Stellar Astrophysics
Lecture 7

* Fission/Fusion
* Nucleosynthesis
* Post main-sequence evolution
* Star death
* Remnants

Early ideas for stellar energy source

* Solar Luminosity is 3.839 x 10^{33} erg s^{-1}
* Compare with approximately 5 x 10^{27} ergs used by the world in 2008
* Time scale is energy budget/luminosity
* Chemical?
  * 10,000 years
* Gravitational?

$$\Delta E_g = \frac{3GM^2}{10R} \sim 1.1 \times 10^{41} J$$

$$t_{KH} = \frac{\Delta E_g}{L_{solar}} \sim 10^7 yr$$

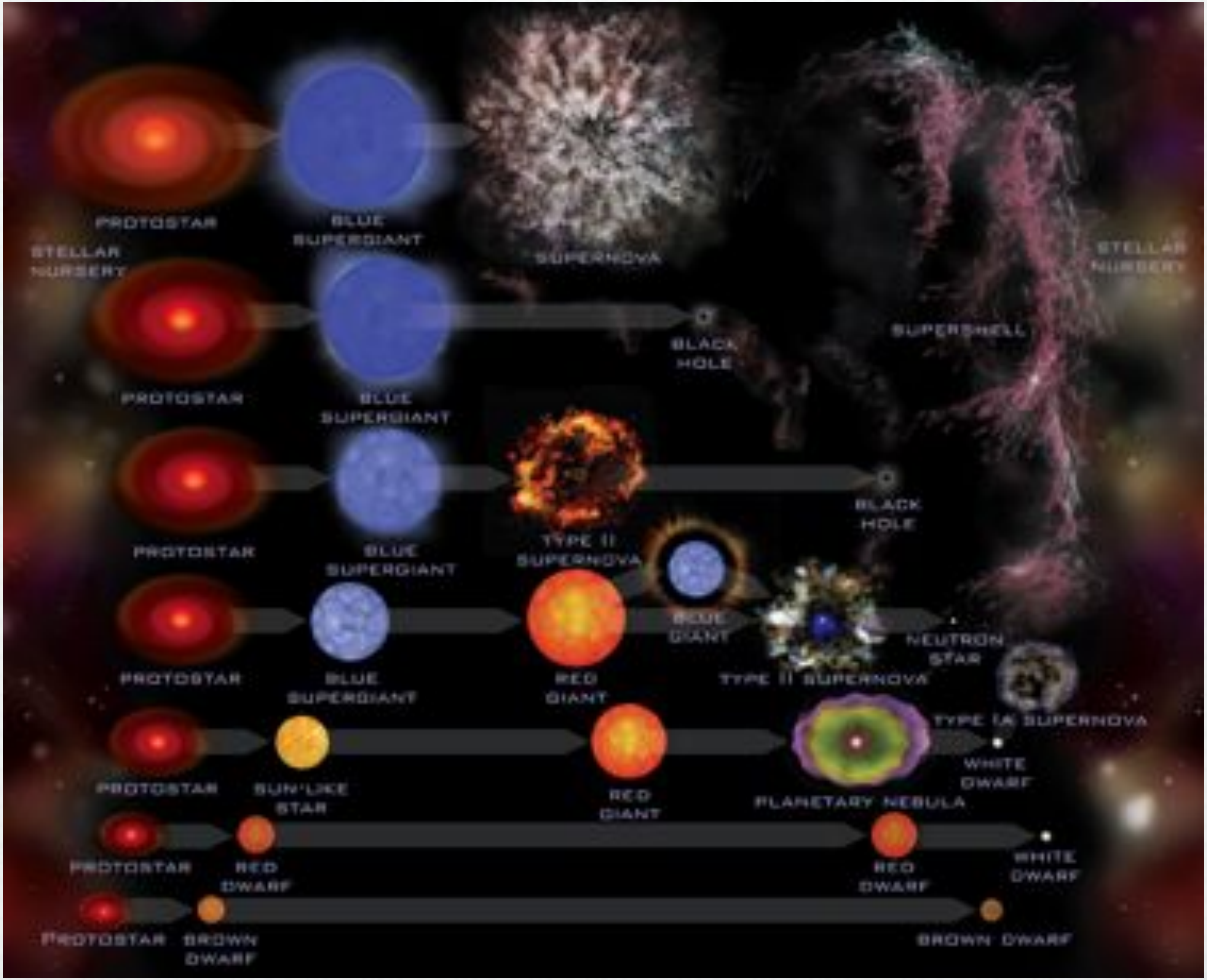
Nuclear?

* $E_{nuclear} = .1 \times .007 \times M_{solar} \times c^2 = 1.3 \times 10^{44} J$
* $T_{nuclear} \sim 10^{10} yr$
* Nuclear is the source, also very finicky
* Life of a 25 solar mass star

Stages in the Life of a 25 Solar Mass Star

Burning phase	Required temperature	Required mean density	Duration
Hydrogen burning	4×10^7 degrees K	5 gm per cubic cm	7,000,000 years
Helium burning	2×10^8 degrees K	700 gm per cubic cm	700,000 years
Carbon burning	6×10^8 degrees K	200,000 gm per cubic cm	600 years
Neon burning	1.2×10^9 degrees K	4 million gm per cubic cm	1 year
Oxygen burning	1.5×10^9 degrees K	10 million gm per cubic cm	6 months
Silicon burning	2.7×10^9 degrees K	30 million gm per cubic cm	1 day

Lives of stars vary dramatically



Nuclear Physics

Fundamentally explains these features of stellar lifetimes

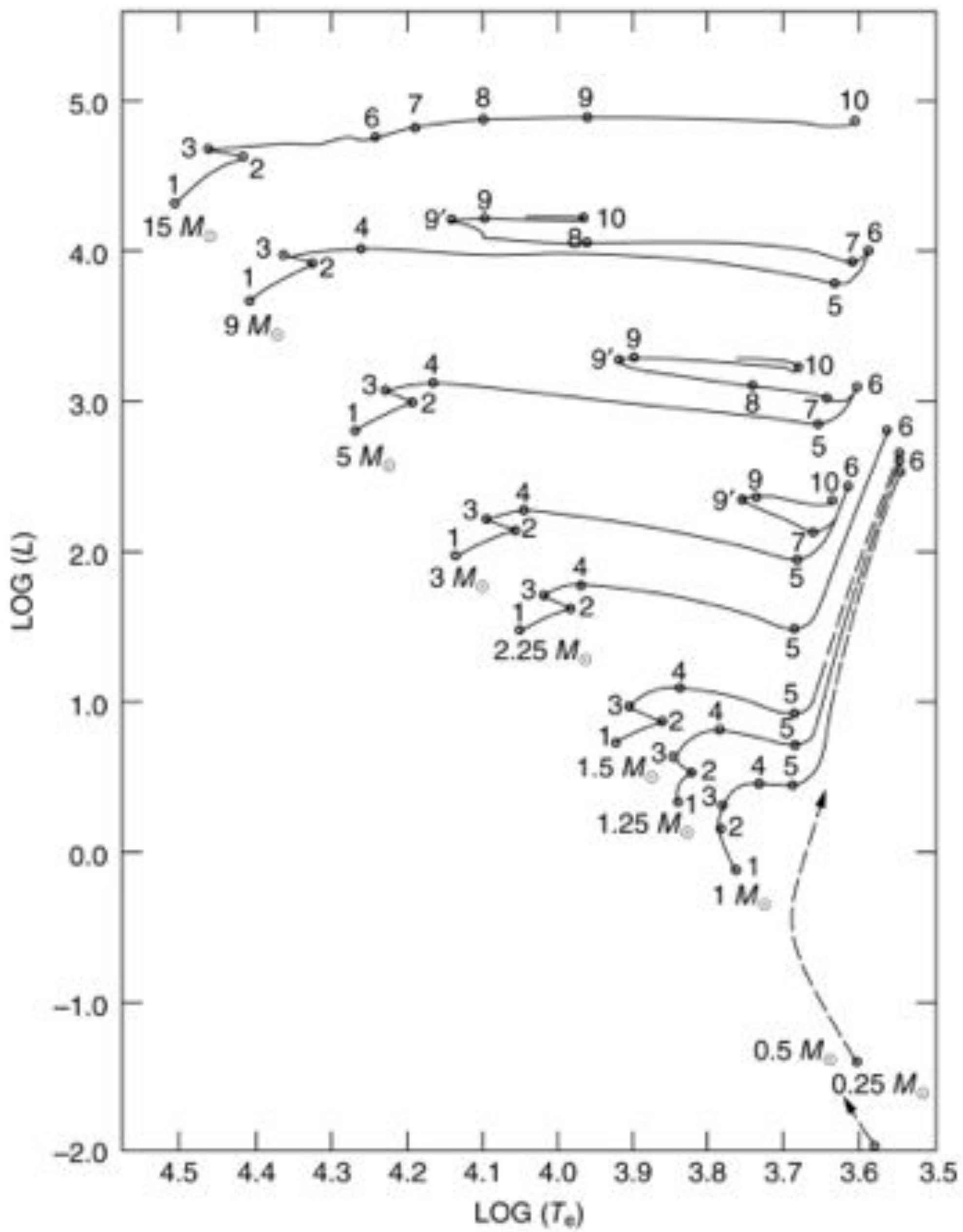


Figure 6.10 Partial evolutionary tracks for stars of various masses. The numbered points on the curves represent various time steps of the evolutionary process. Figure reproduced with permission from Iben, I., *Annual Review of Astronomy and Astrophysics*, 5, 571 (1967).

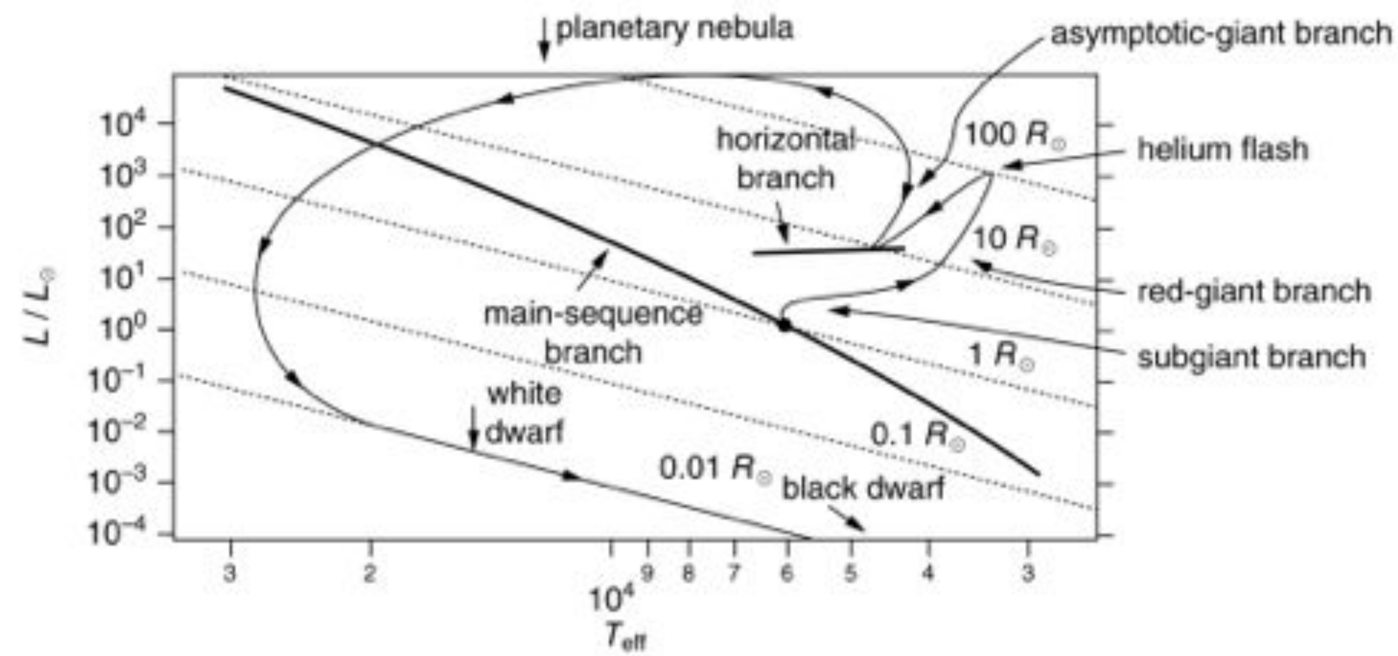


Figure 6.13 Illustration of the approximate evolutionary track of a $1 M_\odot$ star in the H–R diagram. The dotted lines show the position for various values for the radius.

It's all quantum mechanics and electromagnetism vs gravity

How can we explain the lifetimes of stars or why the PP cycle goes as T^4, the CNO as T^{20} and the triple alpha process as T^{41}

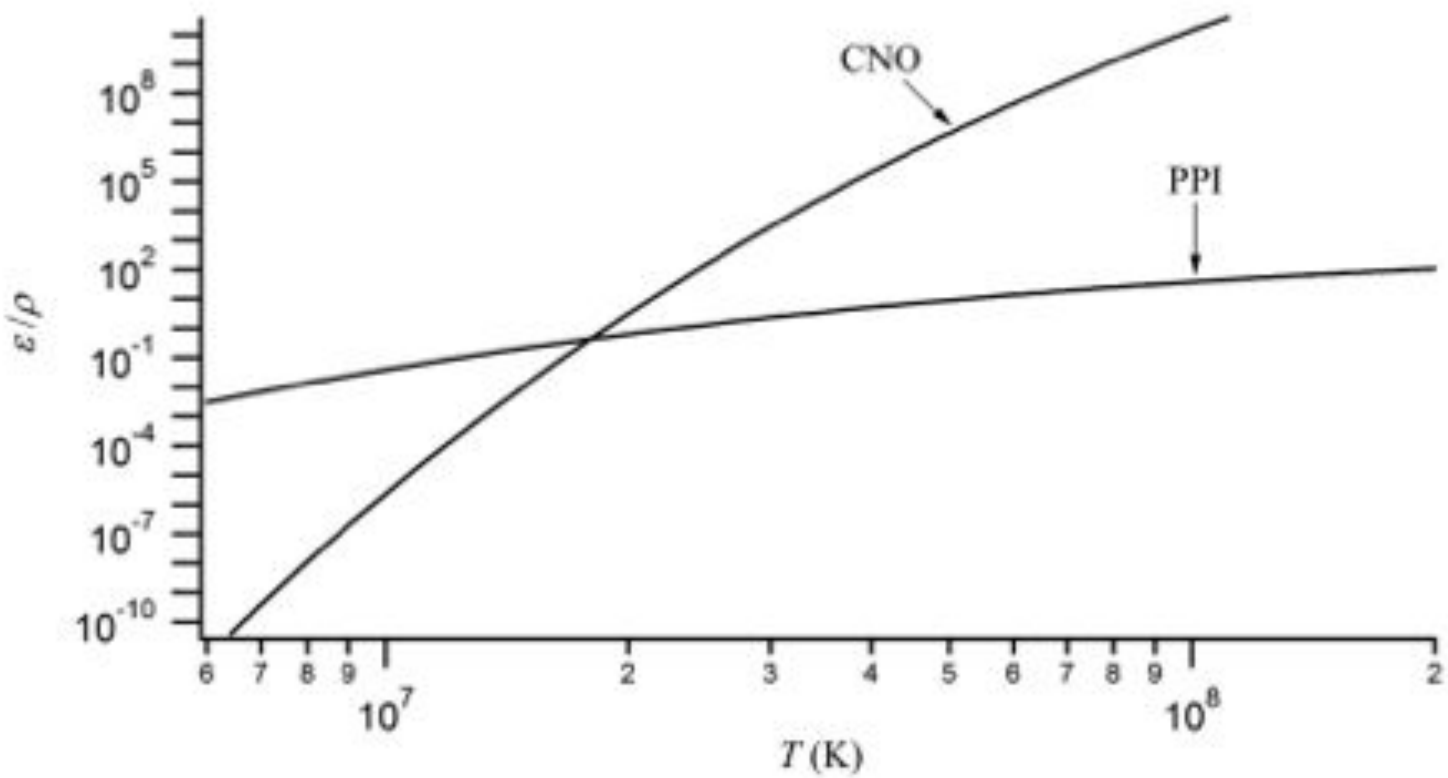
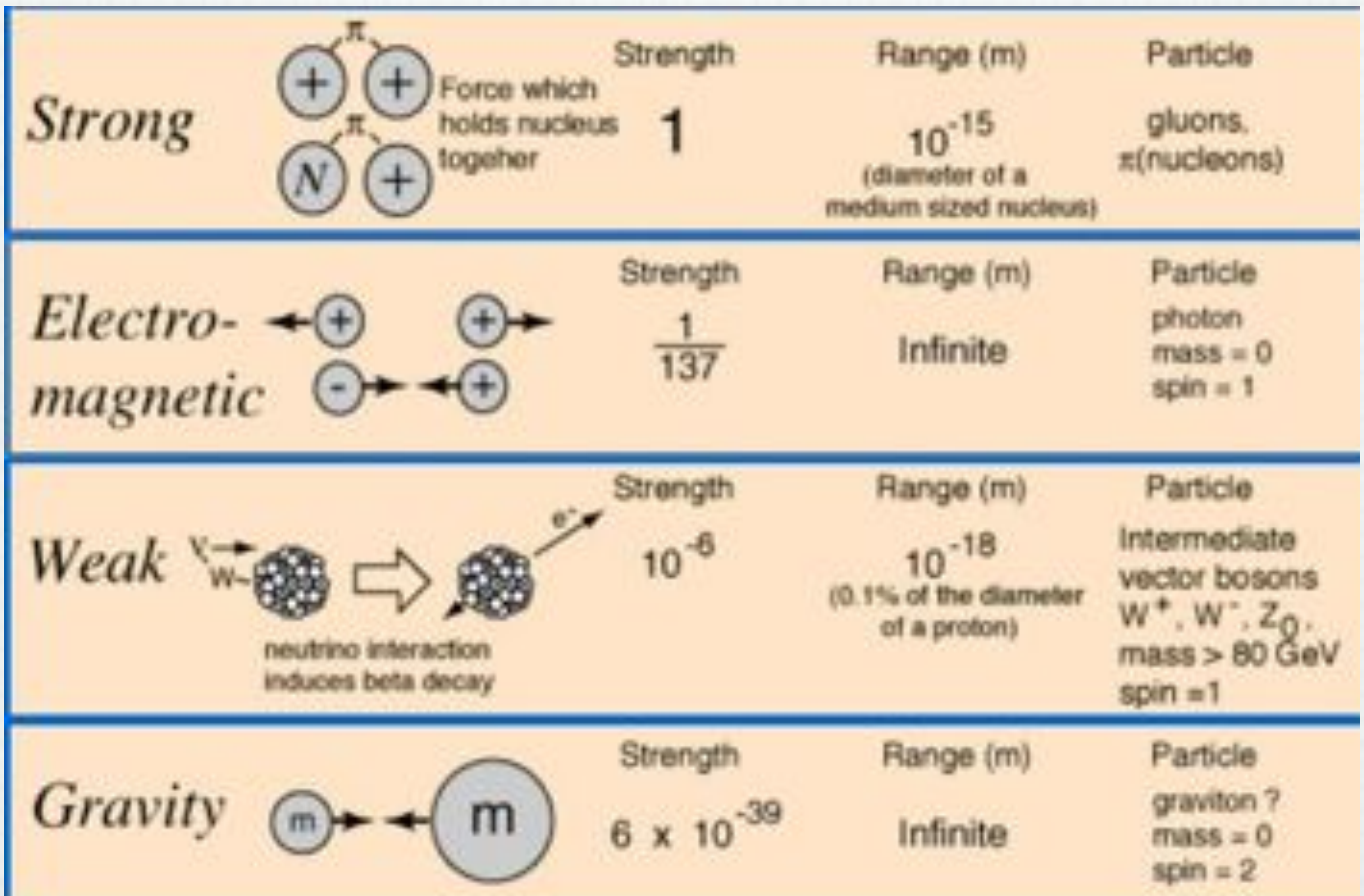


Figure 6.7 The temperature dependence of the nuclear energy production rate per unit mass (ε) for the PPI chain and the CNO cycles.

