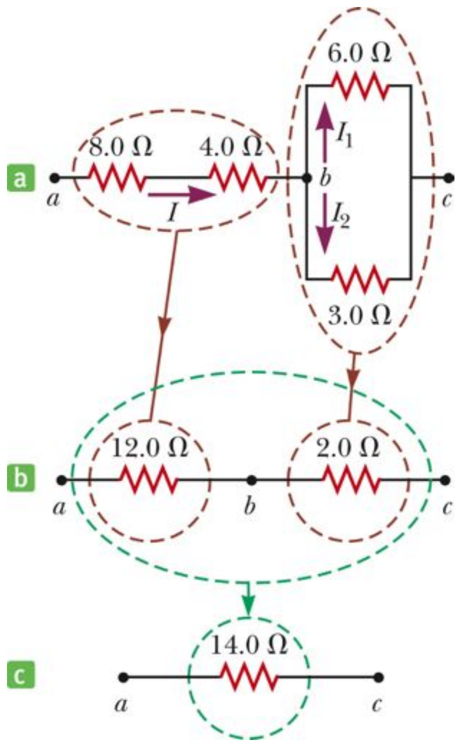
- Repeat the first two steps as necessary.
  - A complicated circuit consisting of several resistors and batteries can often be reduced to a simple circuit with only one resistor.
  - Replace any resistors in series or in parallel using steps 1 or 2.
  - Sketch the new circuit after these changes have been made.
  - Continue to replace any series or parallel combinations.
  - Continue until one equivalent resistance is found.

Problem-Solving Strategy, 4

- Use Ohm's Law.
  - Use $\Delta V = I R$ to determine the current in the equivalent resistor.
  - Start with the final circuit found in step 3 and gradually work back through the circuits, applying the useful facts from steps 1 and 2 to find the current in the other resistors.

Example

- Complex circuit reduction
  - Combine the resistors in series and parallel.
    - Redraw the circuit with the equivalents of each set.
  - Combine the resulting resistors in series.
  - Determine the final equivalent resistance.



Examples

* Problems 5&9, chapter 18

Resistors of values 8.0, 12.0, and 24.0 ohms are connected in series across a battery with a small internal resistance. Which resistor dissipates the greatest power?

a. the 8.0-ohm resistor

b. the 12.0-ohm resistor

c. the 24.0-ohm resistor

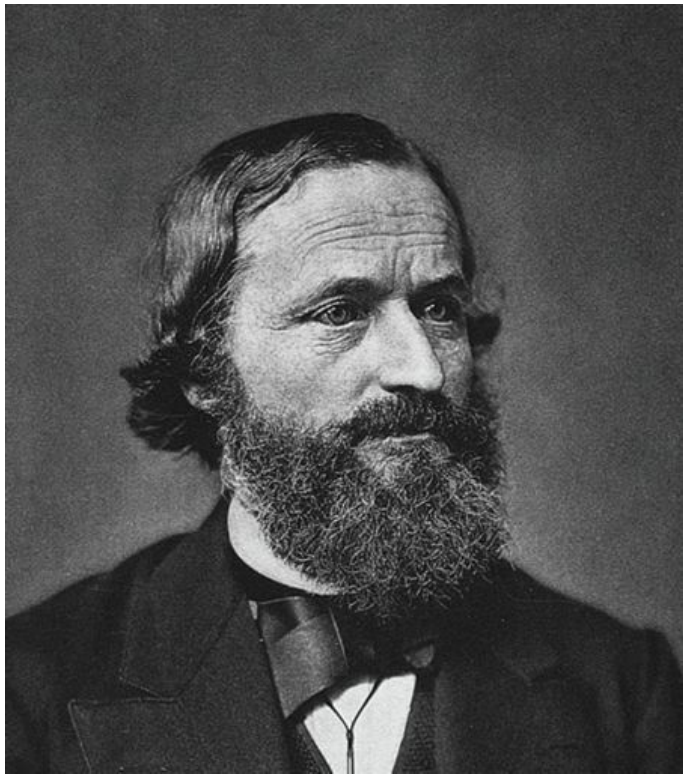
d. The answer depends on the internal resistance of the battery.

