#1. $\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$ $R = 1.097 \times 10^7\ m^{-1}$

3-2 656.3 nm   n=2→1   122 nm UV
4-2 486.1 nm
5-2 434 nm
6-2 410 nm
7-2 397 nm

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#2. The only change is $e^2 \to Ze^2$

$E_{n(Z)} = Z^2 E_n$ $a(Z) = \frac{a}{Z}$

$Z=2$ $E_{1\to\infty}$ = 4, 13, 6 eV = 54.4 eV

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#3. $R_{21} = \frac{C_0}{4a^2} re^{-r/2a}$ $Y_{11} = Y_0 \sin\theta e^{i\phi}$

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$\frac{A}{4a^2} re^{-r/2a} \sin\theta e^{i\phi} = \psi_{211}$

$\frac{A^2}{16a^4} \iiint r^2 e^{-r/a} \sin^2\theta\, r^2 \sin\theta\, dr\, d\theta\, d\phi \Rightarrow -\sqrt{\frac{3}{8\pi}} \sin\theta e^{i\phi} \cdot \frac{1}{\sqrt{\pi a}}\ 8a^2 re^{-r/2a}$

$L^2 = -\hbar^2 \left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]$

$L^2\psi = R_{nl} L^2 Y_{lm}$

$$= R_{n\ell}\ (-\hbar^2)\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right] Y_{\ell m}$$

$$= -\hbar^2 R_{n\ell}\left[e^{i\phi}\,\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta}\sin\theta + \frac{\sin\theta}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}e^{i\phi}\right]$$

$$= -\hbar^2 R_{n\ell}\left[e^{i\phi}\,\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\cos\theta + \frac{1}{\sin^2\theta}\ (-1)\, e^{i\phi}\right]$$

$$= -\hbar^2 R_{n\ell}\left[e^{i\phi}/\sin\theta\left[\cos^2\theta - \sin^2\theta\right] + \frac{1}{\sin^2\theta}(-1)\,e^{i\phi}\right]$$

$$= -\hbar^2 R_{n\ell}\, e^{i\phi}\sin\theta\left[\frac{1}{\sin^2\theta}\left(\cos^2\theta - \sin^2\theta - 1\right)\right]$$

but $\cos^2\theta - 1 = -\sin^2\theta$

$$\text{So} = -\hbar^2 R_{n\ell} Y_{\ell m}(-2) = 2\hbar^2 \psi_{n\ell m} = 1(1+1)\hbar^2\psi_{n\ell m}$$

$$\frac{1}{2mr^2}\left[-\hbar^2\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r}\right]\psi + V\psi$$

4.)

$$S_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad -\lambda^2-(-ii) = -\lambda^2-1=0 \qquad \lambda = \pm 1 \frac{\hbar}{2}$$

$\frac{\hbar}{2}, -\frac{\hbar}{2}$ $\qquad \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \frac{\hbar}{2}\begin{pmatrix} a \\ b \end{pmatrix}$

$-ib = a \qquad \chi_+^y = \begin{pmatrix} 1 \\ i \end{pmatrix} \to \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$

$\frac{a}{-i} = b \quad ia = b$

$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = -\begin{pmatrix} a \\ b \end{pmatrix} \quad -ib = -a \quad ib = a \quad b = \frac{a}{i} \quad b = -ia$

$$\chi_-^y = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}$$

$\chi = c\,\chi_+^y + d\,\chi_-^y \qquad c = \langle \chi_+ | \chi \rangle = \frac{1}{\sqrt{2}}\,\overline{1\ -i}\begin{pmatrix} a \\ b \end{pmatrix} = \frac{1}{\sqrt{2}}(a-ib)$

$d = \langle \chi_- | \chi \rangle = \frac{1}{\sqrt{2}}\,\overline{1\ i}\begin{pmatrix} a \\ b \end{pmatrix} = \frac{1}{\sqrt{2}}(a+ib)$

$$\chi = \frac{1}{\sqrt{2}}\left[(a-ib)\chi_+^y + (a+ib)\chi_-^y\right]$$

5.)