Basics

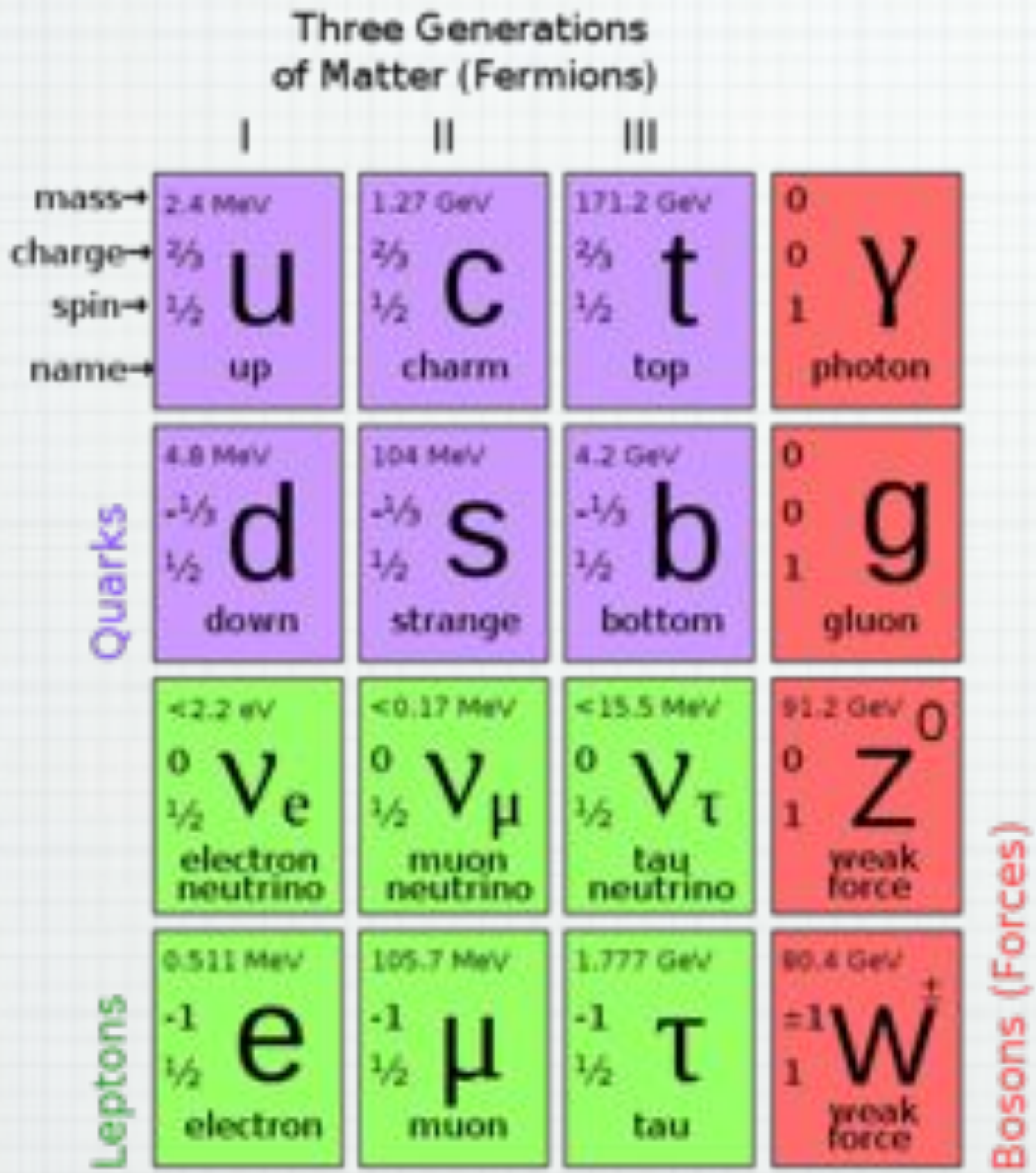
* Four fundamental forces



Force	Description	Strength	Range (m)	Particle
Strong	Force which holds nucleus togeher	1	10^{-15} (diameter of a medium sized nucleus)	gluons, π(nucleons)
Electro-magnetic		$\frac{1}{137}$	Infinite	photon mass = 0 spin = 1
Weak	neutrino interaction induces beta decay	10^{-6}	10^{-18} (0.1% of the diameter of a proton)	Intermediate vector bosons W^+, W^-, Z_0, mass > 80 GeV spin = 1
Gravity		6×10^{-39}	Infinite	graviton ? mass = 0 spin = 2

Two types of particles

* Leptons and Baryons (plus their anti particles)



Baryons qqq and Antibaryons $\bar{q}\bar{q}\bar{q}$

Symbol	Name	Quark content	Electric charge	Mass GeV/c^2	Spin
p	proton	uud	1	0.938	1/2
$\bar{p}$	anti-proton	$\bar{u}\bar{u}\bar{d}$	−1	0.938	1/2
n	neutron	udd	0	0.940	1/2
Λ	lambda	uds	0	1.116	1/2
Ω^-	omega	sss	−1	1.672	3/2

Nuclear Reactions

* obey several conservation laws
  * energy
  * momentum
  * charge
  * baryon and lepton number
  * Example

$$e^{-} + p \rightarrow \nu_e + n$$

Fission vs Fusion

- Squish vs smash
- Why is iron the endpoint for stellar lives?
- Binding energy per nucleon (Z+N=A)

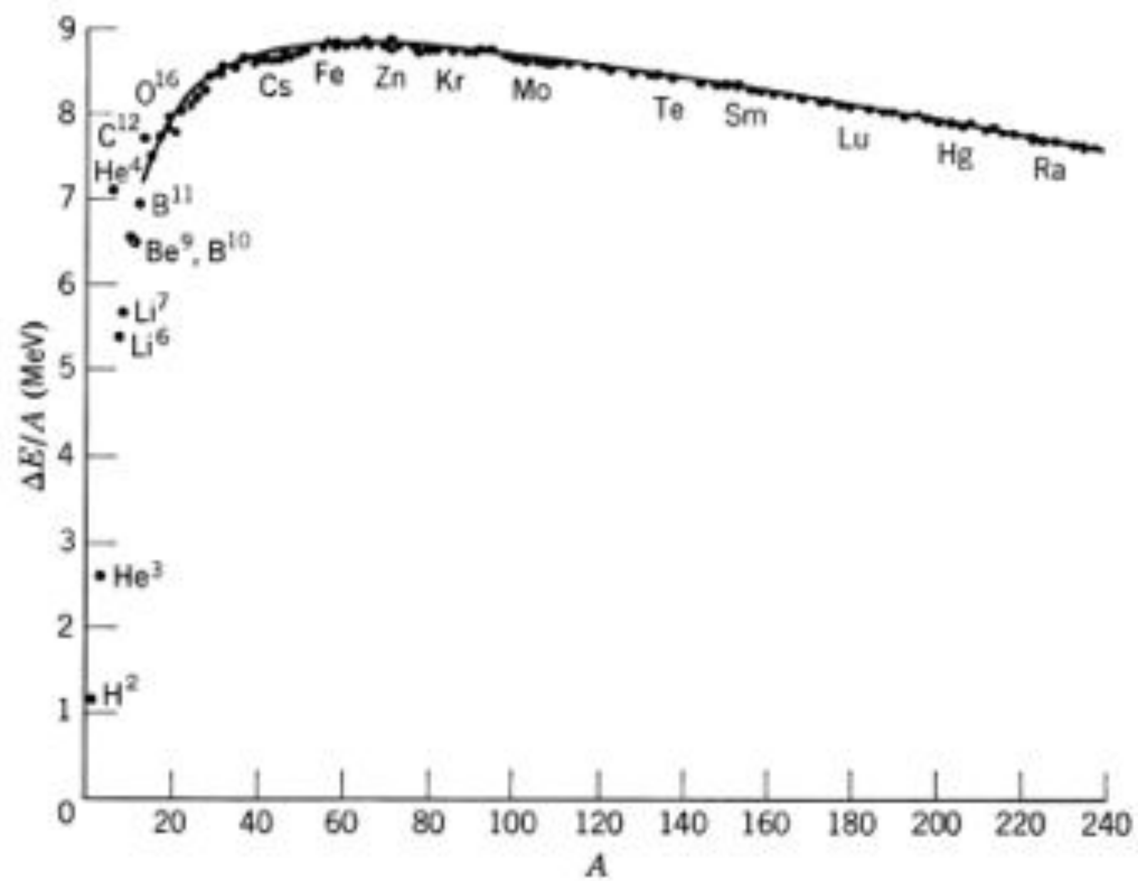
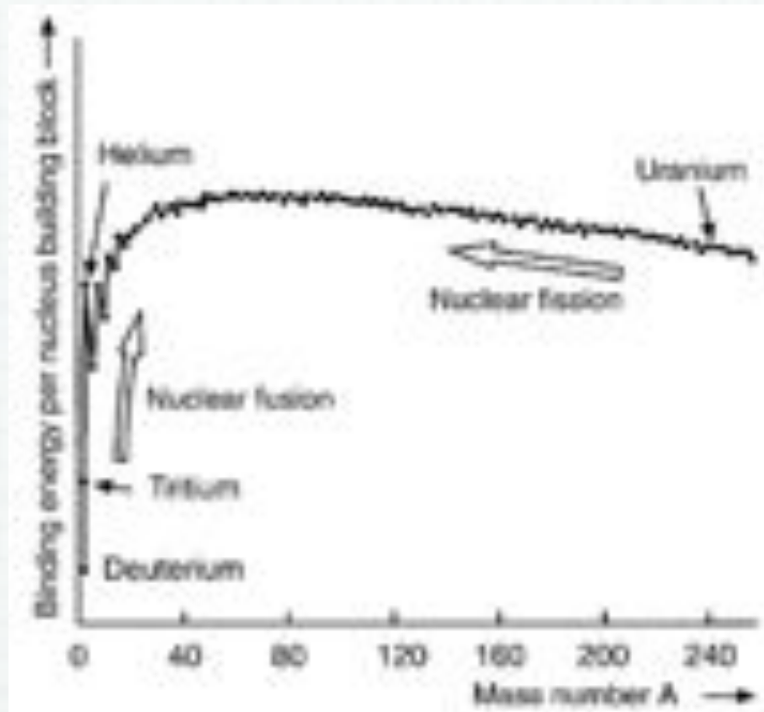


Figure 6.1 The average binding nuclear energy per nucleon $\Delta E/A$ as a function of the number of nucleons A in the various nuclei shown. The solid curve represents the results using the semiempirical mass formula (see (optional) Section 6.3.1). Reproduced with permission from Eisberg, R. and Resnick, R., *Quantum Physics of Atoms, Molecules, Solids, Nuclei and Particles*, John Wiley &



$$\frac{E_b}{A} = \frac{\Delta mc^2}{A} = \frac{[Zm_p + (A-Z)m_n - m_{nucleus}]c^2}{A}$$

Grows up to iron in exothermic reactions, then reactions are endothermic

I like to think of this differently

* Up until iron, two parent nuclei fuse to make a daughter nuclei that is less massive than the parents. The mass deficit goes into radiant and kinetic energy as the parents fall into a well
* After iron the two parent nuclei are LESS massive in total than the daughter. Energy must be added to make up for the mass deficit to fuse them so energy is removed from a system in these fusion processes, not added to it
* Fission however breaks a more massive parent nucleus into less massive daughter nuclei releasing energy.
* Fusing elements more massive than iron removes energy from a system rapidly

Liquid drop model

- Or a first pass at understanding nuclear properties
- u = 1.661 x 10^{-24} g
  - m_p = 1.0072765u
  - m_n = 1.0086649u

$$m(Z,N) = 1.0072765Z + 1.0086649N - \frac{\Delta E}{c^2}$$

- the binding energy is in units of uc^2

Liquid Drop Model

$$\Delta E = E_{vol} + E_{surf} + E_{coul} + E_{asym} + E_{pair}$$

* The volume term is positive binding due to neighboring nucleons
* The surface term is negative as these nucleons feel internal nucleons less
* The coulomb force is repulsive
* The next two terms are also negative, too much detail

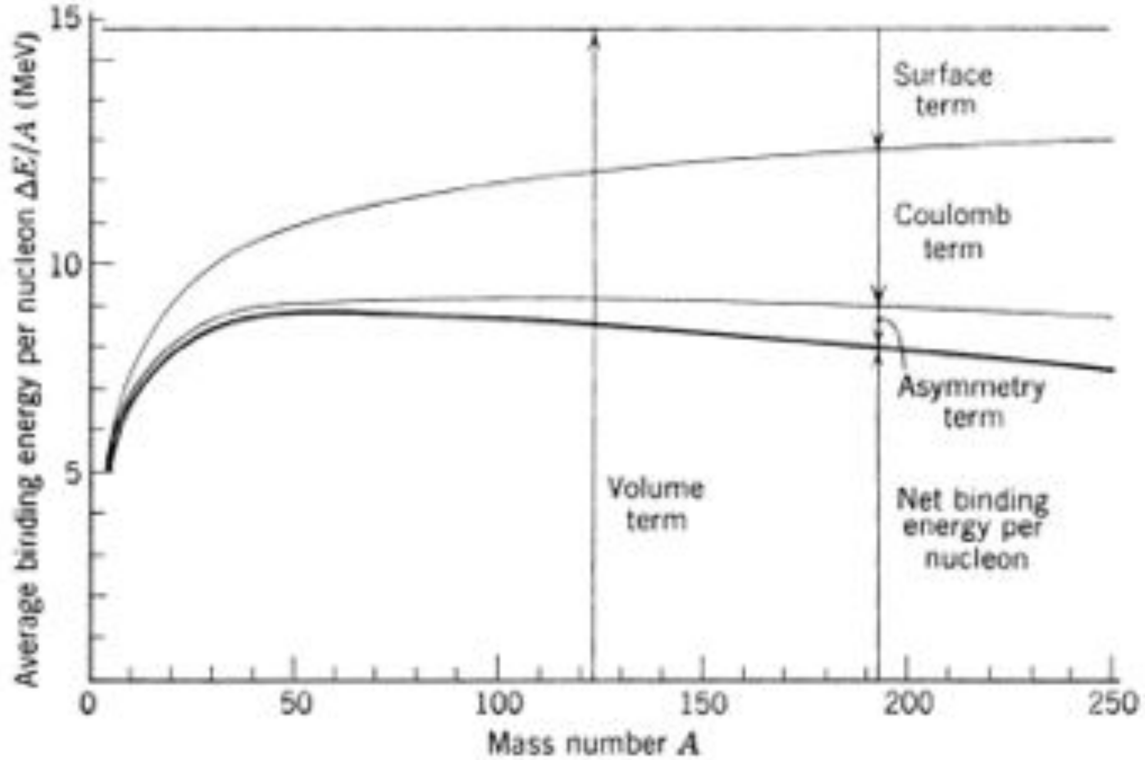


Figure 6.2 The relative importance of the volume, surface, Coulomb and asymmetry term of the semiempirical mass formula for the average binding energy per nucleon ($\Delta E/A$). Reproduced with permission from Eisberg, R. and Resnick, R., *Quantum Physics of Atoms, Molecules, Solids, Nuclei and Particles*, John Wiley & Sons, Ltd, New York (1985).

$$E_B = a_V A - a_S A^{2/3} - a_A \frac{(A-2Z)^2}{A^{1/3}} - a_C \frac{Z(Z-1)}{A^{1/3}} + \delta(A,Z)$$

Volume term, Surface term, Asymmetry term, Coulomb term, Pairing term

For pairing term:

$$\delta(A,Z) = \begin{cases} +\delta_o & \text{A, Z even} \\ 0 & \text{A odd} \\ -\delta_o & \text{A, Z odd} \end{cases}$$

where

$$\delta_o = \frac{a_P}{A^{1/2}}$$

Coefficients:

$a_V = 15.85$ MeV
$a_S = 18.34$ MeV
$a_A = 23.21$ MeV
$a_C = 0.714$ MeV
$a_P = 12.00$ MeV

Fusion in stars

Basics of Nuclear Fusion

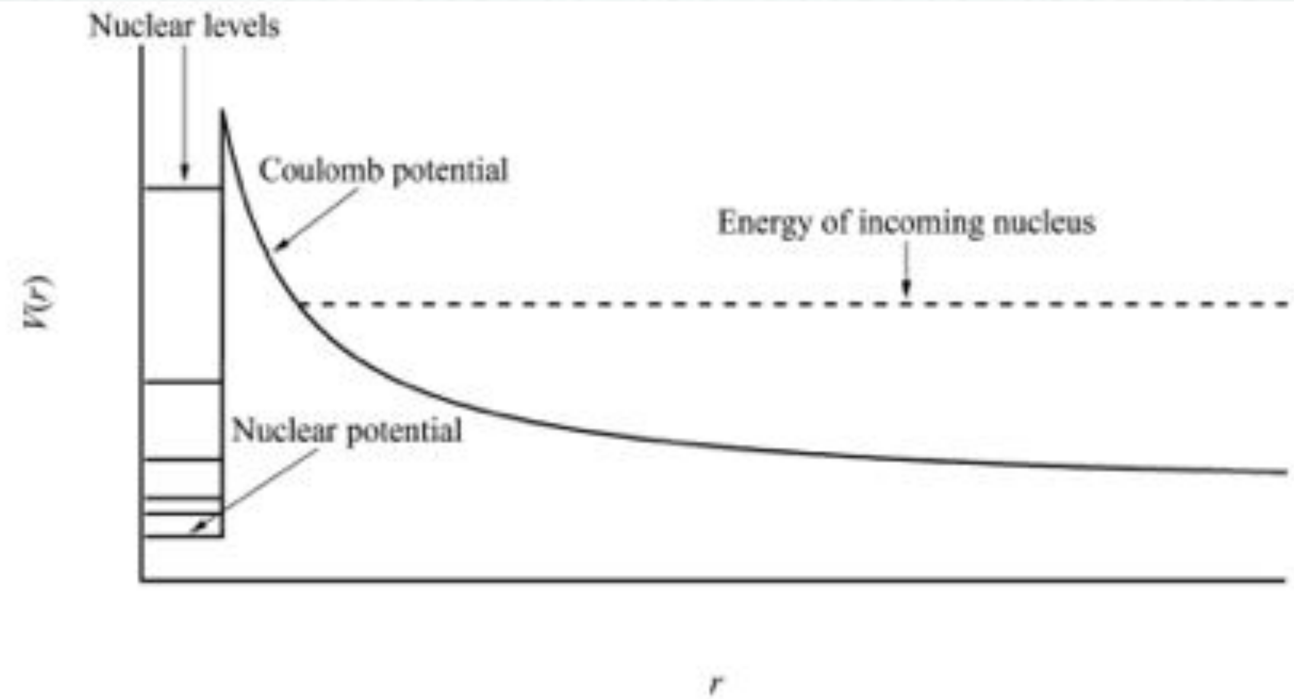


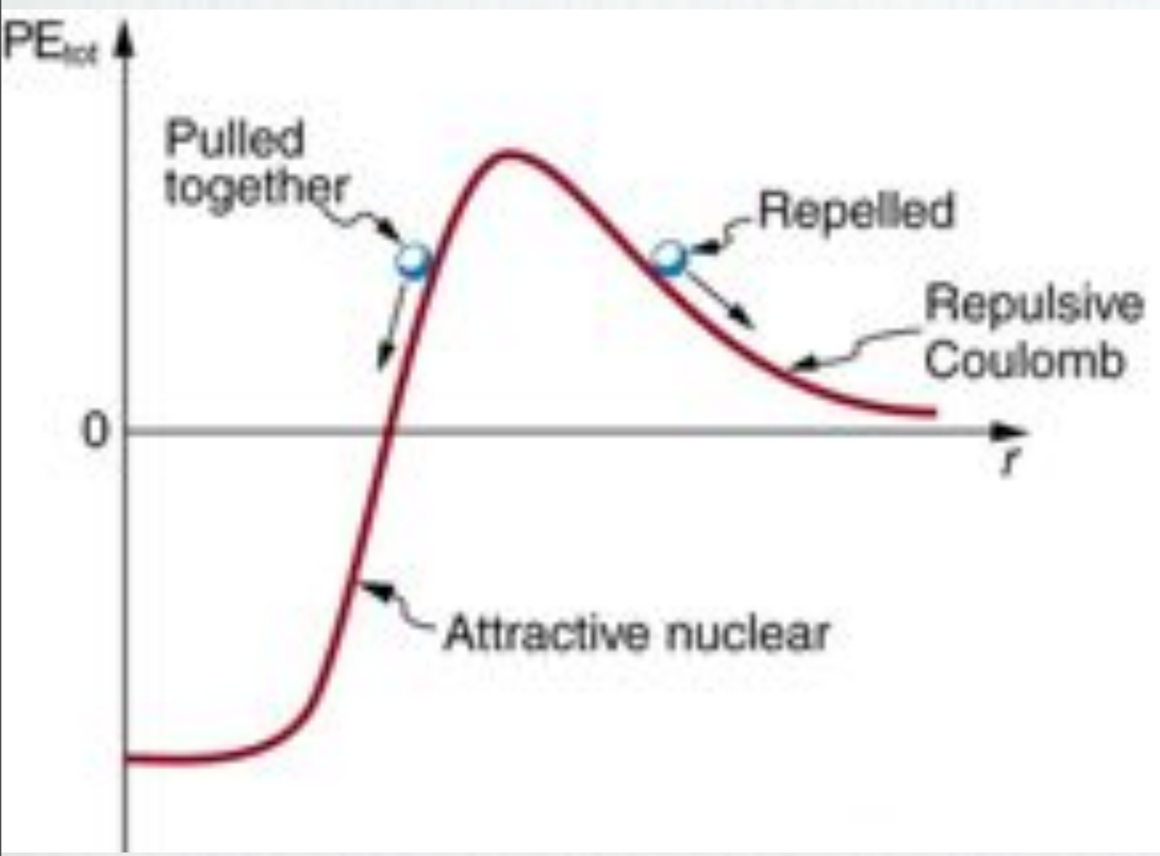
Figure 6.5 A schematic view of the potential of a nucleus (solid line). The potential due to the strong nuclear force is approximated by a square well. Outside this well the potential is due to the Coulomb repulsion force. The dashed line represents the energy of an incoming nucleus that is attempting to fuse with the nucleus, or to tunnel through the potential illustrated in the figure. Also shown in the figure are nuclear energy levels found inside the well.