Two resistors of values 6.0 and 12.0 ohms are connected in parallel. This combination in turn is hooked in series with a 2.0-ohm resistor and a 24-V battery. What is the current in the 2-ohm resistor?

a.2.0 A
b.4.0 A
c.6.0 A
d.12 A

Gustav Kirchhoff

- 1824 – 1887
- Invented spectroscopy with Robert Bunsen
- Formulated rules about radiation



Kirchhoff's Rules

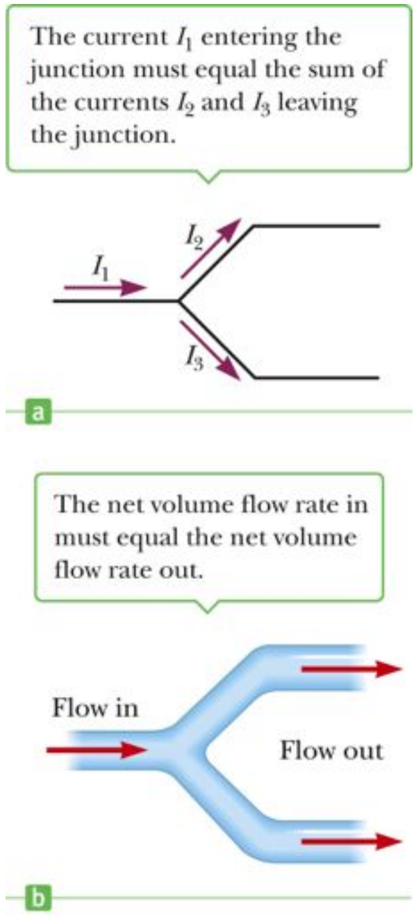
- There are ways in which resistors can be connected so that the circuits formed cannot be reduced to a single equivalent resistor.
- Two rules, called Kirchhoff's Rules, can be used instead.

Statement of Kirchhoff's Rules

- Junction Rule
  - The sum of the currents entering any junction must equal the sum of the currents leaving that junction.
    - A statement of Conservation of Charge
- Loop Rule
  - The sum of the potential differences across all the elements around any closed circuit loop must be zero.
    - A statement of Conservation of Energy

More About the Junction Rule

- $I_1 = I_2 + I_3$
- From Conservation of Charge
- Diagram b shows a mechanical analog.



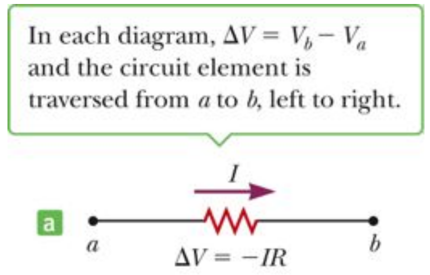
Section 18.4

Loop Rule

- A statement of Conservation of Energy
- To apply Kirchhoff's Rules,
  - Assign symbols and directions to the currents in all branches of the circuit.
    - If the direction of a current is incorrect, the answer will be negative, but have the correct magnitude.
  - Choose a direction to transverse the loops.
    - Record voltage rises and drops.

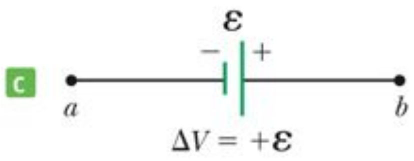
More About the Loop Rule

- Traveling around the loop from a to b
- In a, the resistor is transversed in the direction of the current, the potential across the resistor is –IR.
- In b, the resistor is transversed in the direction opposite of the current, the potential across the resistor is +IR.



Loop Rule, Final

- In c, the source of emf is transversed in the direction of the emf (from – to +), the change in the electric potential is +ε
- In d, the source of emf is transversed in the direction opposite of the emf (from + to -), the change in the electric potential is -ε



Junction Equations from Kirchhoff's Rules

- Use the junction rule as often as needed, so long as, each time you write an equation, you include in it a current that has not been used in a previous junction rule equation.
  - In general, the number of times the junction rule can be used is one fewer than the number of junction points in the circuit.

Loop Equations from Kirchhoff's Rules

- The loop rule can be used as often as needed so long as a new circuit element (resistor or battery) or a new current appears in each new equation.
- You need as many independent equations as you have unknowns.

Problem-Solving Strategy – Kirchhoff's Rules

- Draw the circuit diagram and assign labels and symbols to all known and unknown quantities.
- Assign directions to the currents.
- Apply the junction rule to any junction in the circuit.
- Apply the loop rule to as many loops as are needed to solve for the unknowns.
- Solve the equations simultaneously for the unknown quantities.
- Check your answers.

Key Concepts

* The potential drop across a resistor is equal to the source emf of the battery minus Ir
* The load resistance is equal to IR where R is the total resistance outside of the battery/power source
* Resistors in series add algebraically
* Resistors in parallel add as their inverses
* The current is the same across resistors in series
* The potential drop is the same across resistors in parallel
* Kirchhiff's Rules
    * The sum of the currents entering any junction equals the sum leaving that junction
    * The sum of the potential differences across all elements around any closed circuit loop must be zero

Example

* Chapter 18 13, 26

Key Equations

$$\Delta V = \epsilon - Ir \rightarrow \epsilon = IR + Ir \rightarrow I = \frac{\epsilon}{R + r}$$

$$R_{eq,series} = R_1 + R_2 + R_3 + ...$$

$$R_{eq,parallel} = 1/R_1 + 1/R_2 + 1/R_3 + ...$$