#1)

~~57~~ pts Total

10 pts

10 pts A → an operator $\hat{p} = -\hbar i \frac{\partial}{\partial x}$ +2

B → $\langle \hat{Q} \rangle = \int \Psi^* \hat{Q} \Psi$ +2

C → Average, no +1 +1 +2

D → Average, actual/most likely value +1 +1 +2

E → Wavefunction, normalize it +1 +1 +2

#2)

15 pts

A → $A^2 \int_0^a \frac{x}{a^2} dx \Rightarrow \frac{A^2}{a^2} \frac{x^2}{2} \Big|_0^a = \frac{A^2}{a^2} \frac{a^2}{2} = 1 \quad \frac{A^2}{2} = 1 \quad A = \sqrt{2}$ +2 +1 +3

$\Psi = \frac{\sqrt{2}\sqrt{x}}{a}$

B → $\langle x \rangle = \int_0^a \frac{2x}{a^2} \cdot x \, dx = \frac{2}{a^2} \frac{x^3}{3} \Big|_0^a = \frac{2}{3} a \quad \langle x \rangle^2 = \frac{4}{9} a^2$ +2 +1 +3

$\langle x^2 \rangle = \int_0^a \frac{2}{a^2} x^3 dx = \frac{2}{a^2} \frac{x^4}{4} \Big|_0^a = \frac{1}{2} a^2$ +2 +1 +3

$\sigma_x = \sqrt{\frac{1}{2} a^2 - \frac{4}{9} a^2} = a\sqrt{\frac{1}{18}}$ +2 +1 +3

C → $\int_{a/2}^a |\Psi|^2 dx = \frac{2}{a^2} \int_{a/2}^a x \, dx = \frac{2}{a^2} \left(\frac{x^2}{2} \Big|_{a/2}^a \right) = \frac{2}{a^2} \left[\frac{a^2}{2} - \frac{a^2}{8} \right] =$ +2

$= 1 - \frac{1}{4} = 3/4$ +1 +3

15 pts

3.) $P = \hbar k = \frac{n\pi\hbar}{a}$ +1 $E = \left(\frac{n\pi\hbar}{a}\right)^2 \cdot \frac{1}{2m} = n^2 \epsilon_0$ +1 $\epsilon_0 = \left(\frac{\pi\hbar}{a}\right)^2 \cdot \frac{1}{2m}$

9 pts

So $[2, 4, 6] \frac{\pi\hbar}{a}$ +2 and $[4, 16, 36]\epsilon_0$ +2

9 pts

but, $\hat{p}^4 \neq p^4$ So No exact.

$\langle E \rangle = \sum |c_n|^2 \epsilon_0$ +2 $= \left[\frac{1}{2} \cdot 4 + \frac{1}{4} \cdot 16 + \frac{1}{4} \cdot 36\right] \epsilon_0$ +1

9/9

$= [2 + 4 + 9]\epsilon_0 = 15\epsilon_0$

4.) $\int_0^a \psi^* \psi \, dx = 1 = A^2 \int_0^a dx = aA^2 = 1$ +1 $A = \frac{1}{\sqrt{a}}$ +1 +2

~~8~~ pts

15 pts

$\psi = \frac{1}{\sqrt{a}}$

$\psi(x,0) = \sum c_n \psi_n \qquad c_n = \int \psi_n^* \psi(x,0)\, dx$ +2

$= \frac{1}{\sqrt{a}} \int_0^a \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right) dx$ +2 $\quad u = \frac{n\pi x}{a} \quad du = \frac{n\pi\, dx}{a}$

$x = 0 \quad u = 0 \qquad x = a \quad u = n\pi$

$= \frac{\sqrt{2}}{a} \frac{a}{n\pi} \int_0^{n\pi} \sin(u)\, du = \frac{\sqrt{2}}{n\pi} \left[-\cos(u) \Big|_0^{n\pi}\right]$

$$= \frac{\sqrt{2}}{m\pi}\left[\cos(0) - \cos(n\pi)\right]$$

$$= \frac{\sqrt{2}}{m\pi}\left[1 - (-1, 1, -1, 1 \dots)\right]$$

$m = \quad 1 \quad 2 \quad 3 \quad 4$

$$C_n = \frac{2\sqrt{2}}{m\pi} \quad n \text{ odd}$$

$+4$

$0 \quad$ else

$+3$

$$\Psi(x,t) = \sum_{1,3,5\dots} \frac{2\sqrt{2}}{m\pi} \cdot \sqrt{\frac{2}{a}} \sin(m\pi x) e^{-i\frac{E_n t}{\hbar}}$$

5.) $\hat{p} = -\hbar i \frac{d}{dx}$ $+1$ $\quad \hat{p}\psi = \frac{\sqrt{2}}{\sqrt{a}} \hbar i \frac{n\pi}{a} \cos\left(\frac{n\pi x}{a}\right) \neq \#\cdot\sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right)$ $+2$

3pts

6.) $\hat{E}\Psi = (\hat{p}^2c^2 + m^2c^4)\Psi$ $+1$

5pts

$$i\hbar\frac{\partial}{\partial t}\Psi = (-\hbar i)^2 c^2 \frac{d^2\Psi}{dx^2} + m^2c^4\Psi \quad +2$$

$$i\hbar\frac{\partial\Psi}{\partial t} = -\hbar^2c^2\frac{d^2\Psi}{dx^2} + m^2c^4\Psi \quad +2$$