* Without quantum mechanics

$$\frac{1}{2}\mu\bar{v}^2 = \frac{3}{2}k_B T_{classical} = \frac{1}{4\pi\epsilon_0}\frac{Z_1 Z_2 e^2}{r} \rightarrow T_{classical} \sim 10^{10}K$$

* With quantum mechanics

$$\Delta x \Delta p_x \geq \frac{\hbar}{2} \rightarrow \lambda = \frac{h}{p} \rightarrow \lambda \propto r$$

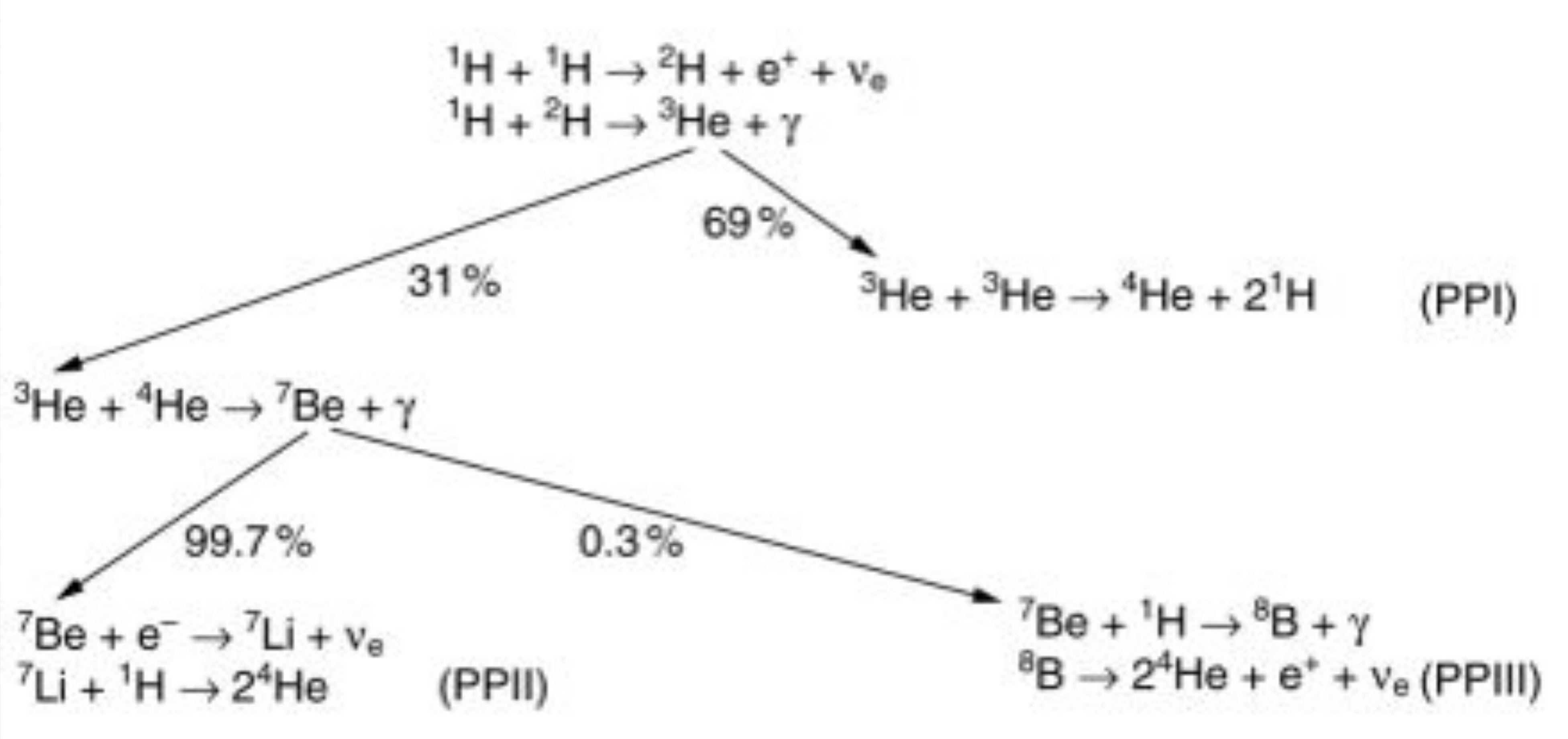


$$\frac{\mu\bar{v}^2}{2} = \frac{p^2}{2\mu} = \frac{(h/\lambda)^2}{2\mu} = \frac{1}{4\pi\epsilon_0}\frac{Z_1 Z_2 e^2}{\lambda}$$

$$T_{quantum} = \frac{Z_1^2 Z_2^2 e^4 \mu}{12\pi^2\epsilon_0^2 h^2 k_B} \sim 10^7 K$$

Two primary processes in main sequence stars

PP cycle



Globally $4^1H \rightarrow {}^4He + 2e^+ + 2\nu_e + 2\gamma$

$$E = (m_{initial} - m_{final})c^2 + 2\gamma - 2\nu_e = (4m_H - m_{He} - 2m_e)c^2 + 2\gamma - 2\nu_e$$

where the neutrinos are counted as a loss term since they leave the star

E=[1-(.02,.04,.279)]26.732 MeV
(PP1, PP2, PP3)

CNO cycle

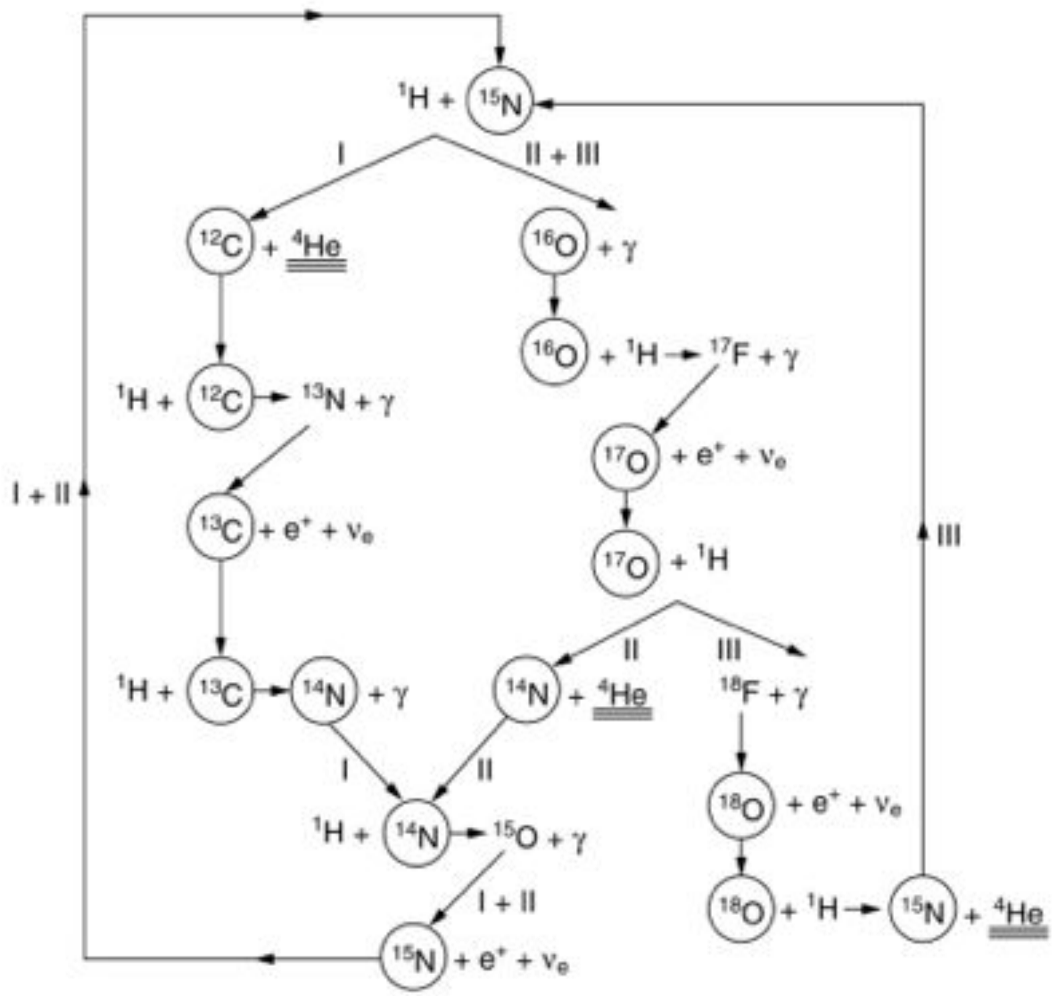


Figure 6.6 Illustration of the CNO cycles. The catalysts are circled. Figure reproduced with permission from Pearson, J.M., *Nuclear Physics: Energy and Matter*, Adam Hilger, Bristol (1986).

$$^{12}C + p \rightarrow {}^{13}N + \gamma$$
$$^{13}N \rightarrow {}^{13}C + e^+ + \nu_e$$
$$^{13}C + p \rightarrow {}^{14}N + \gamma$$
$$^{14}N + p \rightarrow {}^{15}O + \gamma$$
$$^{15}O \rightarrow {}^{15}N + e^+ + \nu_e$$
$$^{15}N + p \rightarrow {}^{12}C + {}^4He$$

$$4\,^1H \rightarrow {}^4He + 2e^+ + 2\nu_e + 3\gamma$$

Almost identical energy generation as PP chain.
Dominant in stars more massive than 1.5 times our sun

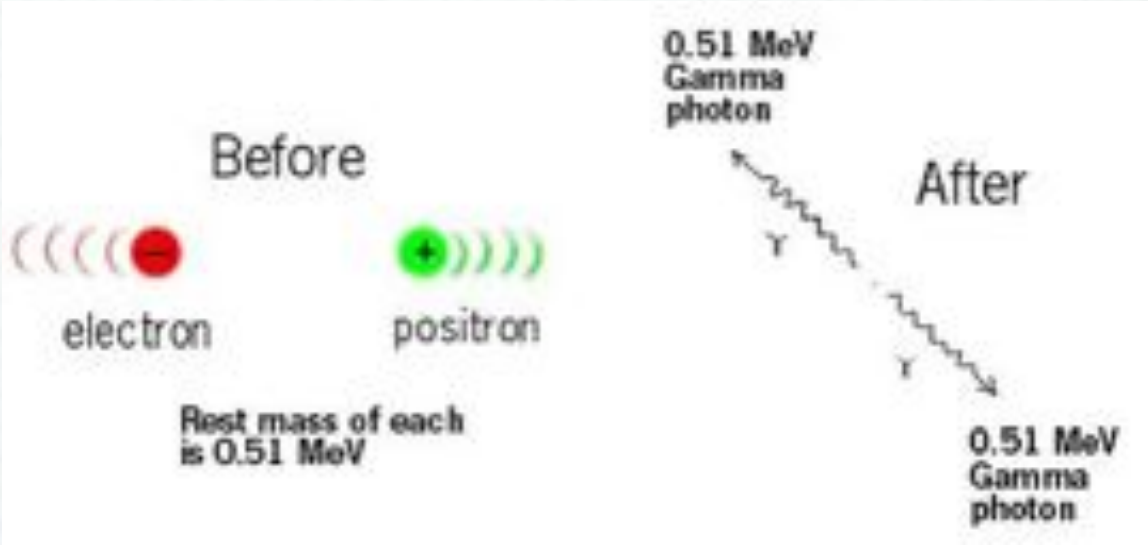
So how do we calculate how this actually happens?

* Reaction rates - r_{ix}
  * total reactions per second per unit volume where i is the incident particles and x is the target particle
* Energy liberated per second per unit mass - ε_{ix}
  * we need it for $\frac{dL(r)}{dr} = 4\pi r^2 \rho(r)\epsilon(r)$

Reaction Rates and Energy Production

* For a two particle interaction $r_{ix} \approx r_0 X_i X_x \rho^{\alpha'} T^{\beta}$
* Where the Xs are mass fractions, r_0 is a constant, and α' and β are power law fits from the reaction rate equations α' is usually around 2 and β can range from 1 to 40
* Then $\epsilon_{ix} = \frac{\xi_0}{\rho} r_{ix} = \xi_0 r_0 X_i X_x \rho^{\alpha'-1} T^{\beta} = \epsilon_0 X_i X_x \rho^{\alpha} T^{\beta}$
* Where ξ_0 is the energy produced in one reaction
* From which we may calculate $dL = \epsilon dm = (\epsilon_{nuclear} + \epsilon_{gravitational}) dm$
* and close our equations of stellar structure

* Nuclear physics tells us what reactions occur and how much energy we get in each reaction

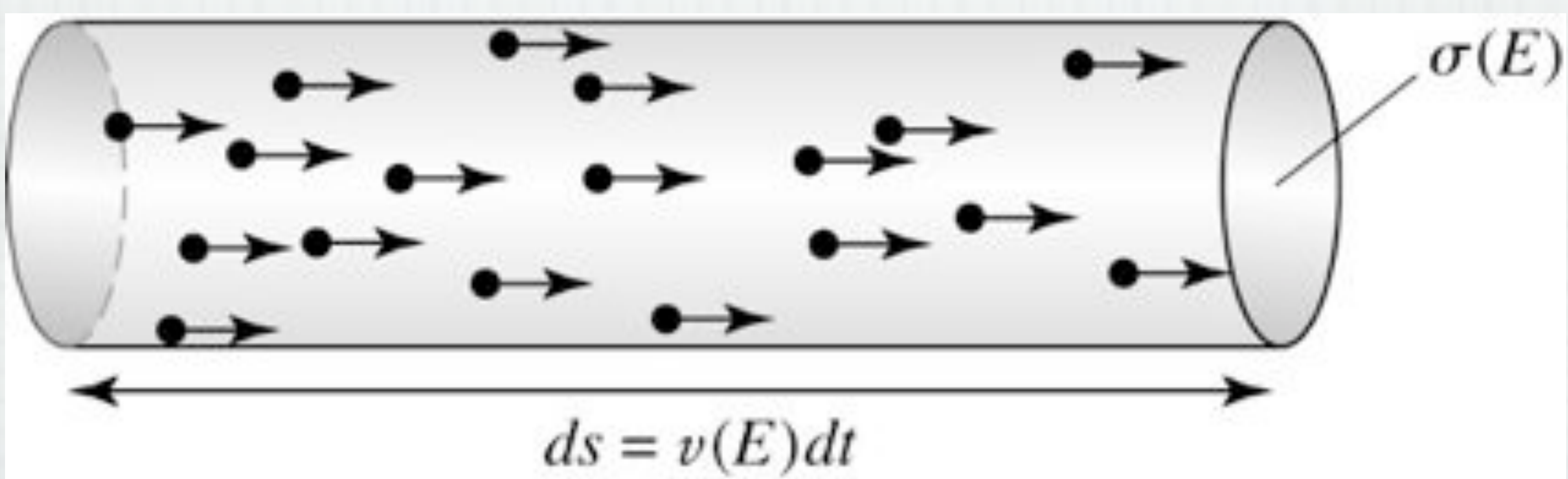


* We still need our reaction rate to get our energy production rate
* Outline as follows
* consider incident (i) particles on target particle (x) with a cross section σ for a fusion reaction to occur within

* For one fusion reaction to occur, two particles must get close enough to each other to fall in each other's strong potential wells
* This depends on tunneling probability and velocity
* Particles in a star take on a distribution of velocities with some being faster than others. Recall the Maxwell Distribution for velocity and make the substitution $K = E = \mu v^2/2$
* The number of particles with energy between E and dE may be written as

$$n_E dE = \frac{2n}{\pi^{1/2}} \frac{1}{(k_B T)^{3/2}} E^{1/2} e^{-E/k_B T} dE$$

* This equation gives the number of particles in a unit volume with energy in the range dE but says nothing about interactions
* We introduce an energy dependent cross section given by $\sigma(E)$=(# reactions/nucleus/time)/(# incident particles/area/time)
* The number of reactions per unit time with target x which has a cross section $\sigma(E)$will depend on the number of incident particles i with velocity v which sweep out a volume $\sigma(E)ds = \sigma(E)v(E)dt$ in time dt



* Strictly speaking the cross section is a measure of probability of tunneling but we may roughly approximate it as the target particle's cross sectional area

* The number of incident particles per unit volume with energy between E and dE is $n_{iE}dE$

* The number of reactions dN_E is given by the number of particles that can strike x in time interval dt with a velocity $v(E) = \sqrt{2E/\mu}$

* The number of reactions is then just the number/volume with the appropriate energy

$$dN_E = \sigma(E)v(E)n_{iE}dEdt$$

* Now the fraction of incident particles with appropriate energy for a reaction is also $n_{iE}dE = \frac{n_i}{n}n_E dE$

* Now the reactions per target nucleus X per unit time is dN_E/dt =

$$\sigma(E)v(E)\frac{n_i}{n}n_E dE$$

* For n_x targets the reaction rate r_{ix} is given by

$$r_{ix} = \int_o^\infty n_x n_i \sigma(E)v(E)\frac{n_E}{n}dE$$

* Simple right? What's the real form for the cross section?
* Varies rapidly with energy and reaction
* Generally solved for numerically
* Roughly, the cross section depends on two factors
  * the size of the target nucleus
  * the likelihood of the the incident and targets to tunnel across each other's coulomb barrier

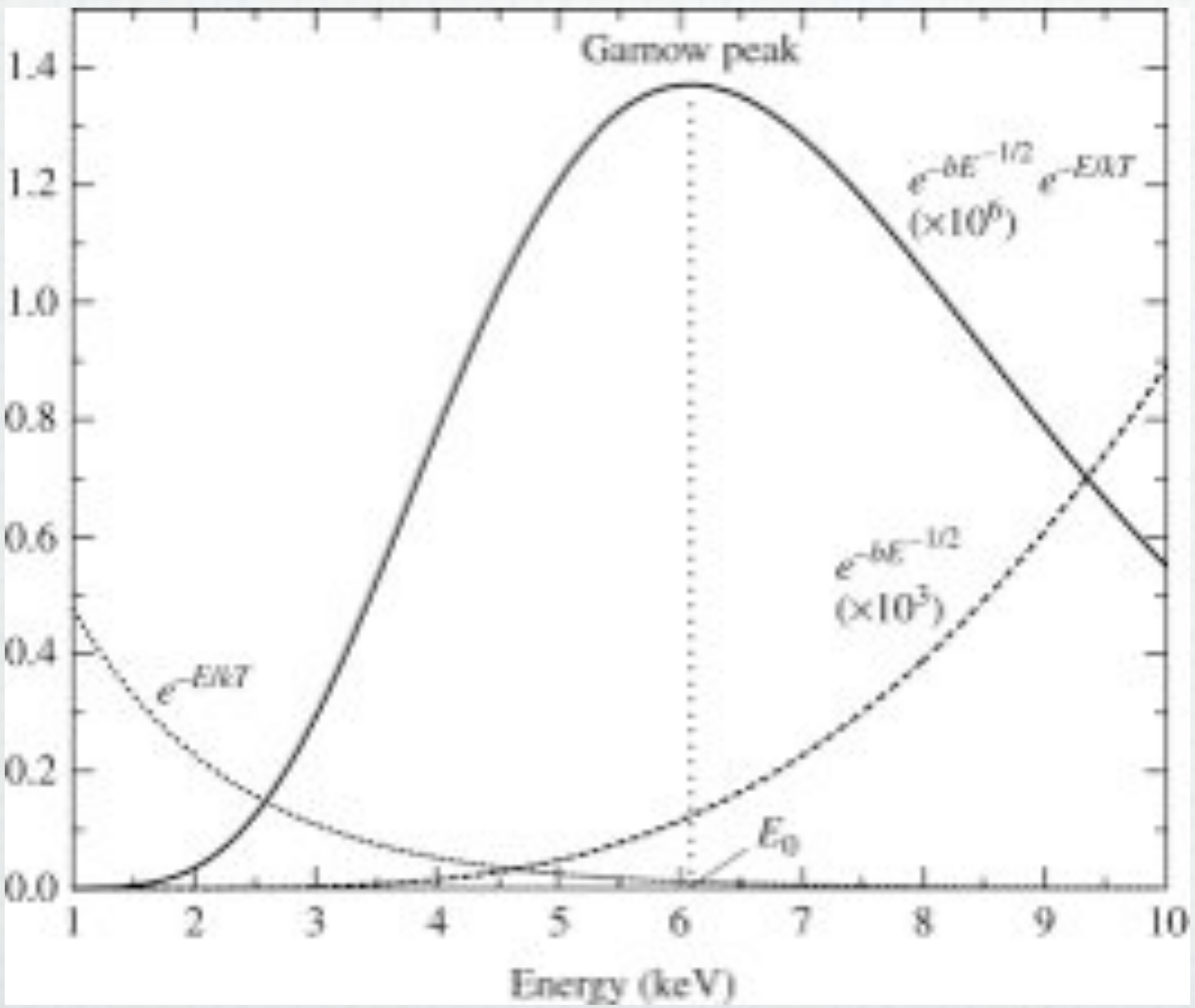
- The size of the target nucleus is of order a de Broglie wavelength $\sigma(E) \propto \pi\lambda^2 \propto \pi(\frac{h}{p})^2 \propto \frac{1}{E}$
- The tunneling probability gives $\sigma(E) \propto e^{-2\pi^2 U_C/E}$
- Where U_c is the coulomb barrier $\frac{U_c}{E} = \frac{Z_1 Z_2 e^2}{2\pi\epsilon_0 hv}$
- Leading to $\sigma(E) \propto e^{-bE^{-1/2}}$
- Where $b = \frac{\pi\mu^{1/2} Z_1 Z_2 e^2}{\sqrt{2}\epsilon_0 h}$
- Putting all this together leads to

$$\sigma(E) = \frac{S(E)}{E} e^{-bE^{-1/2}}$$

- Or finally

$$r_{ix} = (\frac{2}{k_B T})^{3/2} \frac{n_i n_x}{(\mu\pi)^{1/2}} \int_0^\infty S(E) e^{-bE^{-1/2}} e^{-E/k_B T} dE$$

* In general there is an energy around which reactions are most likely to occur. The product of the two exponential functions in the integral is non-zero only in certain places. S(E) is generally taken to be constant in the region of overlap and is taken outside of the integral as a constant



In practice S(E) is measured or numerically calculated

The Gamow peak is the region of maximum overlap

$$E_0 = (\frac{bk_BT}{2})^{2/3}$$

- So how do we do this integral?

$$r_{ix} = (\frac{2}{k_B T})^{3/2} \frac{n_i n_x}{(\mu\pi)^{1/2}} S_0 \int_0^\infty e^{-bE^{-1/2}} e^{-E/k_B T} dE$$

- Recalling that the product of exponentials is zero except at the maximum overlap given by $E=E_0$ we define $F(E) = -bE^{-1/2} - \frac{E}{k_B T}$
- and taylor expand this function about E_0 so that

$$F(E) = F(E_0) + F'(E_0)(E - E_0) + \frac{1}{2} F''(E_0)(E - E_0)^2$$

- But the expansion is about a maximum so the second term is zero

* Now $$e^{(-bE^{-1/2}-\frac{E}{k_BT})} \approx Ce^{-(\frac{E-E_0}{\Delta/2})^2}$$
* Where $$C = e^{(-bE_0^{-1/2}-\frac{E_0}{k_BT})} = e^{-3E_0/k_BT} = e^{-\tau}$$
* and $$\Delta = 4(\frac{E_0k_BT}{3})^{1/2}$$
* Now the integral can be trivially done

$$r_{ix} = (\frac{2}{k_BT})^{3/2}\frac{n_in_x}{(\mu\pi)^{1/2}}S_0\frac{\Delta}{4}\sqrt{\pi}e^{-3E_0/k_BT}$$

* Delta is roughly the standard deviation around the Gamow peak where interactions occur

Table 12.1 Parameters of the thermally averaged reaction rates at $T_6 = 15$.

