Physics 321, Fall 2014 Exam 3

RULES

Use whatever you like except another human being. I intentionally made this a bit harder. Use wikipedia, google, your book, whatever you want.

Problem 1:

Using the results for energy levels in the hydrogen atom determine which transitions are in the optical portion of the wavelength. Write down the transitions and the corresponding wavelengths for the first three.. Hint - they are the Balmer lines. What wavelength range is the n=2 to n=1 transition in?

Problem 2:

The solution for the hydrogen atom may be trivially extended to any atom with Z protons so long as the atom posesess only one electron. Hydrogen is simply Z=1 By following through the derivation determine the formula for the allowed energy levels of singly ionized helium, the formula for the radius of a singly ionized helium as a function of energy level, and the ionization potential of singly ionized helium. Hint - if you look at equation 4.52 and follow the derivation in the book the anwwer may be simply written down.

Problem 3:

Explicitly construct and normalize $\Psi_{2,1,1}(r, \theta, \phi, t)$ Use either 4.76 or 4.87 and 4.88 for $R_{n,l}$. Normalize this function by hand - no integrators. What is the degeneracy in energy for this wave function? Show explicitly that the $Y_{l,m}$ portion is an eigenfunction of the L^2 operator. What is the eigenvalue? Is this consistent with equation 4.118? Hint —- $cos^2(\theta) - 1 = -sin^2(\theta)$

Problem 4:

Calculate the eigenspinors and eigenvalues of $\hat{S}_y$. Write the eigenspinors of $\hat{S}_z = \begin{pmatrix} a \\ b \end{pmatrix}$ as a linear combination of the eigenspinors of $\hat{S}_y$.

Problem 5:

Consider $\psi = A\begin{pmatrix} 1-2i \\ 2 \end{pmatrix}$. Determine A. What is the probability of measuring spin up and down for the z, x, and y components of spin? Hint, you have the x part from the book (equation 4.152) and you just did the y part. See example 4.2.

Problem 6:

Consider the Hamiltonian $H = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 2 & i \\ 0 & -i & 2 \end{pmatrix}$. Determine the possible energies and their corresponding eigenvalues For $\psi = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$ determine the probabilities of measuring each energy for this wavenfunction. Show that they add to 1,

Problem 7:

Determine the most likely radial location for an electron in the second excited state with $R_{2,1}$.

Challenge Problem - meaning extra credit

The three dimensional momentum space wave function is given by $\phi(\vec{p}) = \frac{1}{(2\pi\hbar)^{3/2}} \int e^{-i(\vec{p}\cdot\vec{r})/\hbar}\psi(\vec{r})d^3r$ Taking the ground state as your wave function $\psi = \frac{1}{\sqrt{\pi a^3}}e^{-r/a}$

Align the polar axis along P so that $\vec{p} \cdot \vec{r} = prcos\theta$. Find $\phi(\vec{p})$. Use this to calculate $< p^2 >$ and from this the kinetic energy. Express your answer for kinetic energy in terms of E_1 . Comment on the result. Hint - do the theta integral first and express the resulting sin as a combination of exponentials. Note, you may use integrators or tables for this problem but I want to see the integral set up correctly. You will only gain extra credit from this problem so don't worry if you have no clue how to do it.