$$A^2 \overline{(1+2i \quad 2)} \begin{pmatrix} 1-2i \\ 2 \end{pmatrix} = A^2 (1+4+4) = 1 \quad A = \frac{1}{\sqrt{9}}$$

$$\frac{1}{\sqrt{9}} \begin{pmatrix} 1-2i \\ 2 \end{pmatrix} \quad P(S_{z+}) = \frac{5}{9} \quad P(S_{z-}) = \frac{4}{9}$$

a^2 $\qquad$ b^2

$$P(S_{x+}) = \left|\frac{a+b}{\sqrt{2}}\right|^2 = \frac{(1-2i+2)(1+2i+2)}{18} = \frac{(3-2i)(3+2i)}{18} \quad \frac{13}{18}$$

$$P(S_x^-) = \left|\frac{a-b}{\sqrt{2}}\right|^2 = \frac{(1-2i-2)(1+2i-2)}{18} \quad \frac{(-1-2i)(-1+2i)}{18}$$

$$= 1-2i+2i+4 = \frac{5}{18}$$

$$P(S_+^y) = \left|\frac{1}{\sqrt{2}}(a-ib)\right|^2 = \frac{1}{18}(1-2i-2ib)(1+2i+2ib) = (1-4i)(1+4i)$$

$$= \frac{17}{18}$$

$$P(S_-^y) = \frac{1}{18}$$

#6.) $\lambda = 5, 3, 1 = 1, 3, 5$

$\psi = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix}$

$\lambda = 1 \quad \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ i \\ i \end{pmatrix} \qquad \lambda = 3 \quad \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ i \\ i \end{pmatrix} \qquad \lambda = 5 \quad \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$\lambda = 1 \quad \left| \frac{1}{2} \; \overline{(0 \; 1 \; -i)} \begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix} \right|^2 = \frac{(-1)^2}{4} = \frac{1}{4}$

$\lambda = 3 \quad \frac{1}{4}$

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$\lambda = 5 \quad \frac{1}{2}$

#7.) $4\pi r^2 R_{2,1}^2 = 4\pi r^2 \cdot \frac{1}{24} a^{-3} \frac{r^2}{a^2} e^{-r/a}$

$= \frac{\pi}{6a^5} r^4 e^{-r/a}$

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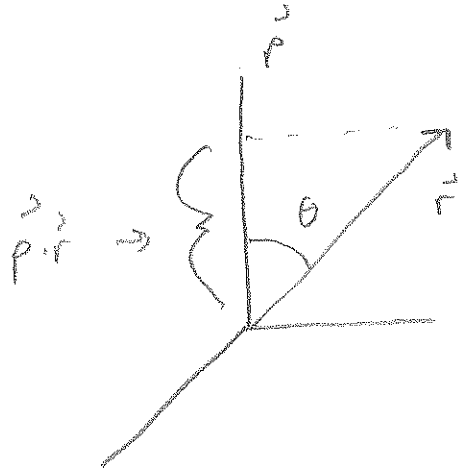
$\frac{dP}{dr} = \frac{\pi}{6a^5} \left[4r^3 e^{-r/a} - \frac{1}{a} r^4 e^{-r/a} \right] = 0$

$4r^3 - \frac{r^4}{a} = 0 \qquad 4 = \frac{r}{a} \qquad r = 4a$

Challenge



$$\psi = \frac{1}{(\pi a^3)^{1/2}} e^{-r/a} \qquad \phi(p) = \frac{1}{(\sqrt{2\pi\hbar})^3} \frac{1}{\sqrt{\pi a^3}} \int_0^{2\pi}\int_0^{\pi}\int_0^{\infty} e^{-r/a} r^2 \sin\theta \, e^{ipr\cos\theta/\hbar} \, dr\, d\theta\, d\phi$$

$$\int_0^{\pi} \sin\theta \, e^{ipr\cos\theta/\hbar} \, d\theta = \frac{2\hbar}{pr} \sin\left(\frac{pr}{\hbar}\right)$$

$$\int_0^{2\pi} d\phi = 2\pi$$

$$\int_0^{\infty} r e^{-r/a} \sin\left(\frac{pr}{\hbar}\right) dr = \frac{(2p/\hbar)a^3}{\left(1+(ap/\hbar)^2\right)^2}$$

$$\phi(p) = \frac{1}{\pi}\left(\frac{2a}{\hbar}\right)^{3/2} \frac{1}{\left[1+(ap/\hbar)^2\right]^2}$$

$$\langle p^2 \rangle = \frac{\hbar^2}{a^2} \qquad \langle T \rangle = -E_1$$