Reaction	Coulomb barrier (MeV)	Gamow peak (E_0) (keV)	I_{max} ($e^{-3E_0/kT}$)	Δ (keV)	$(\Delta)I_{max}$
p + p	0.55	5.9	1.1×10^{-6}	6.4	7×10^{-6}
p + N	2.27	26.5	1.8×10^{-27}	13.6	2.5×10^{-26}
$\alpha + C^{12}$	3.43	56	3×10^{-57}	19.4	5.9×10^{-56}
$O^{16} + O^{16}$	14.07	237	6.2×10^{-239}	40.4	2.5×10^{-237}

Finally

$$\epsilon = C\rho X_1 X_2 \frac{1}{T^{2/3}} e^{[-3(\frac{e^4 Z_1^2 Z_2^2}{32 k_B \epsilon_0^2 \hbar^2 T})^{1/3}]}$$

- for pp

$$\epsilon_{pp} = 2.4 \times 10^{-1} \rho X^2 (\frac{10^6}{T})^{2/3} e^{[-33.8(\frac{10^6}{T})^{1/3}]} W kg^{-1}$$

- for CNO

$$\epsilon_{CNO} = 8.7 \times 10^{20} \rho X_{CNO} X (\frac{10^6}{T})^{2/3} e^{[-152.28(\frac{10^6}{T})^{1/3}]} W kg^{-1}$$

- For triple alpha

$$\epsilon_{3\alpha} = 50.9 \rho^2 Y^3 (\frac{10^8}{T})^3 e^{[-44.027(\frac{10^8}{T})]} W kg^{-1}$$

Some energy production rates

$$\epsilon_{pp} \approx \epsilon_{pp,0}\rho X^2 T_6^4 \rightarrow \epsilon_{pp,0} = 1.08 \times 10^{-12} W m^3 kg^{-2}$$

$$\epsilon_{CNO} \approx \epsilon_{CNO,0}\rho X X_{CNO} T_6^{19.9} \rightarrow \epsilon_{CNO,0} = 8.24 \times 10^{-31} W m^3 kg^{-2}$$

15 million K CNO kicks in

$$\epsilon_{3\alpha} \approx \epsilon_{3\alpha,0}\rho^2 y^3 T_8^{41}$$

100-200 million K triple alpha kicks in

For a ten percent increase in temperature the energy output in triple alpha increases 50 times

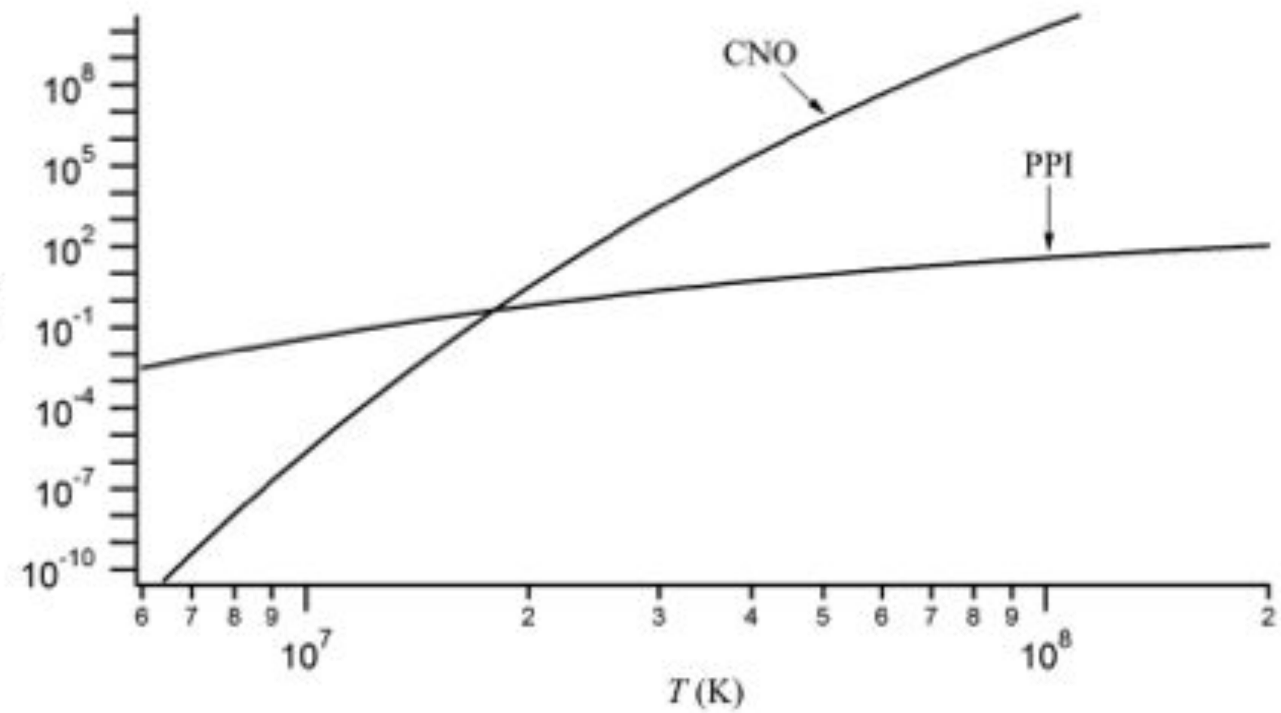
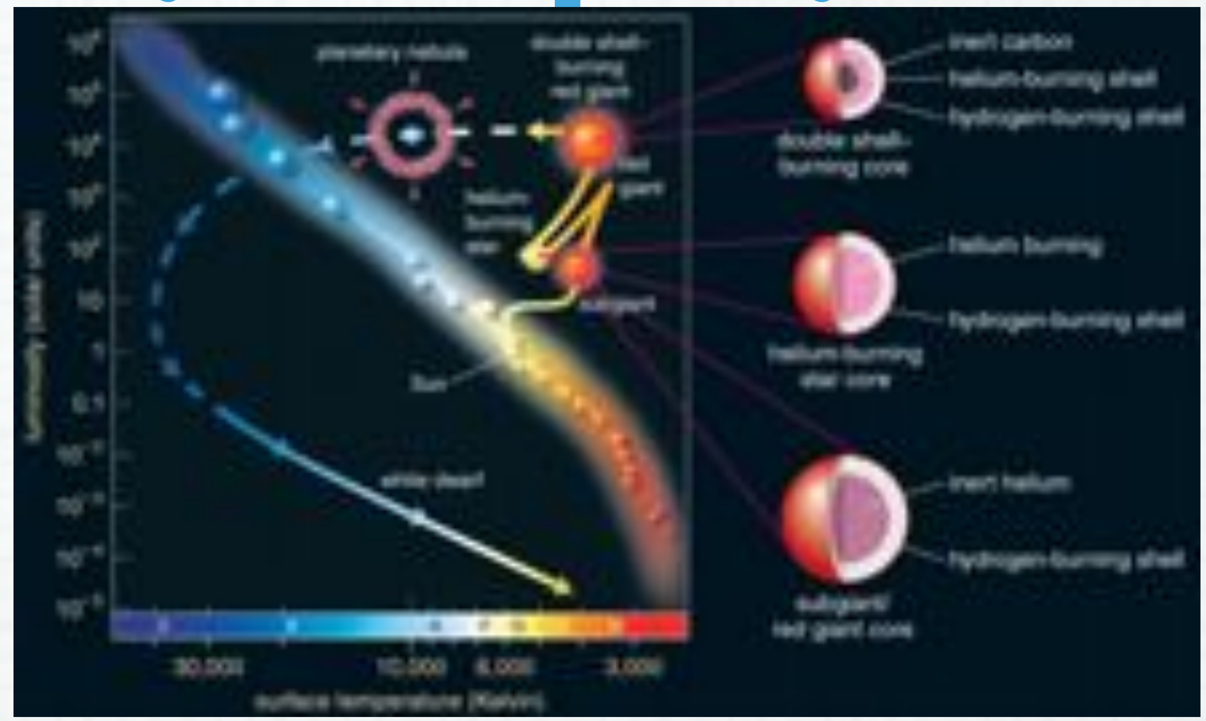


Figure 6.7 The temperature dependence of the nuclear energy production rate per unit mass (ε) for the PPI chain and the CNO cycles.

Off the main sequence

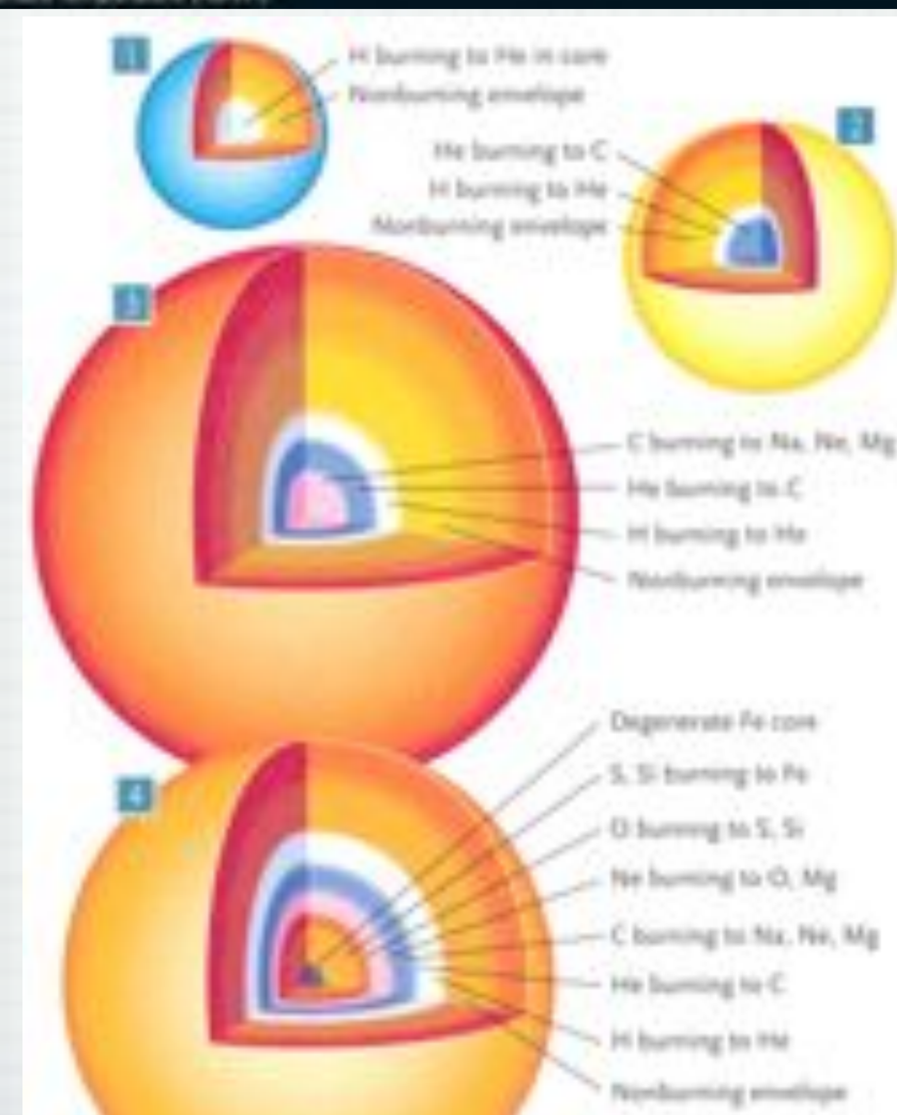


* Further stages of nuclear burning occur

* These are progressive but not exclusionary

* More than one can occur at once

* Generally a primary burning is occurring in the core with various others occurring in shells



Stellar Lifetimes

* Stars can fuse about 10 percent of their mass into helium
* .7% of mass in hydrogen goes into energy
* T_{ms} about $.007M_{star}c^2$
* More generally $t_{ms} = \frac{E_{tot}}{L_*} =\approx 10^{10}\frac{M_* L_\odot}{M_\odot L_*} yr$
* How about for a B or M star?
* For laughs let's see what the timescale would be if the sun was burning chemically, say 10eV energy/event

How about during helium burning?

$$3^4He \rightarrow {}^{12}C + 3\gamma \quad E \approx 2.275 MeV$$

$$\frac{E}{3m_{^4He}c^2} \approx 6.5 \times 10^{-4} = F_{burnt}$$

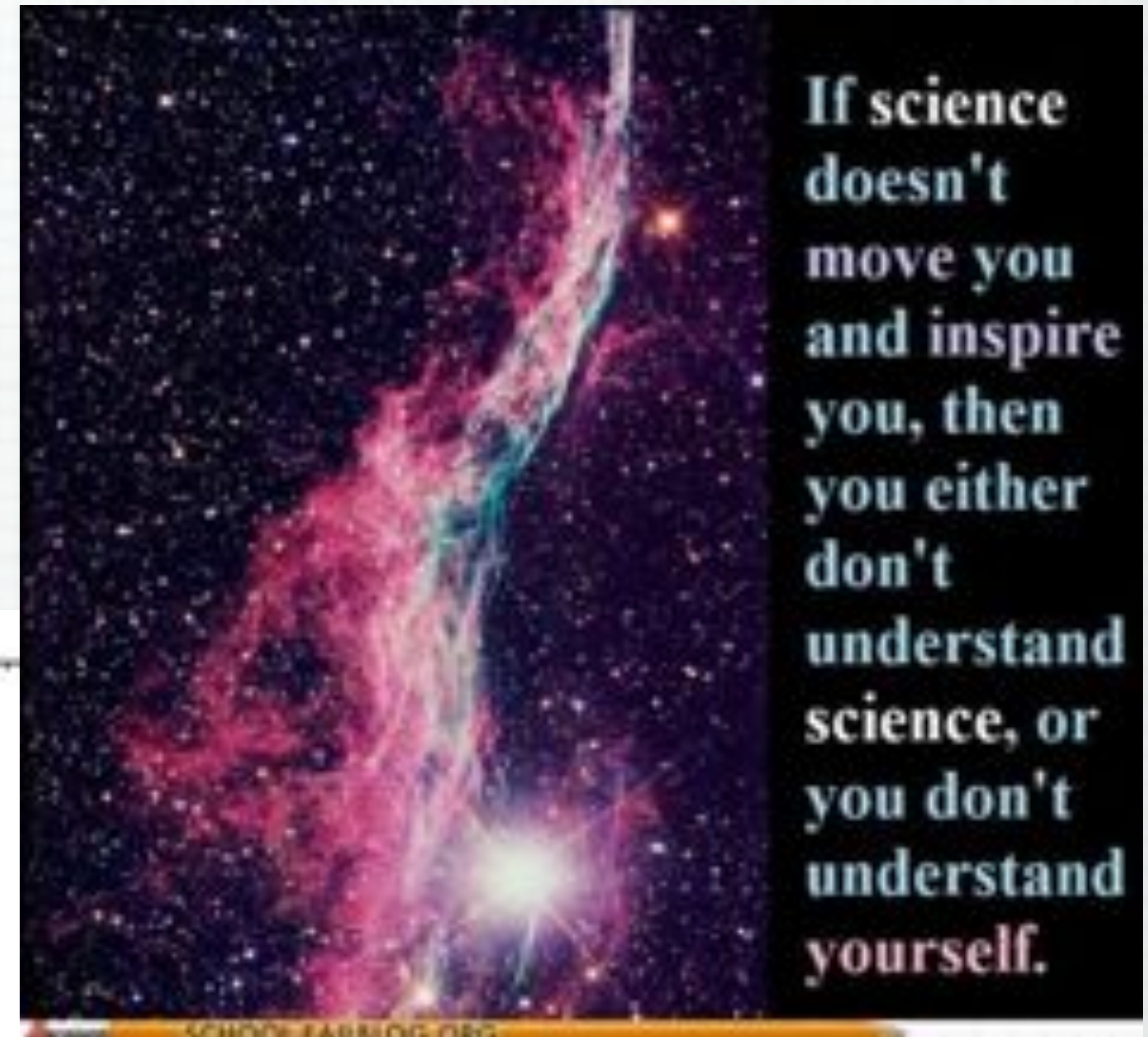
$$E_{HB} = .1FM_{\odot}c^2 \approx 1.162 \times 10^{50} erg$$

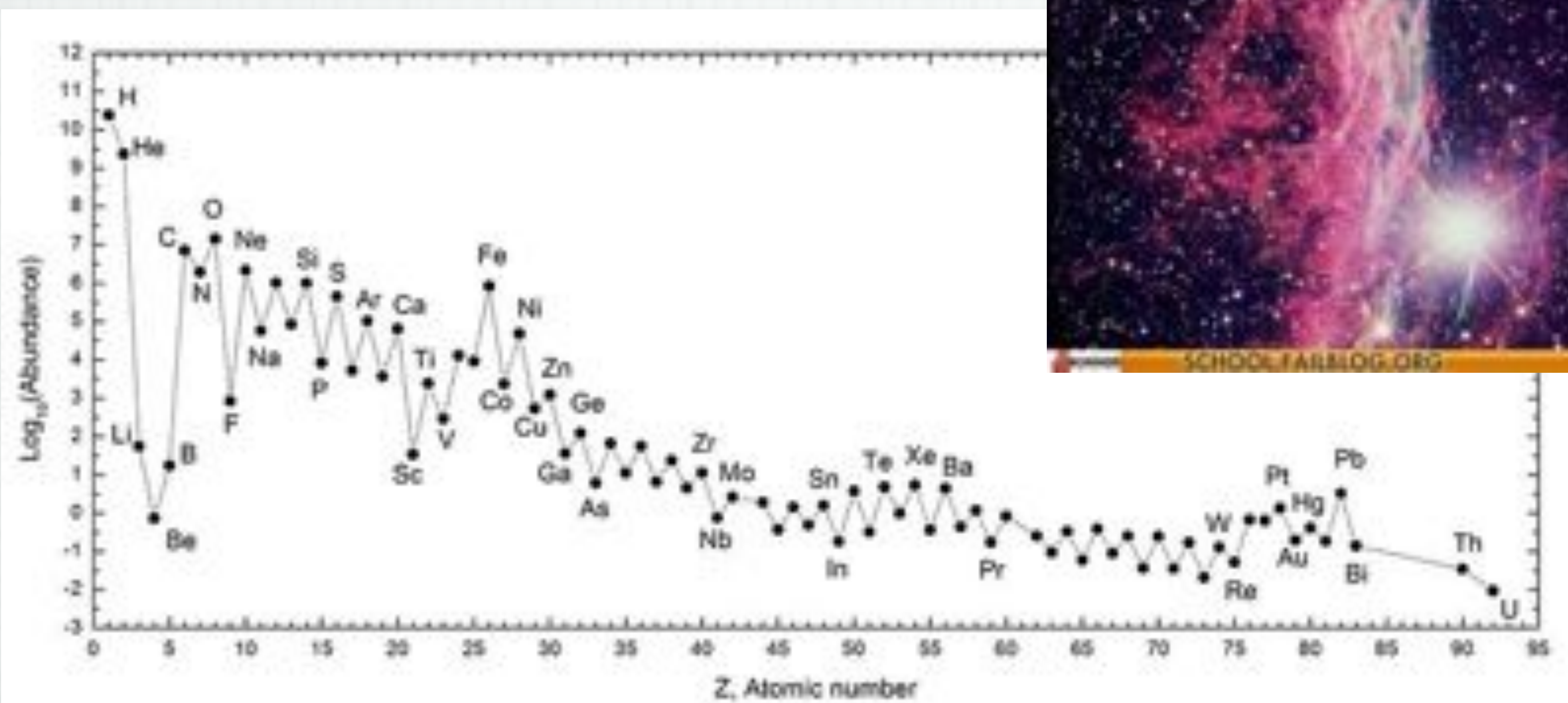
$$L_{HB} \approx 100L_{\odot} \rightarrow T_{HB} \approx 10^7 years$$

Each successive stage burns faster because there is less energy per unit mass in fusion towards iron and the temperature required for each successive fusion reaction speeds up the rate for each reaction

Making Elements

We're all star stuff





From here on

* I'll assume we are talking about a star that is sufficiently massive enough to keep contracting and burning more massive elements
* I'll go back to specific examples of varying stellar masses later

