Physics 321, Fall 2012 Exam 1

Dr. Jared Workman, Wednesday, October 3rd 9:00-10:50

RULES

You may use an equation sheet. This sheet may NOT have solutions to problems but may contain any formulae or integrals you choose to include.

Problems

1: SHORT ANSWER

A) In quantum mechanics an observable quantity is represented by what?

B) Write down the generic formula for calculating the expectation value of a quantity. Write it out explicitly for kinetic energy.

C) What does the expectation value tell you about the system? In what way does the expectation value obey classical mechanical laws?

D) What is the distinction between the expectation value and the most probable value? Are both always possible outcomes of the measurement of a system?

E) A measurement is performed on a function that is an eigenfunction of the operator corresponding to the measurement. Describe what the result physically tells you about the state of the system. Describe what you can and can't know if another operator is applied to a function that is not an eigenfunction of that operator.

F) What exactly is Schröedinger's Equation solving for? What needs to be done to the solution to make it physically meaningful? Once you've done that how do you use it to find the probability a particle is between $x = a$ and $x = b$? Write out the formula explicitly.

2: For a particle moving an appreciable fraction of the speed of light the energy is no longer given by $p^2/2m$ rather it is given by $E^2 = p^2c^2 + m^2c^4$. Using the operator representation of observables "derive" the equation which describes the evolution of a spinless, relativistic particle. Use the wave function $\Psi = e^{i(\vec{k}\cdot\vec{r}-\omega t)}$. Remember - $\vec{k}\cdot\vec{r} = k_x x + k_y y + k_z z$. I want the fully three dimensional solution.

3: A particle incident from the left encounters a potential step at $x = 0$ with $V = V_0$. The particle has $E > V_0$. A short while later the potential becomes infinite at $x = a$. Write down the solution to Schröedinger's equation in the two regions. (see the illustration on the back)

A) Use continuity of the wave function and its derivative at $x = 0$ and $x = a$ to generate a system of equations which would allow you to solve for the magnitudes in terms of one unknown.

B) write down the reflection probability R in terms of coefficients. DO NOT SOLVE IT, simply indicate which combination of coefficients would be required to calculate it. What do you think the value of R will be?

4: THE LONG ONE - Particle in an infinite square well

A) Solve it and write down your solution basis functions in terms on "n".

B) write down but do not integrate the orthogonality condition in terms of your basis functions and the Kronecker Delta function.

C) Write down what the momentum is and what the energy is for this system.

D) Show explicitly that the basis functions are eigenfunctions of the energy operator.

E) Show explicitly that the basis functions are NOT eigenfunctions of the momentum operator. Think back to 1 E, comment on whether you can have determinate states of momentum associated with the particle in an infinite square well.

F) Calculate σ_x and σ_p. Show explicitly that their product satisfies the Heisenberg Uncertainty Principle.

G) A particle's wave function is $(x-a)^2$ at $t = 0$. Write down the full time dependent solution in terms of the expansion coefficients and temporal portion. This should include a sum and an integral. Do not evaluate the integral.

Equations

$sin^2(\theta) = \frac{1-cos(2\theta)}{2}$
$cos^2(\theta) = \frac{1+cos(2\theta)}{2}$
$\int x \cdot sin(x)dx = sin(x) - x \cdot cos(x)$
$\int x \cdot cos(x)dx = x \cdot sin(x) + cos(x)$
$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$