Physics 321, Fall 2013 Exam 2

Dr. Jared Workman, Due By Friday December 6th

RULES

Use whatever you want to except each other. Do not compare answers, do not collaborate, work on your own.

Problems

1: SHORT ANSWER

A) Describe the physical meaning of the generalized uncertainty principle. Describe what conditions must be met for an uncertainty in the measurement of two physical properties to exist. When can two states be measured exactly? (3 pts)

B) Orthonormality and completeness take on a different form for operators whose spectrums are discreet and operators whose spectrums are continuous. Expound on this and demonstrate completeness and orthogonality for an operator of each type. Use real examples. (4 pts)

C) Explain how n, l, and m each come about in the solution to the hydrogen atom. Write down the restrictions on each one. (3pts)

1) An electron is localized to .01nm. What is the uncertainty in its momentum, its velocity? What is its energy if the uncertainty in its momentum is its velocity. $M_e = 9.11 * 10^{-31} kg$ and $1eV = 1.6 * 10^{-19} Joules$. Do you need to do this relativistically? (3 pts)

2) Construct and normalize R_{21} (4 pts)

$$R_{n,l} = \frac{\rho^{l+1} e^{-\rho} v(\rho)}{r} \tag{1}$$

where $\rho = r/(an)$ and $v(\rho) = \sum c_j \rho^j$ with

$$c_{j+1} = c_j \frac{2(j + 1 + l - n}{(j + 1)(j + 2l + 1)}. \tag{2}$$

3)The angular equation for the three dimensional Schroedinger equation is given by

$$\frac{1}{Y}\left(\frac{1}{sin(\theta)} \frac{\partial}{\partial\theta} sin(\theta) \frac{\partial Y}{\partial\theta} + \frac{1}{sin^2(\theta)} \frac{\partial^2 Y}{\partial\phi^2}\right) = -l(l + 1) \tag{3}$$

Separate it and solve for the ϕ component. Under what conditions can you always separate the radial, temporal, and angular components in Schroedinger's equation? (5 pts)

4) Solve for $\frac{\partial <xp_x>}{\partial t}$ (5 pts)

5) Show $< \hat{Q} > = \sum_n q_n |c_n|^2$ (3 pts)

6) A system evolves due to a Hamiltonian given by

$$\hat{H} = \frac{-\gamma B_0 \hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{4}$$

Solve for the eigenvalues and eigenfunctions. Write down the most generic wavefunction and include its temporal dependence. Solve for the expectation value of the Hamiltonian. Show that the basis functions form an orthonormal set. (10 pts)

7) Construct $P_{1,1}, P_{1,0}, P_{1,-1}$ and from this construct the full solution for $\Psi_{n,l,m} = \Psi_{211}, \Psi_{210}, \Psi_{21-1}$ using your results from problem 2.

$$P_l(x) = \frac{1}{2^l l!}(\frac{d}{dx})^l(x^2-1)^l \tag{5}$$

$$P_l^m(x) = (1-x^2)^{|m|/2}(\frac{d}{dx})^{|m|}P_l(x) \tag{6}$$

$$Y_l^m(\theta,\phi) = \epsilon\sqrt{\frac{(2l+1)}{4\pi}\frac{(l-|m|)!}{(l+|m|)!}}e^{im\phi}P_l^m(\cos\theta) \tag{7}$$

The raising and lowering operators for angular momentum are given explicitly by

$$L_\pm = \frac{\hbar}{i}[(-sin(\phi) \pm icos(\phi))\frac{\partial}{\partial\theta} - (cos(\phi) \pm isin(\phi))cot(\theta)\frac{\partial}{\partial\phi}] \tag{8}$$

or

$$L_\pm = \pm\hbar e^{\pm i\phi}[\frac{\partial}{\partial\theta} \pm icot(\theta)\frac{\partial}{\partial\theta}] \tag{9}$$

Show explicitly that the successive application of the raising operator, starting at Ψ_{21-1} increases the order the m in th Y_{lm} and eventually takes it to zero when applied to Ψ_{211}. Hint, do you need to worry about the r dependent terms? (15 pts)

8) Derive the formula which gives the wavelength of a photon transmitted when an electron undergoes a downward transition between energy levels. $E_n = -13.6eV/n^2$ (4 pts)

9) Infinite well: superposition of energy eigenstates

Suppose that, at one instant, a particle of mass m in an infinite square well of width L is in the state

$$\Psi(x) = \frac{1}{2}\,\phi_1(x) + \frac{1}{3}\,e^{i\pi/2}\,\phi_3(x) + \frac{1}{2}\,e^{i\pi}\,\phi_4(x) + \sqrt{\frac{7}{18}}\,e^{i\pi/2}\,\phi_6(x)$$

where $\phi_n(x)$ are the energy eigenstates.

A) Suppose that the energy of a single particle in this state is measured. What are the possible outcomes of this measurement? (2 pts)

B) Is it possible that the energy measurement yields $E = 5\,\frac{\hbar^2\pi^2}{2mL^2}$? (1 pt)

C) What are the probabilities with which the various outcomes of the energy measurement occur? (3 pts)

D) Suppose that 10000 identical particles are each prepared in the this state and that the energy of each is measured. List the outcomes of this measurement and the number of times that you expect to get each outcome. (3 pts)

E) Determine the expectation value of the energy and the uncertainty in the energy. (4 pts)

10) If one is given a generic function describing the state of a system at t=0 given by $\Psi(x,0)$ demonstrate how one can find the expansion coefficients in terms of the infinite square well eigenfunctions $\sqrt{\frac{2}{a}}\sin(n\pi x/a)$ and the momentum space eigenfunctions $e_p = \frac{1}{\sqrt{2\pi\hbar}}e^{ipx/\hbar}$. What do the expansion coefficients tell you? (4 pts)