Continued Nuclear Burning

* Helium Burning if a star is more massive than .5 M_{solar}

$$^4He + {}^4He \rightarrow {}^8Be + \gamma$$

$$^8Be + {}^4He \rightarrow {}^{12}C^* + \gamma$$

$$^{12}C^* \rightarrow {}^{12}C + \gamma$$

* or

$$3\,{}^4He \rightarrow {}^{12}C + 3\gamma$$

* What's the energy in the process?

$$E = (m_{initial} - m_{final})c^2$$

* How to balance your equation and lets calculate all steps
* Oxygen is also produced $^{12}C + {}^4He \rightarrow {}^{16}O + \gamma$
* Temperatures around 2×10^8 K

Continued Nuclear Burning

* Energy is NEEDED for the first step, energy ~92KeV
* What T do we get from $2/3k_BT$?
* Endothermic
* Heavier elements can be made

$$^{16}O + {^4}He \rightarrow {^{20}}Ne + \gamma$$

$$^{20}Ne + {^4}He \rightarrow {^{24}}Mg + \gamma$$

* How massive an element can be made depends on initial star mass

Continued Nuclear Burning

Stages in the Life of a 25 Solar Mass Star

Burning phase	Required temperature	Required mean density	Duration
Hydrogen burning	4×10^7 degrees K	5 gm per cubic cm	7,000,000 years
Helium burning	2×10^8 degrees K	700 gm per cubic cm	700,000 years
Carbon burning	6×10^8 degrees K	200,000 gm per cubic cm	600 years
Neon burning	1.2×10^9 degrees K	4 million gm per cubic cm	1 year
Oxygen burning	1.5×10^9 degrees K	10 million gm per cubic cm	6 months
Silicon burning	2.7×10^9 degrees K	30 million gm per cubic cm	1 day

* Carbon Burning

$$^{12}C + {}^{4}He \rightarrow {}^{16}O + \gamma$$

$$^{12}C + {}^{12}C \rightarrow {}^{20}Ne + {}^{4}He$$

$$^{12}C + {}^{12}C \rightarrow {}^{24}Mg + \gamma$$

Exothermic

$$^{12}C + {}^{12}C \rightarrow {}^{16}O + 2{}^{4}He$$

$$^{12}C + {}^{12}C \rightarrow {}^{23}Mg + n$$

Endothermic

Wait

* A neutron was produced
* What might this do to the situation?
* Or how do you really start to big up the bigger guys if stellar burning stops at iron?
* More later

Continued Nuclear Burning

- Neon Burning
- Occurs before Oxygen burning because photodisintegration of Neon generates alpha particles and the fusion of helium with neon has a lower energy threshold than reactions related to oxygen

$$^{20}Ne + \gamma \rightarrow {}^{16}O + {}^{4}He$$

Photonuclear, endothermic reaction, E lost = 4.73 MeV

$$^{20}Ne + {}^{4}He \rightarrow {}^{24}Mg + \gamma$$

Exothermic, +9.316 MeV, also

$$^{24}Mg + {}^{4}He \rightarrow {}^{28}Si + \gamma$$

$$^{20}Ne + {}^{20}Ne \rightarrow {}^{16}O + {}^{24}Mg$$

Now we have Oxygen and Silicon

$$^{16}O + {}^{16}O \rightarrow {}^{30}Si + 2{}^{1}H$$

$$^{16}O + {}^{16}O \rightarrow {}^{28}Si + {}^{4}He$$

$$^{16}O + {}^{16}O \rightarrow {}^{31}P + {}^{1}H$$

And a sulfur producing reaction which generates more neutrons for s process nucleosynthesis

$$^{16}O + {}^{16}O \rightarrow {}^{31}S + n$$

Now we are in trouble

Once we hit the iron peak the star is dead

Final Nuclear Burning

* Final stages, temps above a billion degrees, photodisintegration of nuclei produces alpha particles.

$$^{28}Si + {}^{4}He \rightarrow {}^{32}S + \gamma$$

$$^{32}S + {}^{4}He \rightarrow {}^{36}Ar + \gamma$$

$$^{36}Ar + {}^{4}He \rightarrow {}^{40}Ca + \gamma$$

$$^{40}Ca + {}^{4}He \rightarrow {}^{44}Ti + \gamma$$

$$^{44}Ti + {}^{4}He \rightarrow {}^{48}Cr + \gamma$$

$$^{48}Cr + {}^{4}He \rightarrow {}^{52}Fe + \gamma$$

Now you can make nickel as well but it is radioactive and the process is endothermic

$$^{52}He + {}^{4}He \rightarrow {}^{56}Ni + \gamma$$

$$^{28}Si + {}^{28}Si \rightarrow {}^{56}Ni + \gamma$$

But these decay

$$^{56}Ni \rightarrow {}^{56}Co + e^{+} + \nu_e$$

$$^{56}Co \rightarrow {}^{56}Fe + e^{+} + \nu_e$$

Iron builds up

* Reactions are increasingly endothermic
* Pressure supports decreases
* core collapses
* temperature increases
* fusion rate increases
* RAPID PROCESS ~ 1 day
* Heavier elements built up from neutron flux and neutron decay

Building more massive elements

R and S processes

- Rapid and slow
- slow when neutrons pop up in later stage burning
- rapid when there is a large neutron flux due to end stage nucleosynthesis and dying star
- Slow process first, solid line shows path

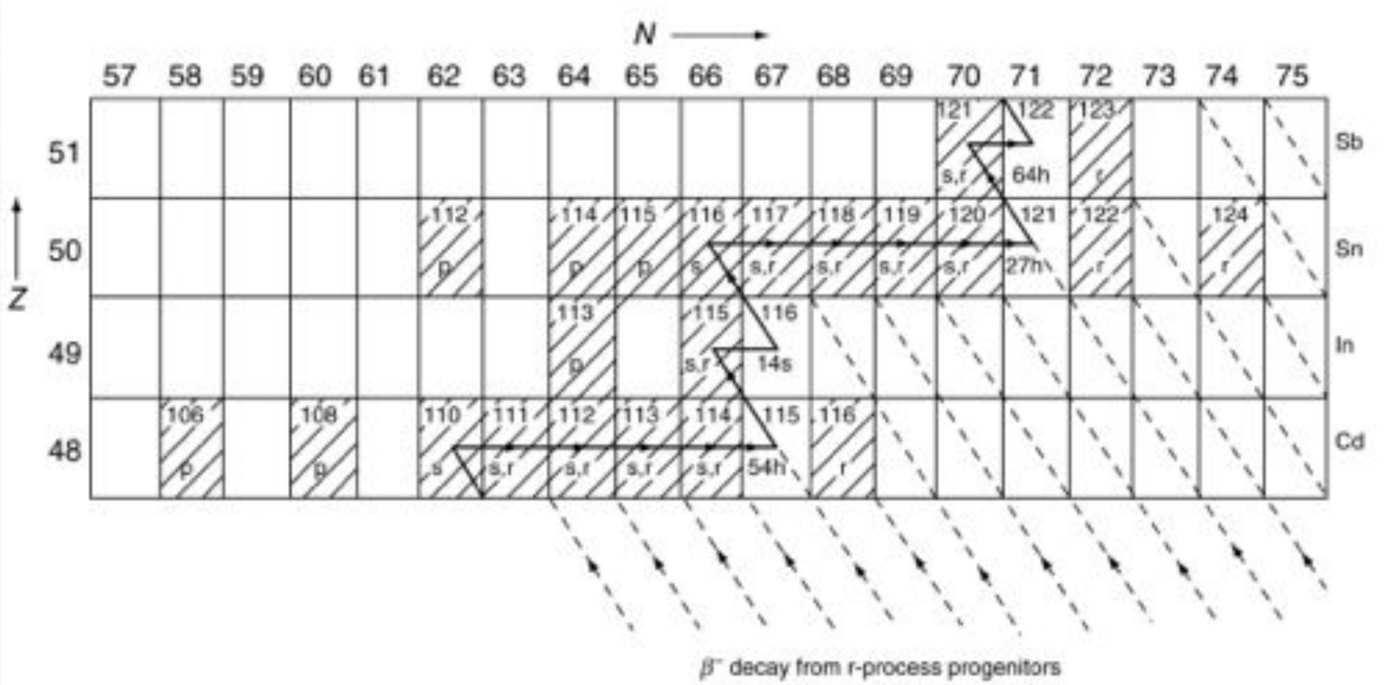


Figure 6.36 Synthesis of the elements Cd through Sb. The stable isotopes are hatched. The solid line shows the path of the *s* process. Figure reproduced and adapted with permission from Pearson, J.M., *Nuclear Physics: Energy and Matter*, Adam Hilger, Bristol (1986).

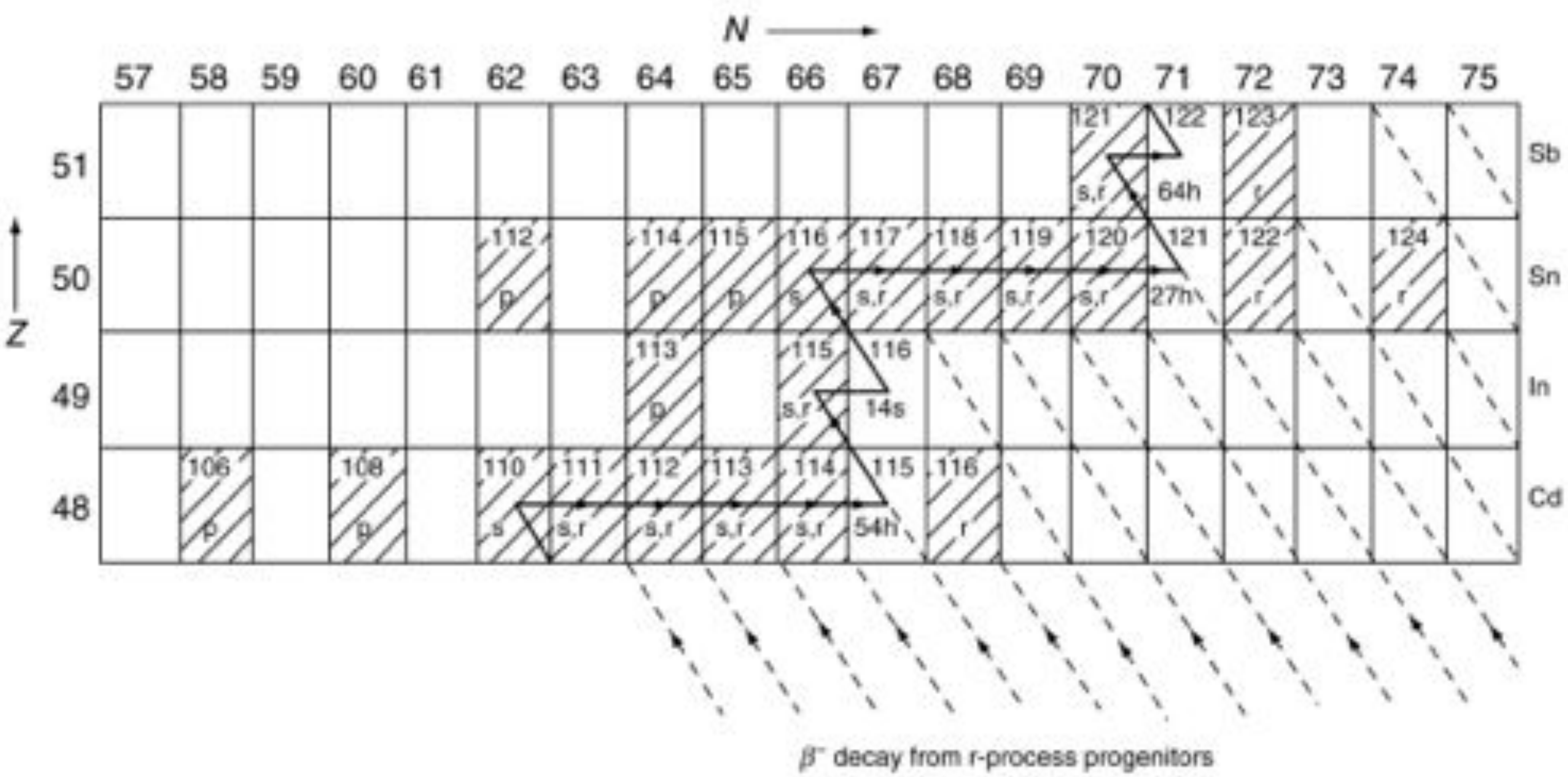


Figure 6.36 Synthesis of the elements Cd through Sb. The stable isotopes are hatched. The solid line shows the path of the *s* process. Figure reproduced and adapted with permission from Pearson, J.M., *Nuclear Physics: Energy and Matter*, Adam Hilger, Bristol (1986).

$$^{114}Cd + n \rightarrow \, ^{115}Cd + \gamma$$

For a slow flux of neutrons this will beta minus decay with a half life of 54 hours

$$^{115}Cd \rightarrow \, ^{115}In + e^- + \bar{\nu}_e$$

Which is stable but may absorb another neutron

$$^{115}In + n \rightarrow \, ^{116}In + \gamma$$

Which also beta minus decays

$$^{116}In \rightarrow \, ^{116}Sn + e^- + \bar{\nu}_e$$

Rapid process

- S process ends with Bismuth 209
- Easiest to quickly build up more massive elements through rapid neutron flux
- No coulomb barrier
- Two major channels
- Photons when T > 1 billion degrees

$$^{m}El(Z) + \gamma \rightarrow {}^{m-1}El(Z) + n$$

- Neutronization as a star finally dies

$$p + e^{-} \rightarrow n + \nu_e$$

P process(es)

* 33 stable proton rich isotopes not formable by r or s process
* s and r form seeds for these though
* 4 processes
  * photodisintegration $^{m}El(Z) + \gamma \rightarrow {}^{m-1}El(Z) + n$

$$^{116}Sn + \gamma \rightarrow {}^{115}Sn + n$$

* u=(116) 115.901744, (115) 114.903346, (n) 1.008665, u=931.5MeV/c^2
* Photon energy? Temperature?

P Process

* Proton absorption

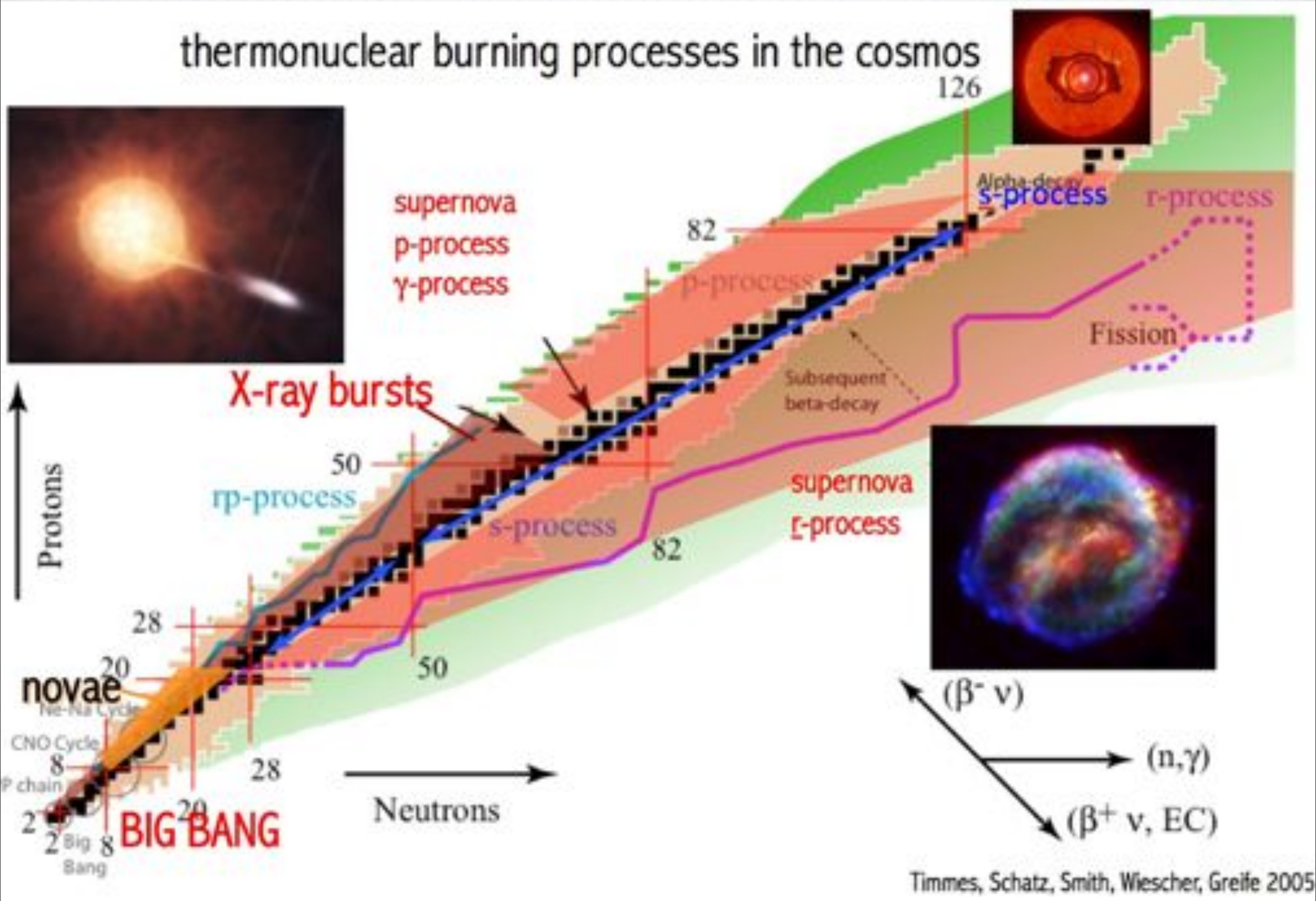
$$^{m}El(Z) + p \rightarrow {}^{m+1}El(Z+1)$$

* Proton fusion plus neutron emmission

$$^{m}El(Z) + p \rightarrow {}^{m}El(Z+1) + n$$

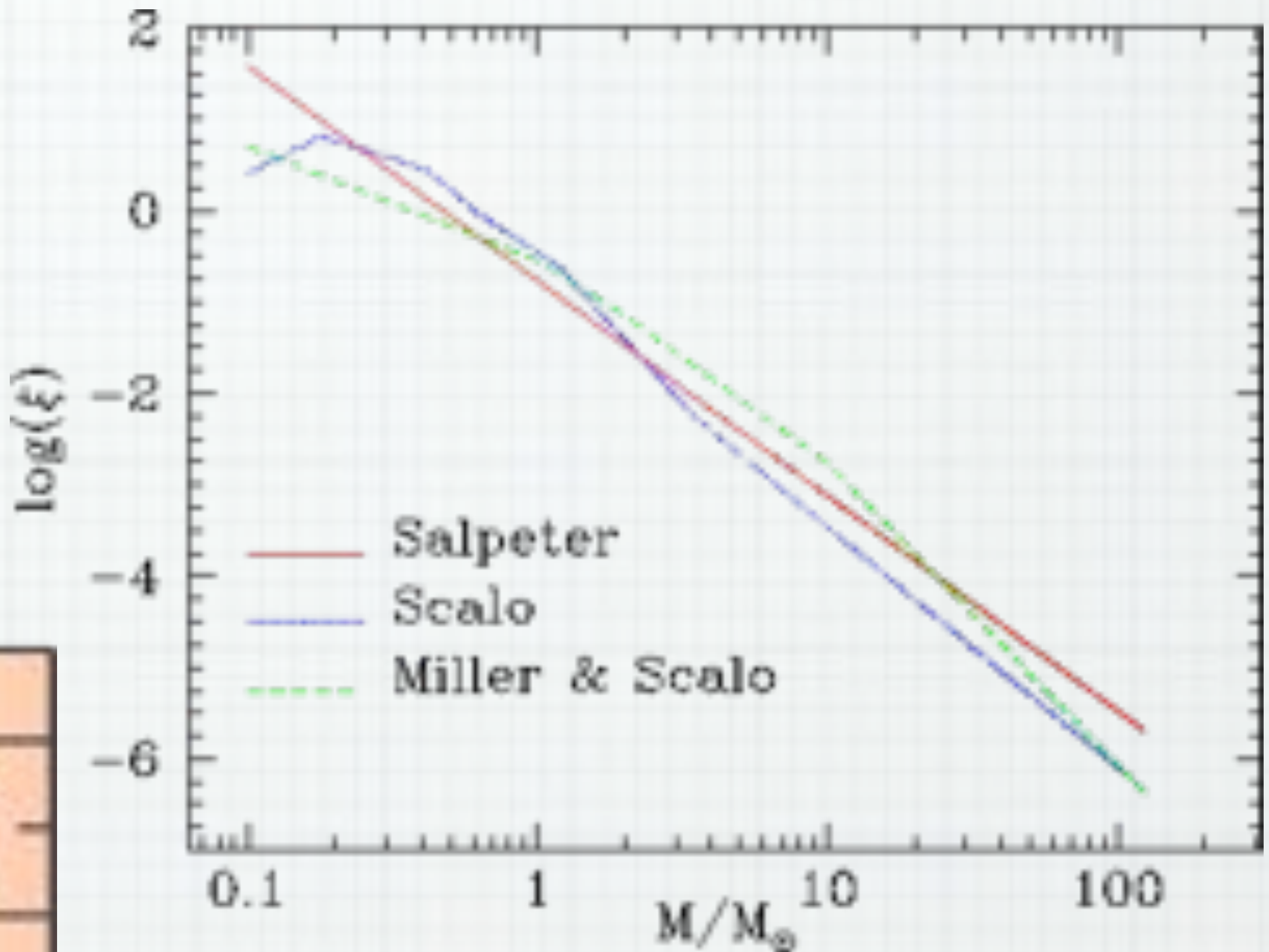
* Positron capture

$$^{m}El(Z) + e^{+} \rightarrow {}^{m}El(Z+1) + \bar{\nu}_e$$

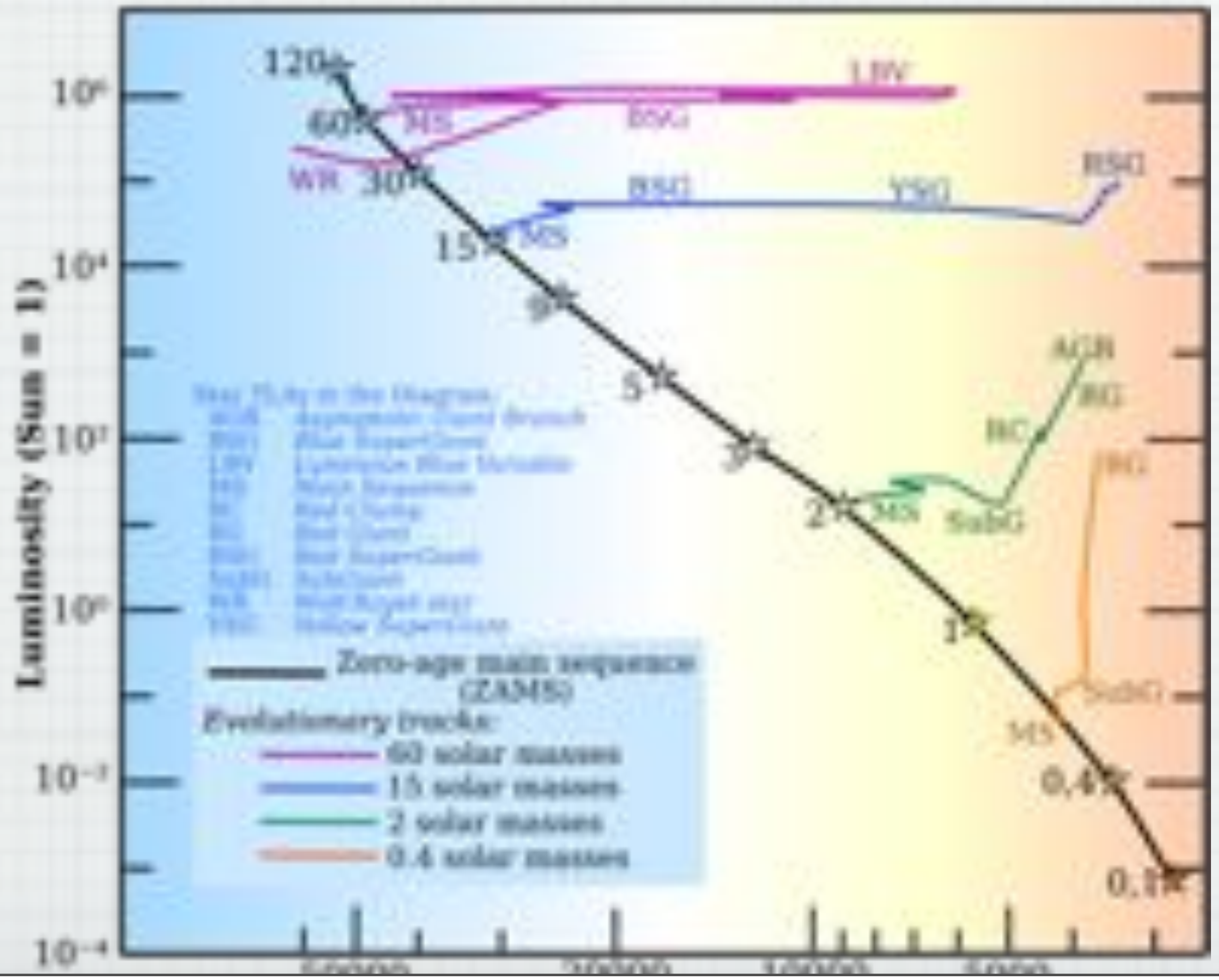

thermonuclear burning processes in the cosmos
126
Alpha-decay
s-process
r-process
supernova
p-process
γ-process
82
p-process
Fission
Subsequent
beta-decay
X-ray bursts
50
rp-process
s-process
supernova
r-process
82
Protons
28
20
50
novae
Ne-Na Cycle
CNO Cycle
8
P chain
28
Neutrons
2
2
Big
Bang
8
20
BIG BANG
(β⁻ ν)
(n,γ)
(β+ ν, EC)
Timmes, Schatz, Smith, Wiescher, Greife 2005

Stellar life cycles

* Very Low mass
* Solar mass
* Intermediate mass
* High mass



It's all mass



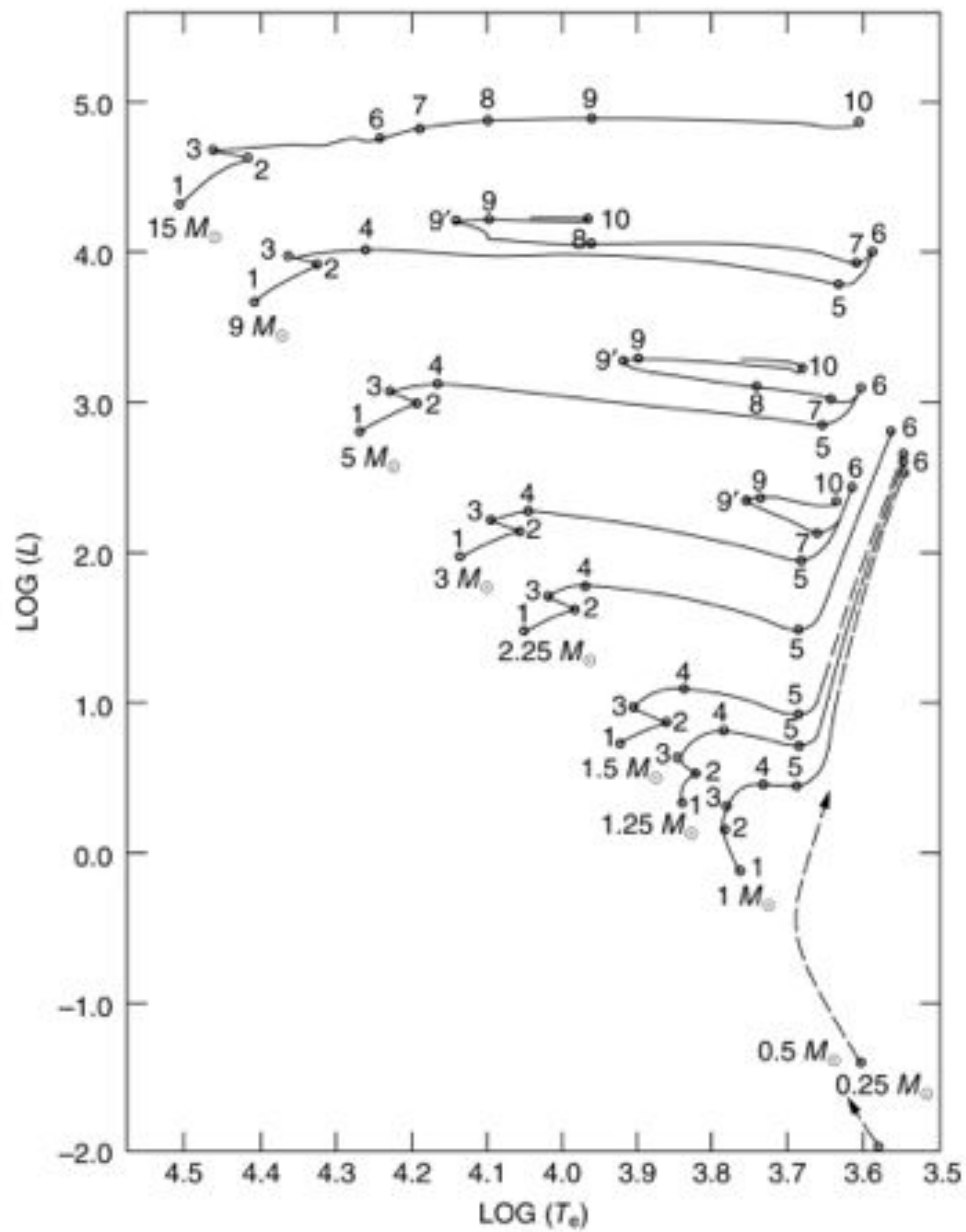


Figure 6.10 Partial evolutionary tracks for stars of various masses. The numbered points on the curves represent various time steps of the evolutionary process. Figure reproduced with permission from Iben, I., *Annual Review of Astronomy and Astrophysics*, 5, 571 (1967).

http://articles.adsabs.harvard.edu//full/1967ARA%26A...5..571I/0000616.000.html

Very low mass stars

* Stars below half a solar mass
* lifetimes of 10^{12} years for 1/4 solar mass stars, 10^{13} years for .08 solar mass stars
* Most stars are small
* These stars never reach helium fusion temperatures
* Many do not even attain the red giant phase (sub .16 solar masses)
* Primarily convective for most of their lives

Very low mass stars

* Efficient at burning hydrogen, may burn 90% of their hydrogen over their life
* As helium builds up in the core degeneracy pressure becomes primary support against gravitational collapse
* Core temperature continues to increase, degeneracy pressure independent of temperature
* Hydrogen burning continues in shells, luminosity increases, for > .16 solar masses a red giant phase begins as increased luminosity drives layers outward
* Higher temperatures due to core contraction can drive shell burning rates which increase luminosity by 1000-10000 times

Very low mass stars

* http://iopscience.iop.org/0004-637X/482/1/420/pdf/35131.pdf
* Will never reach helium flash
* Eventually simply stop burning and leave behind white dwarfs

Most galactic stars are small
Cooling white dwarfs will be majority of remnants
Eventually our galaxy will be no brighter than our sun

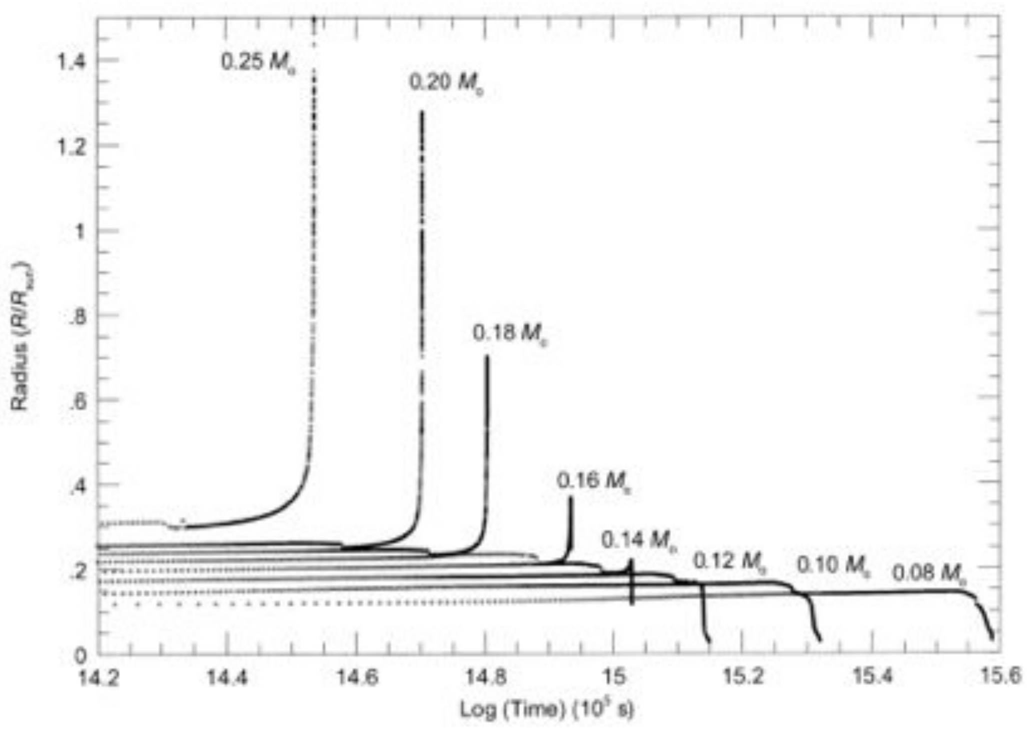
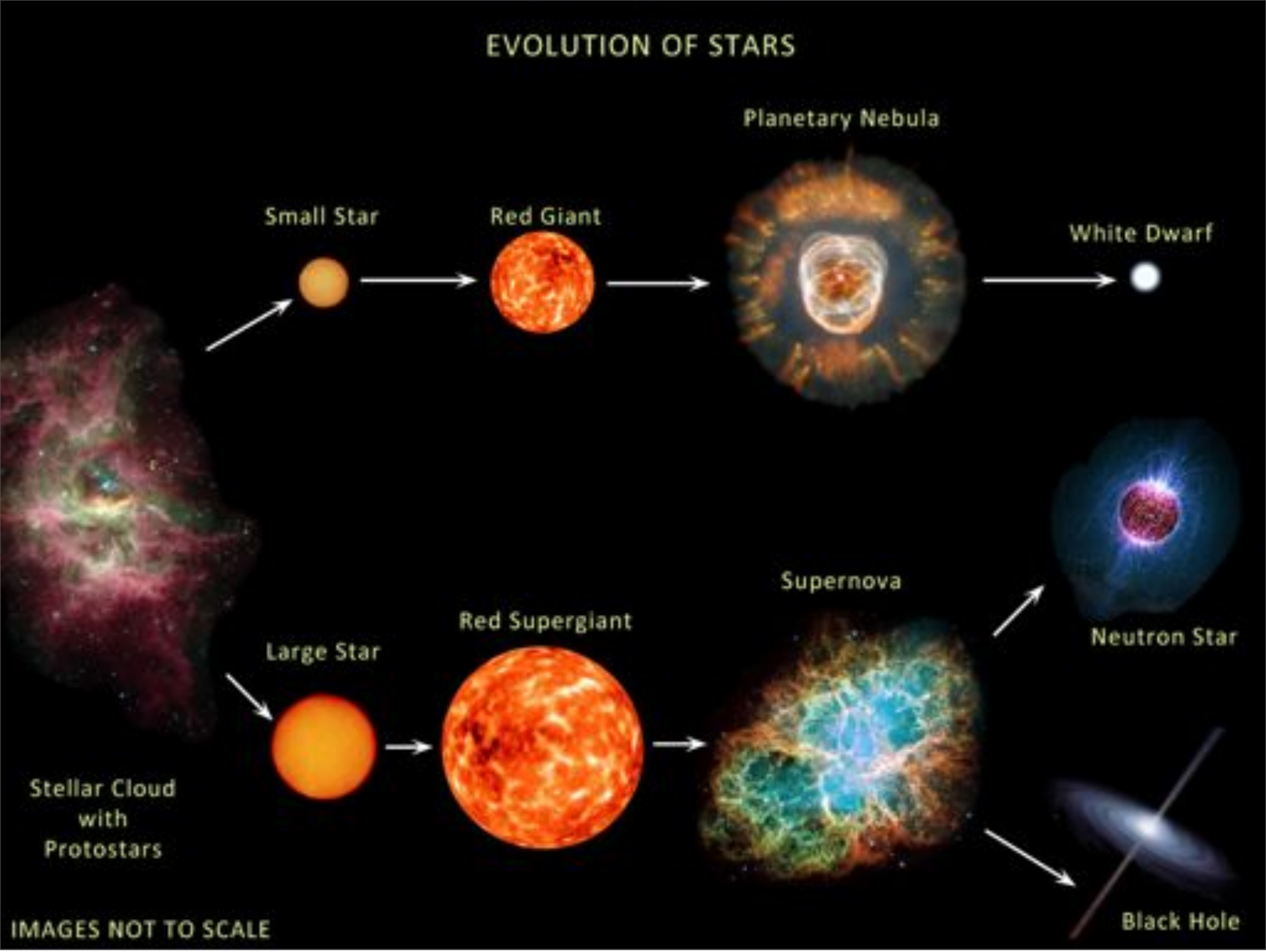


Figure 6.11 The radius of stars with masses between 0.08 and 0.25 $M_\odot$ as a function of time during their evolution. In this figure, a maximum radius is reached for all stars except for the 0.25 $M_\odot$ star. Reproduced by permission of the AAS from Laughlin, G., Bodenheimer, P. and Adams, F.C., *The Astrophysical Journal*, 482, 420 (1997).


EVOLUTION OF STARS
Planetary Nebula
Small Star
Red Giant
White Dwarf
Supernova
Red Supergiant
Neutron Star
Large Star
Stellar Cloud with Protostars
Black Hole
IMAGES NOT TO SCALE

Low to Intermediate mass stars

* Around 1-10 or so solar masses
* 10 is approximately the mass for iron production
* Star depletes hydrogen over its lifetime

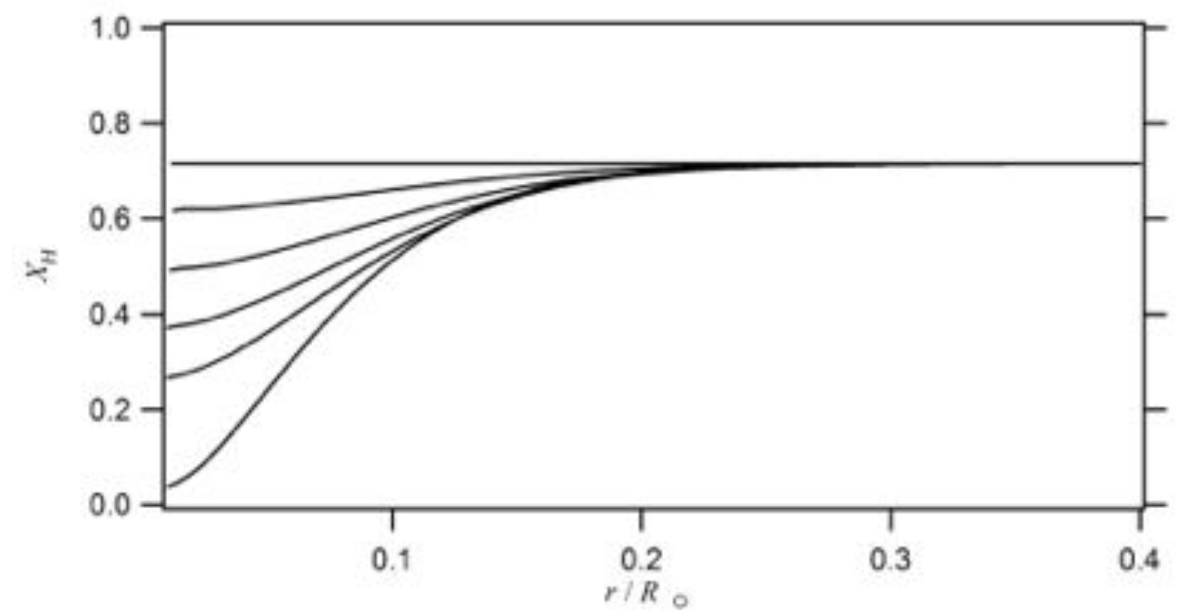


Figure 6.12 Hydrogen mass fraction (X_H) as a function of distance from the centre of the star (written in units of $r/R_\odot$) for times of 0.0, 1.39, 3.02, 4.53, 5.75 and 8.07 Gyr after the onset of hydrogen burning for a $1 M_\odot$ star. Data courtesy of Mathieu Vick, Jacques Richer and Georges Michaud obtained with the Montréal stellar evolution code.

* Once useable hydrogen is gone fusion in core ends
* Star leaves main sequence

Low to Intermediate mass stars

* Stages

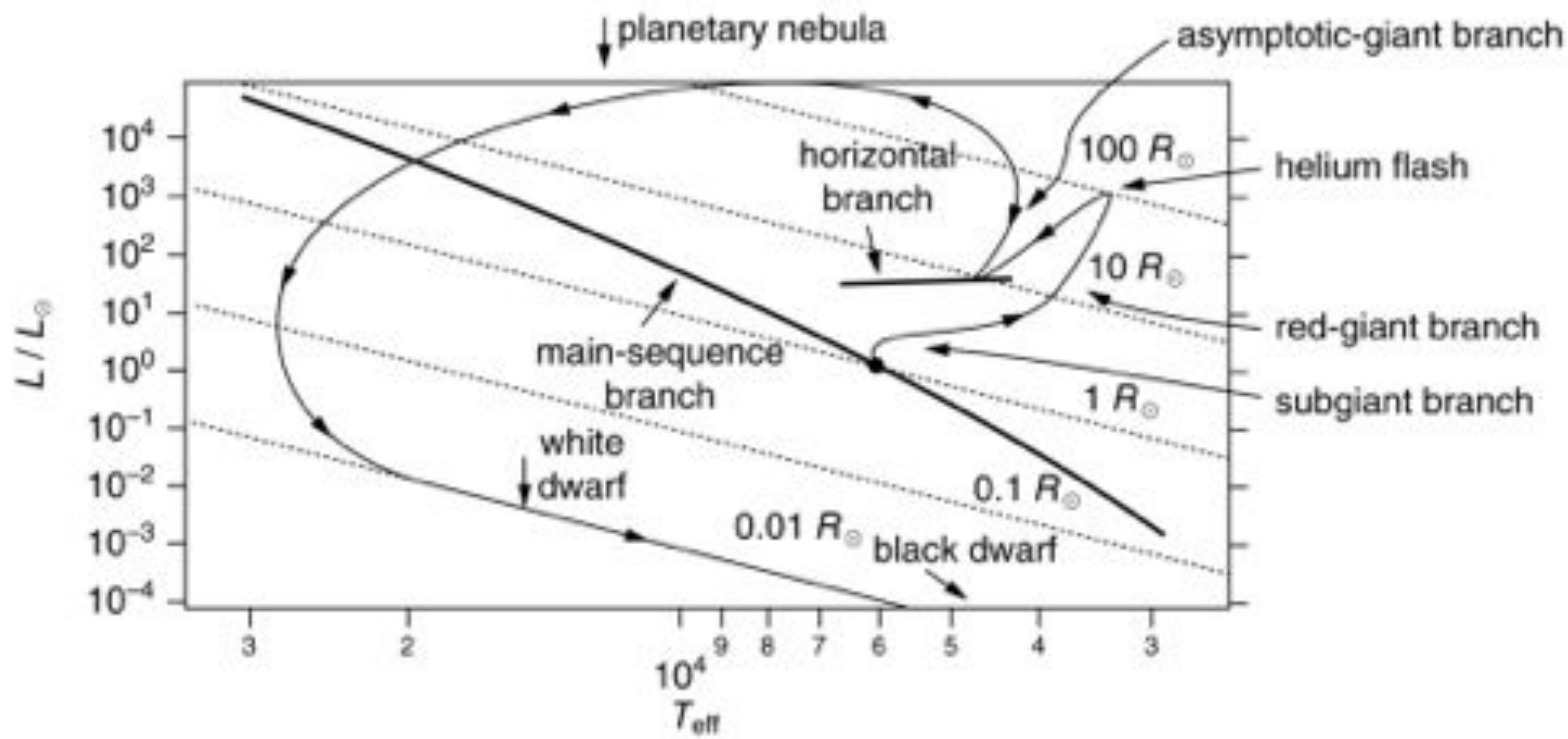


Figure 6.13 Illustration of the approximate evolutionary track of a 1 $M_\odot$ star in the H–R diagram. The dotted lines show the position for various values for the radius.

Low to Intermediate mass stars

* Red sub-giant phase when large layers around non fusing core participate in hydrogen fusion
* More material fusing farther from core drives increased luminosity
* New pressure/gravity equilibrium occurs for a larger radius star
* Core continues to contract, temperature rises, core supported against complete collapse by degeneracy pressure
* The red giant phase lasts around 10^9 years for a star the sized of our sun

Low to Intermediate mass stars

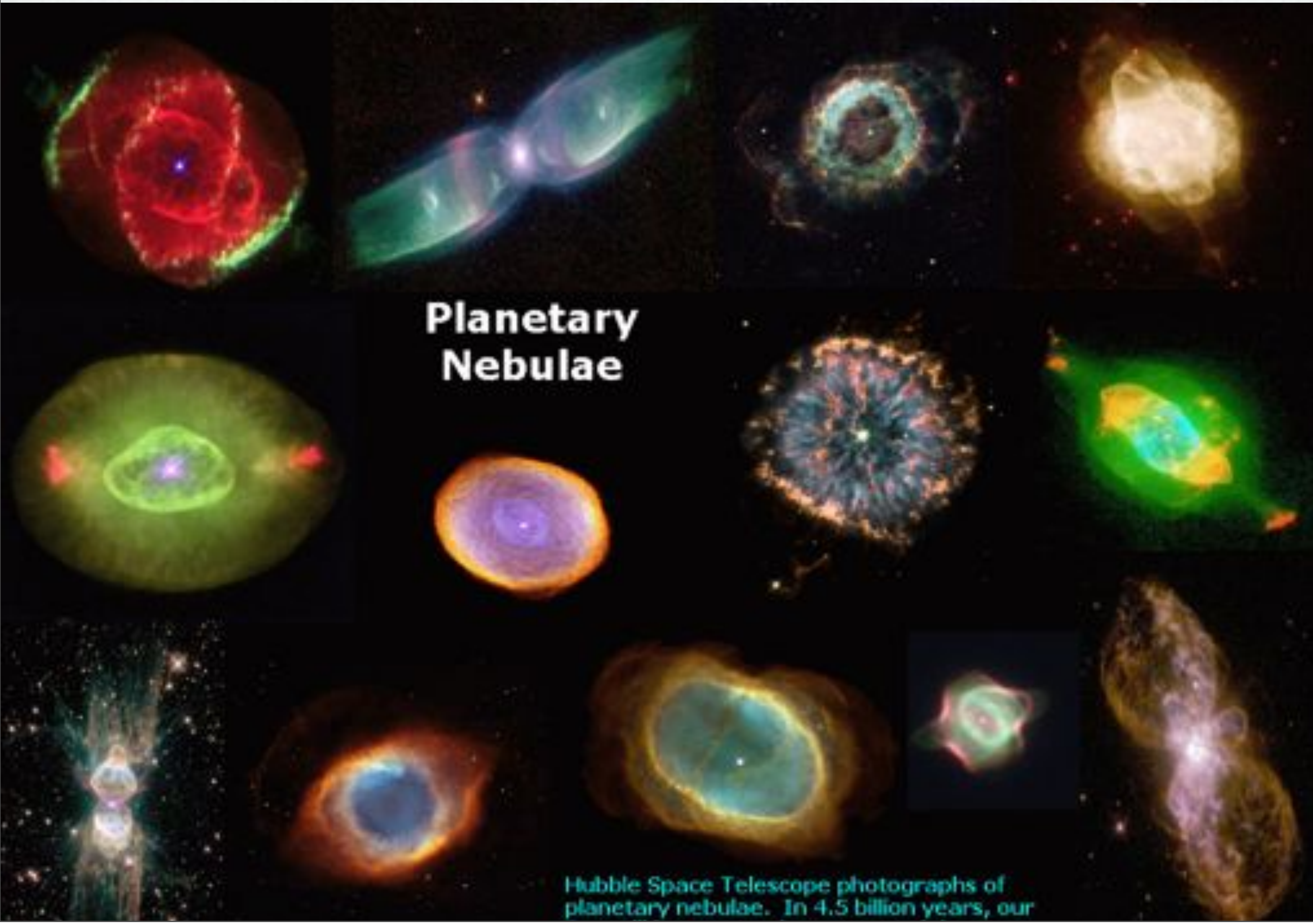
* Eventually the core temperature increases to the point of helium fusion which is highly sensitive to temperature
* No added pressure support up until helium fusion
* Helium flash, brief, short lived
* New equilibrium between gravity and fusion
* A second helium 'main sequence' 10^8 years or so
* Helium runs out, now helium fusion in shells, hydrogen in shells, giant phase, 2 x 10^7 years
* Consecutive phases of fusion for more massive stars, we will stick with the details for 1 solar mass

Low to Intermediate mass stars

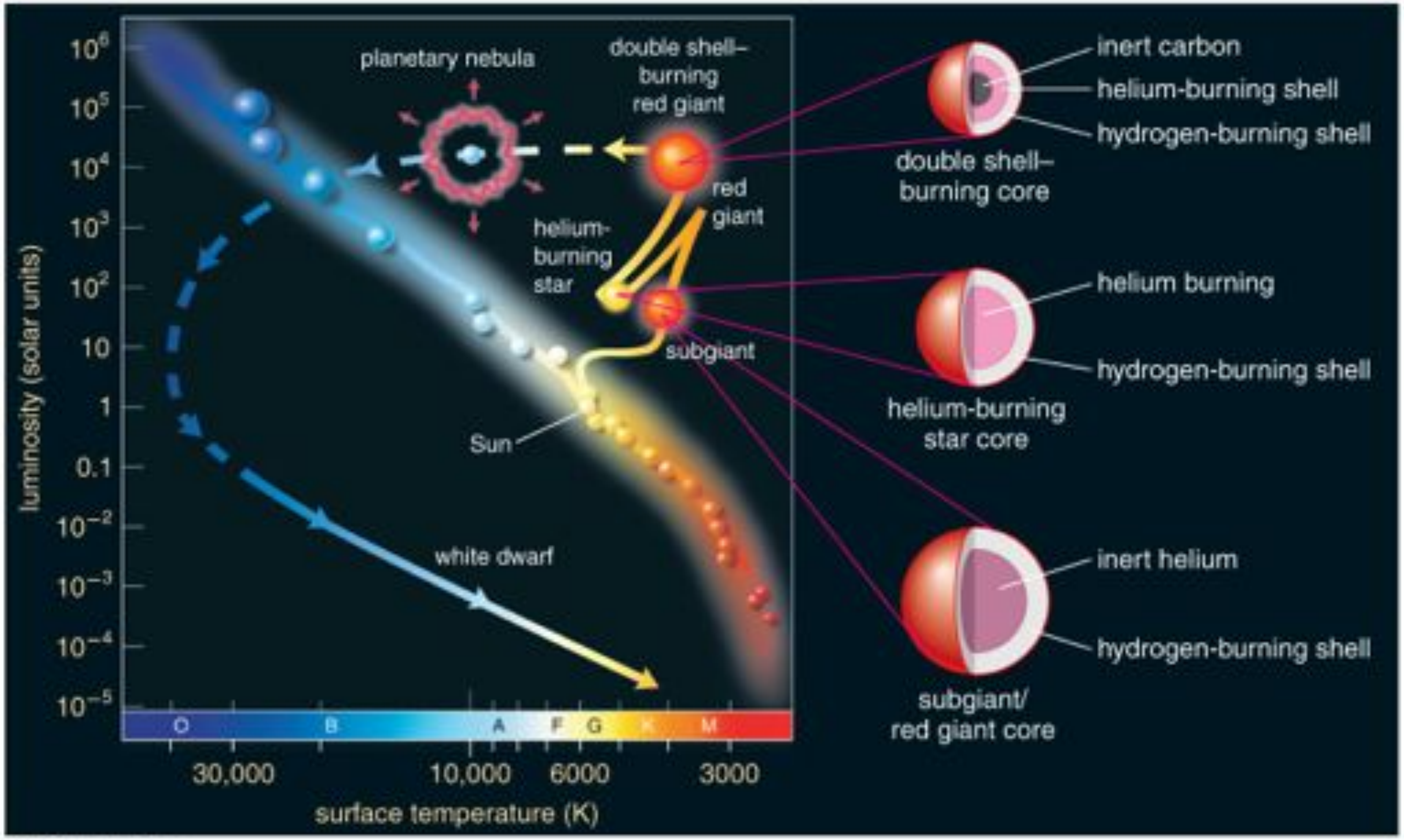
* This phase really is the AGB phase of stellar evolution
* Star swells to about 1 AU in early part when helium shell burning is occurring (Early AGB)
* After helium burning ceases there is primarily hydrogen shell burning (thermal pulse AGB)
* Build up of helium causes intermittent helium shell flashes increasing the star's luminosity by a factor of thousands
* Flashes occur every 10,000-100,000 years and are accompanied by the dredge up of core material
* Flashes lead to mixing and s-process nucleosynthesis

Low to Intermediate mass stars

* The AGB phase is sort of a stellar sputtering engine, it ends eventually
* Much material is lost in the thermal pulses, a 1 solar mass star may leave behind a 1 solar mass core
* Once the engine is turned off we are left with a carbon/helium core with a temperature around 10,000 K
* Still hot enough to drive off outer atmosphere
* Planetary nebula phase
  * Material is driven off over about 50,000 years at about 10 kms-1


Planetary
Nebulae
Hubble Space Telescope photographs of
planetary nebulae. In 4.5 billion years, our

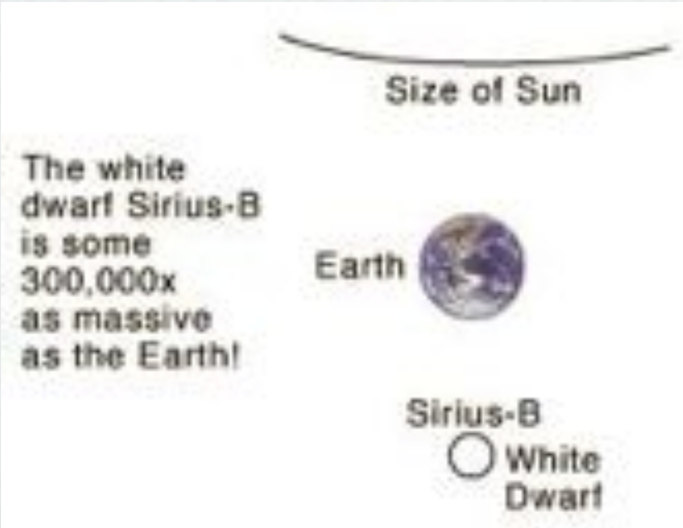
Review

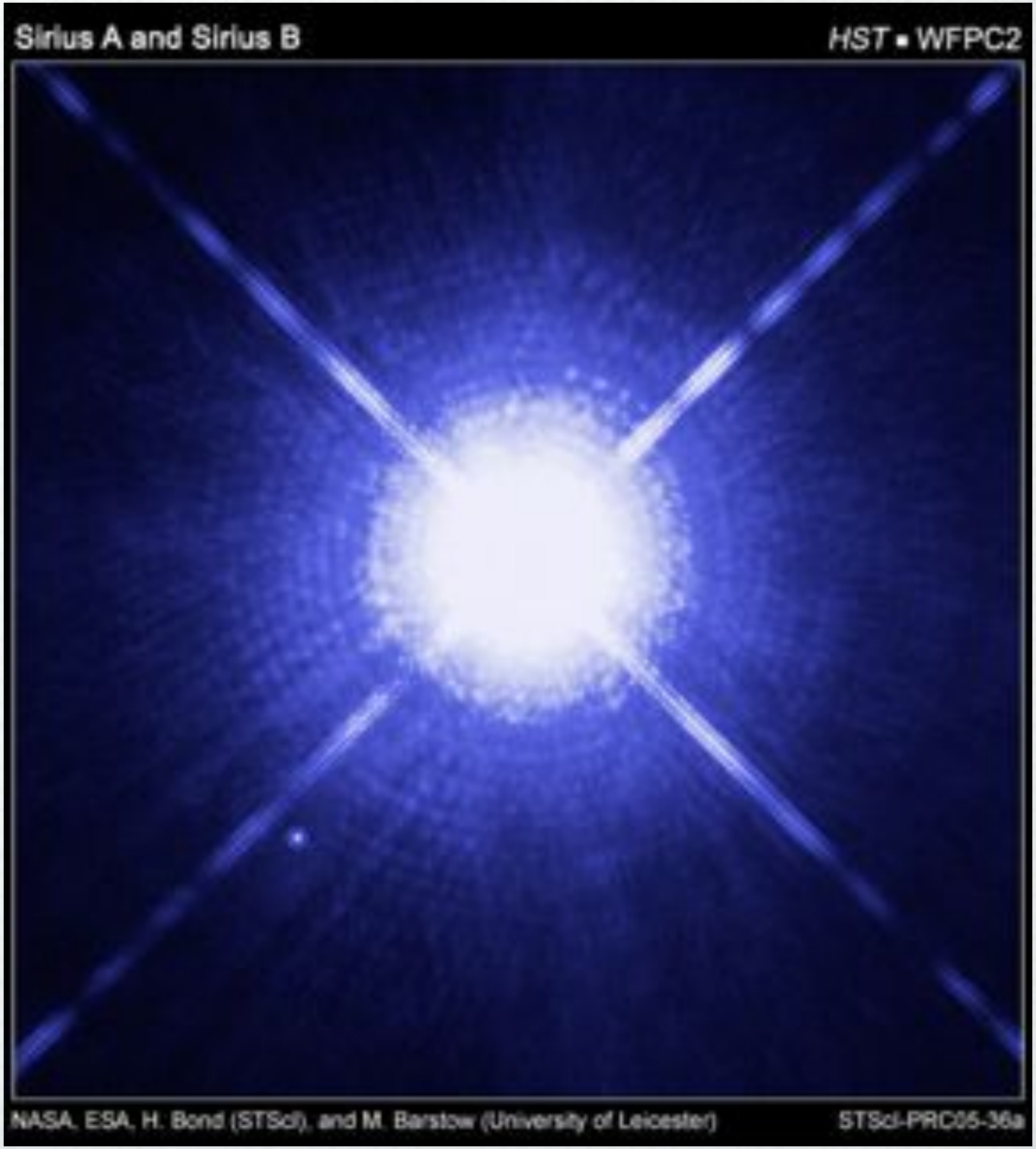


planetary nebula
double shell–
burning
red giant
red
giant
helium-
burning
star
subgiant
Sun
white dwarf
luminosity (solar units)
10^6
10^5
10^4
10^3
10^2
10
1
0.1
10^{-2}
10^{-3}
10^{-4}
10^{-5}
O
B
A
F
G
K
M
30,000
10,000
6000
3000
surface temperature (K)
inert carbon
helium-burning shell
hydrogen-burning shell
double shell–
burning core
helium burning
hydrogen-burning shell
helium-burning
star core
inert helium
hydrogen-burning shell
subgiant/
red giant core

The remnant

* A white dwarf
* About one solar mass
* About the radius of the earth
* Google Procyon B and Sirius B, the B means the second star in each system, Friedreich Bessel predicted them before they were observed





White Dwarfs & Degeneracy Pressure

* Some numbers for Sirius B
  * density about 2×10^9 kg m^{-3}
    * one teaspoon is about 10 tons
  * central pressure about 10^{18} atm

Degeneracy

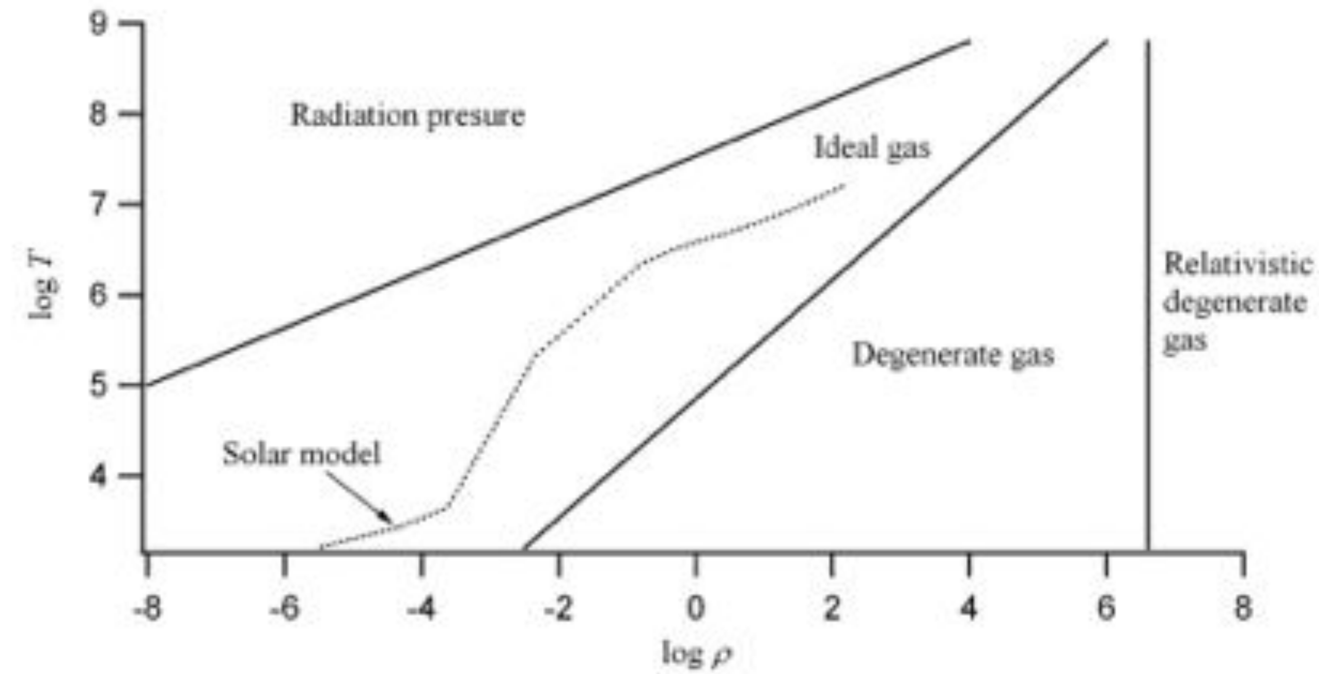
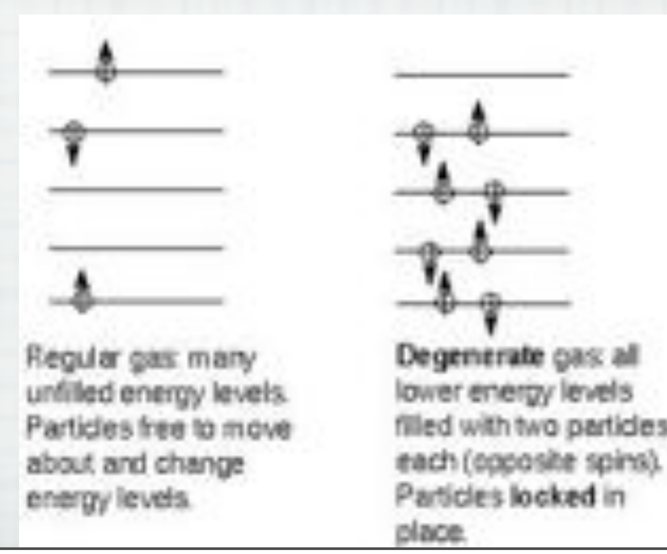


Figure 5.10 The approximate domains of the validity of the ideal-gas approximation, radiation pressure (see Section 5.6.4), degenerate and relativistic degenerate gases on a log T, log ρ diagram. The solid lines show the delimitations of the various regions. Also shown in this figure is the position of the solar model.



Degeneracy pressure

* For high densities the equation of state given by the ideal gas law breaks and quantum mechanical effects come into play
* As more and more particles are packed closely together the Pauli Exclusion Principle forces electrons to occupy increasing energy levels meaning their velocities are increasing
* Thermal energy can not be extracted as the lower energy states are filled
* More pressure can only be generated by increasing the mass which increases the gravitational energy and decreases the size
* Exactly opposite regular pressure
* Essentially $\Delta X \Delta P \geq \hbar/2 \qquad \Delta X \downarrow \Delta P \uparrow$
* We will now derive the conditions in a white dwarf consisting of a carbon core that is completely ionized

Degeneracy Pressure

* In 6 dimensional quantum mechanical phase space

$$\Delta x \Delta y \Delta z \Delta p_x \Delta p_y \Delta p_z = h^3$$

* If we take the physical volume to be $1 cm^3$ then the number of available quantum cells in the momentum range [p,p+dp] is given by $N_q = 2 \times 4\pi p^2 dp/h^3$
* Where the two accounts for spin up and down
* The maximum number of electrons that can be accommodated in a volume in phase space is then

$$n_e(p_o) = \int_0^{p_0} \frac{8\pi p^2 dp}{h^3} dp = \frac{8\pi}{3} \frac{p_0^3}{h^3}$$

* Where p_0 is the maximum allowable momentum

Degeneracy Pressure

* We want to connect the maximum momentum with the density
* The mass of heavy particle per electron is $m_H\mu_e = \frac{\rho}{n_e}$
* Then $\frac{\rho}{\mu_e} = \frac{8\pi}{3}\frac{m_H}{h^3}p_0^3$
* The momentum transfer δp per electron with momentum p_x is given by $\delta p_x = 2p_x = 2p(p_x/p)$
* The number of electrons with p and p_x arriving at a wall per cm^2 per second is given by

$$n_e = dn_e(p,p_x)v_x = dn_e(p,p_x)\frac{p}{m_e}cos(\theta)$$

* The number of electrons with momentum between p and p+dp passing through a ring of thickness pdθ radius sinθ is given by

$$dn_e(p,p_x) = \frac{4\pi}{h^3}p^2 sin(\theta)d\theta dp$$

Degeneracy Pressure

* Then

$$n_e \delta p_x = n_e 2 p_x = n_e 2 p cos(\theta) = \delta P_e(p, p_x) = \frac{8\pi}{m_e h^3} p^4 cos(\theta)^2 sin(\theta) dp d\theta$$

* Integration over all angles over the half sphere gives

$$\Delta P_e(p) = \frac{8\pi}{m_e 3h^3} p^4 dp$$

* Now we must integrate over momenta to get the pressure
* Recall $n_e = \frac{8\pi}{3h^3} p_0^3 \to p_0 = (\frac{3h^3 n_e}{8\pi})^{1/3}$
* and $n_e = \frac{\rho}{m_H} \frac{Z}{A} = \frac{\rho}{\mu_e m_H} \to \mu_e = A/Z$

Degeneracy Pressure

* to evaluate the integral we recast it as

$$P_e = \frac{8\pi}{3h^3}\int_0^{p_0} vp^3 dp$$

* The generic case has $v = \frac{\frac{p}{m_e}}{\sqrt{1 + (\frac{p}{m_e c})^2}}$

* or $P_e = \frac{8\pi}{3h^3}\int_0^{p_0} \frac{\frac{p^4}{m_e}}{\sqrt{1 + (\frac{p}{m_e c})^2}} dp$

* Using $x = \frac{p_0}{m_e c}$ tis can be evaluated directly yielding

* $$P_e = \frac{\pi c^5 m_e^4}{3h^3}[x\sqrt{1+x^2}(2x^2 - 3) - sinh^{-1}(x)]$$

Degeneracy Pressure

* Not too illuminating
* Let's try limits
* Non-relativistic $v = \frac{p}{m_e}$

$$P_e = \frac{8\pi}{3h^3 m_e}\int_0^{p_0} p^4 dp = P_e = \frac{8\pi}{3h^3 m_e}\frac{p_0^5}{5} = k_1'(\frac{\rho}{\mu_e})^{5/3}$$

* Relativistic $v = c$

$$P_e = \frac{8\pi}{3h^3}\int_0^{p_0} cp^3 dp = \frac{8\pi}{3h^3}\frac{p_0^4}{4} = k_2'(\frac{\rho}{\mu_e})^{4/3}$$

* Wait! Pressure goes as $M^{(-4/3),(-5/3)}$?
* No temperature dependence
* What's the upper limit for this process?

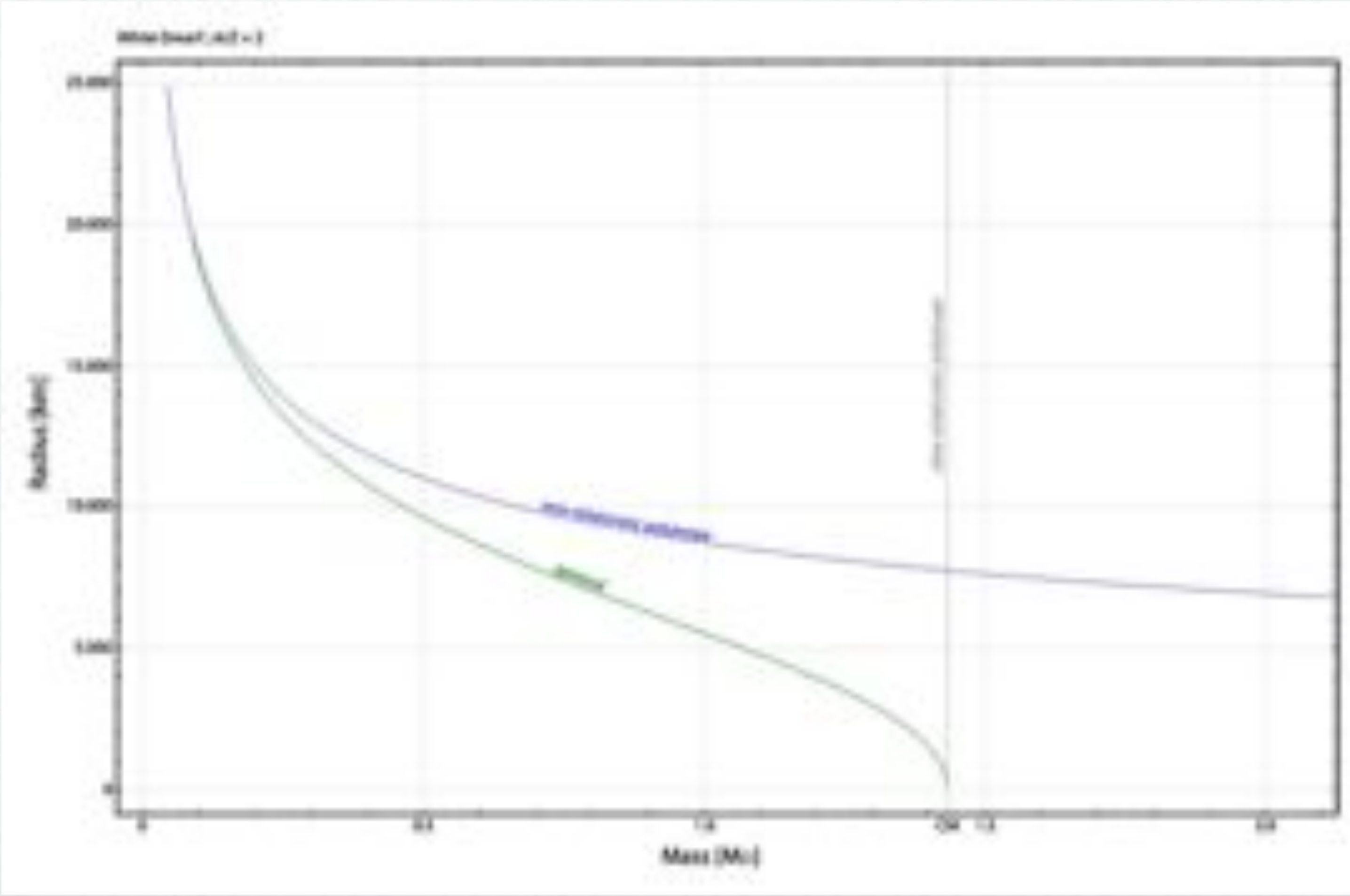
Degeneracy Pressure

* Electrons can not move faster than the speed of light
* Let's balance electron pressure and gravitational contraction in the non-relativistic case

$$P_e = -P_G \rightarrow k_1 \rho^{5/3} = (\frac{4\pi}{3})^{1/3} \frac{G}{5} M^{2/3} \rho^{4/3}$$

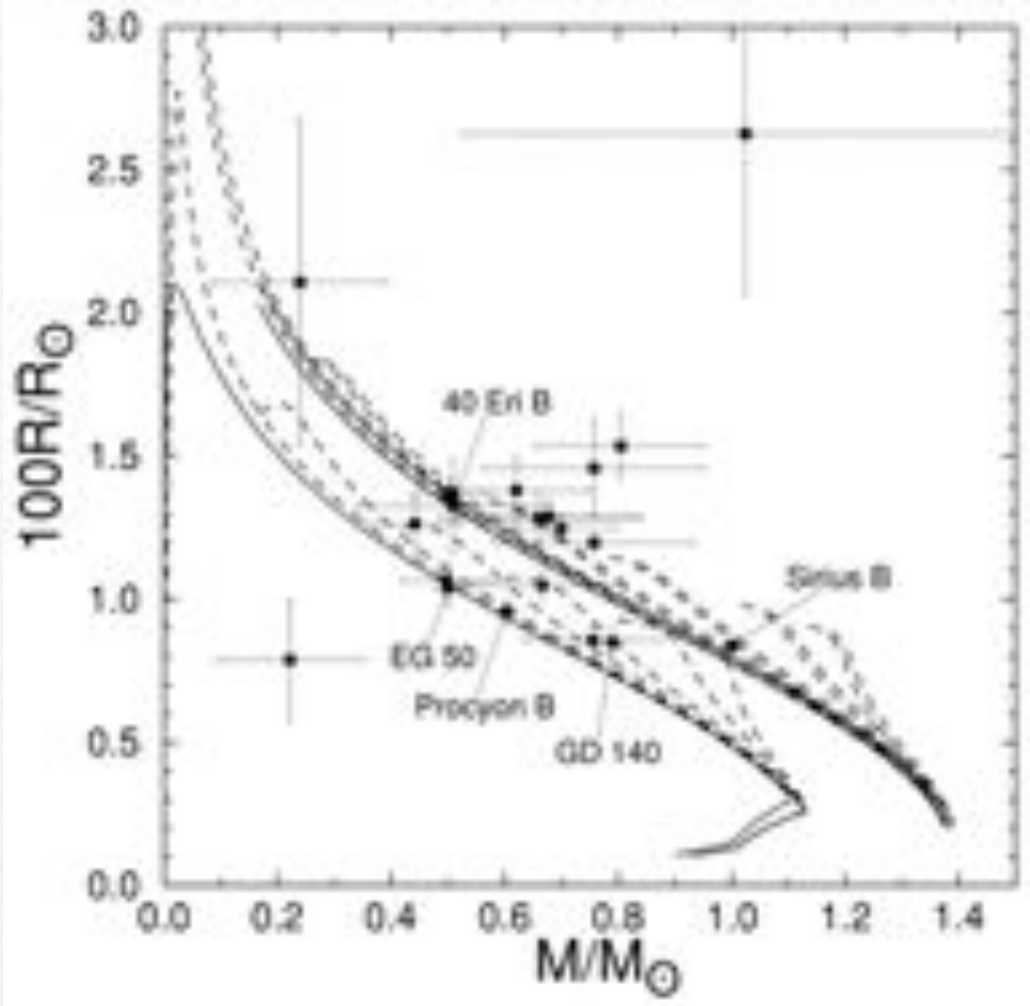
* Solve for R then V when gravity and degeneracy are in equilibrium

$$V = \frac{3}{4\pi} (\frac{5k_1}{G})^3 \frac{1}{M}$$



Degeneracy Pressure

* observed
* What about when v=c?
* NO EQUILIBRIUM SOLUTION!!



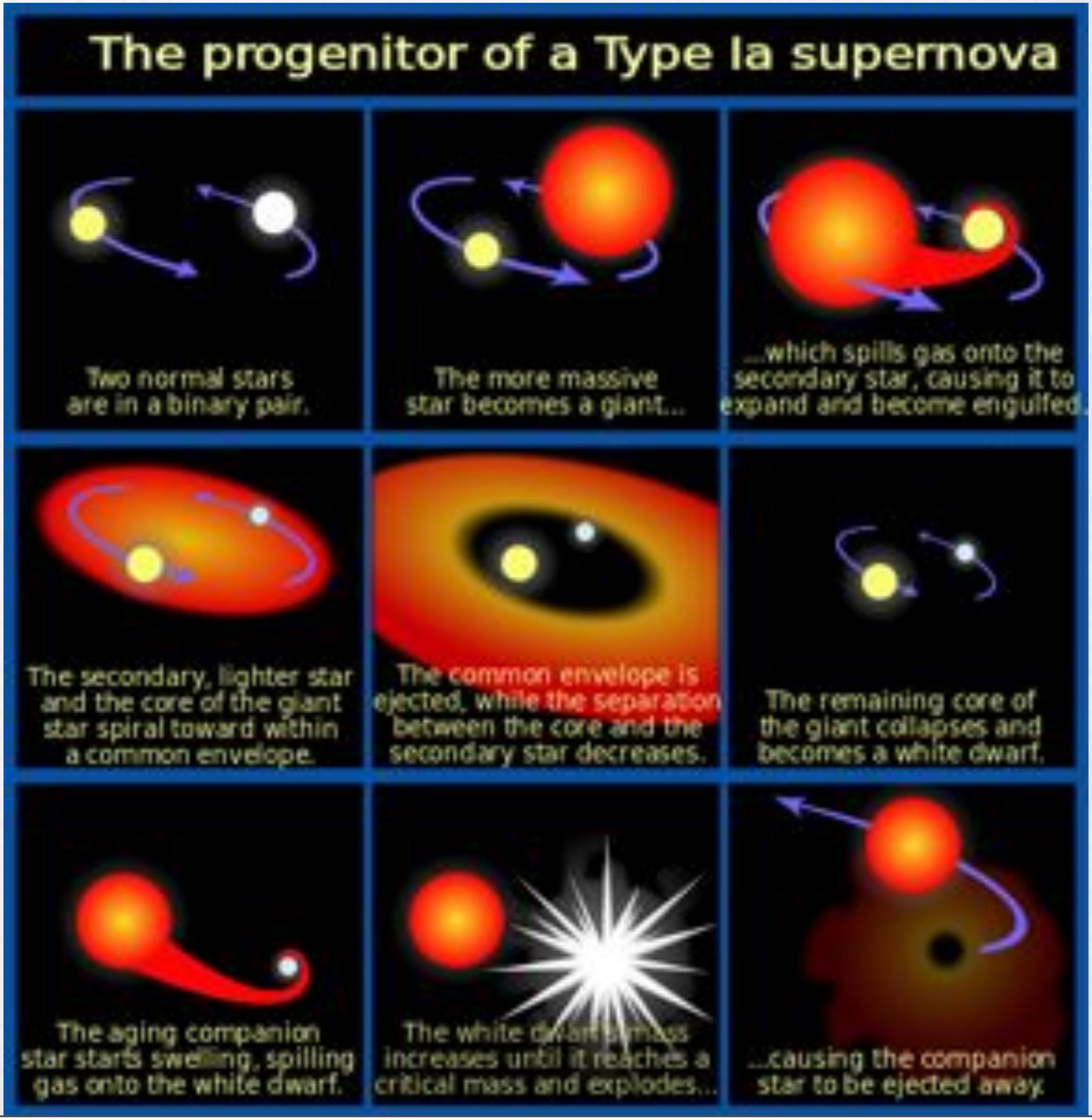
$$k_2\rho^{4/3} = (\frac{4\pi}{3})^{1/3}\frac{G}{5}M^{2/3}\rho^{4/3}$$

$$M = (\frac{5k_2}{G})^{3/2}\sqrt{\frac{3}{4\pi}} = M_{Ch} = 1.7M_{\odot}$$

* Really 1.44 masses, calculation assumed constant density
* Star can no longer use degeneracy pressure to support against gravity, begins to collapse, hits carbon fusion temperature, detonates in type 1A supernovae

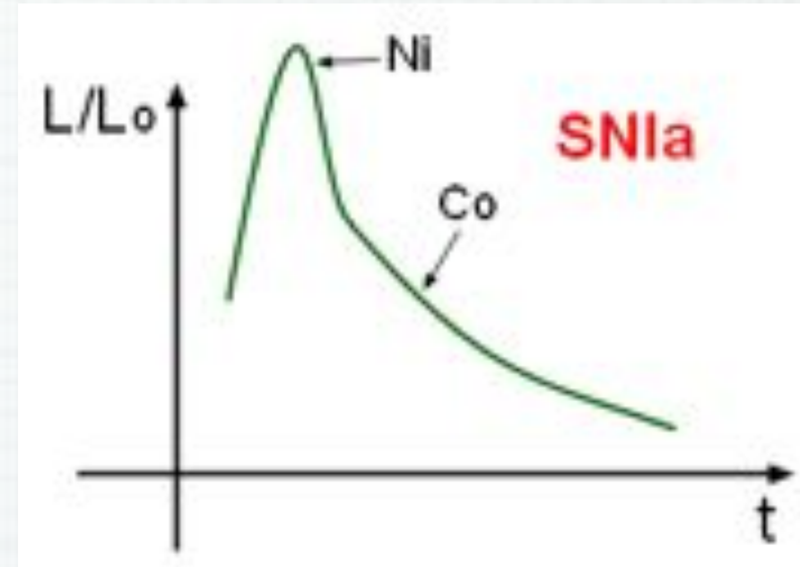
Type Ia supernovae

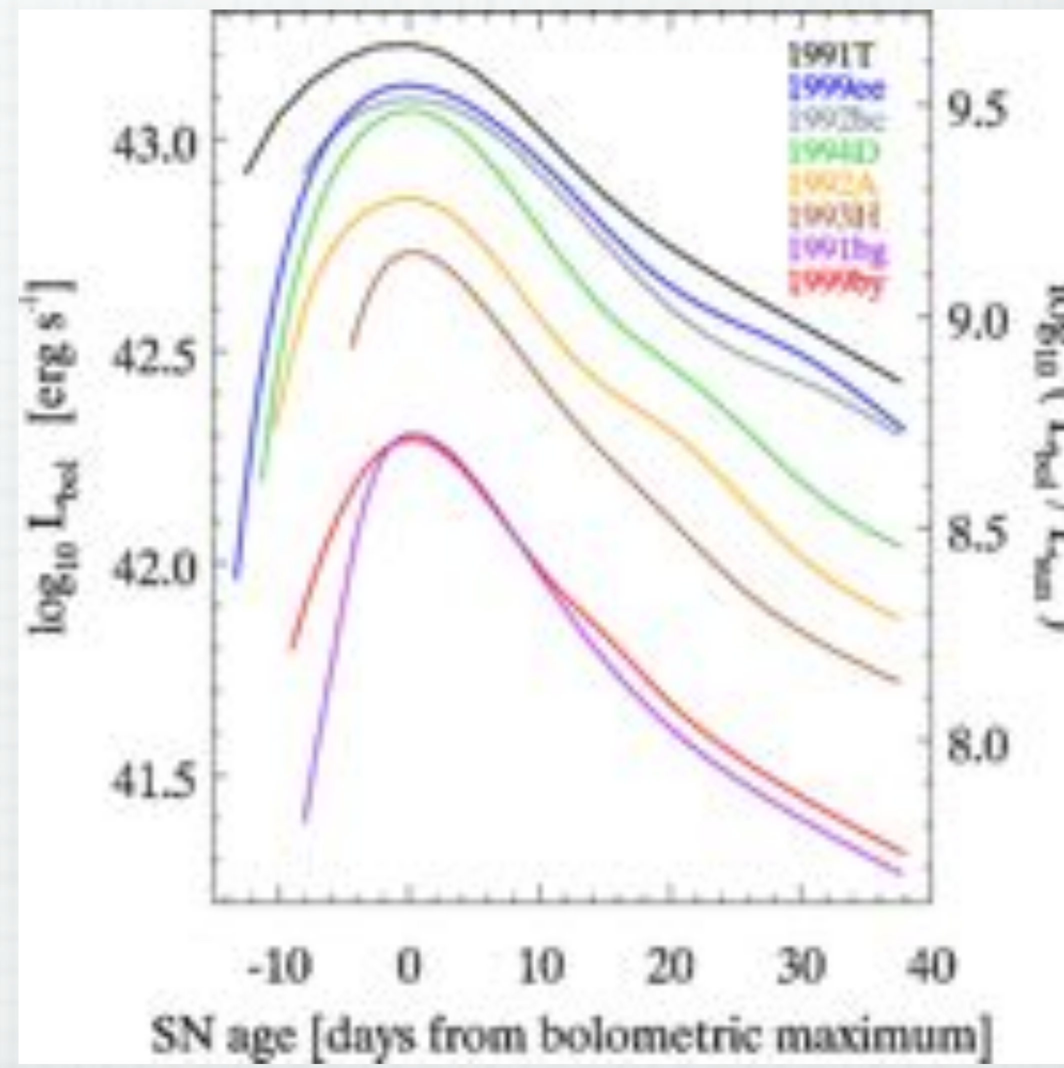
* Either deflagration or detonation
* 5 billion times more luminous than sun
* http://www.youtube.com/watch?v=9BPxc5-9M-4
* 20% accretion
* 80% merger
* 1-2 x 10^{44} joules moving at 5000-20,000 kms^{-1}



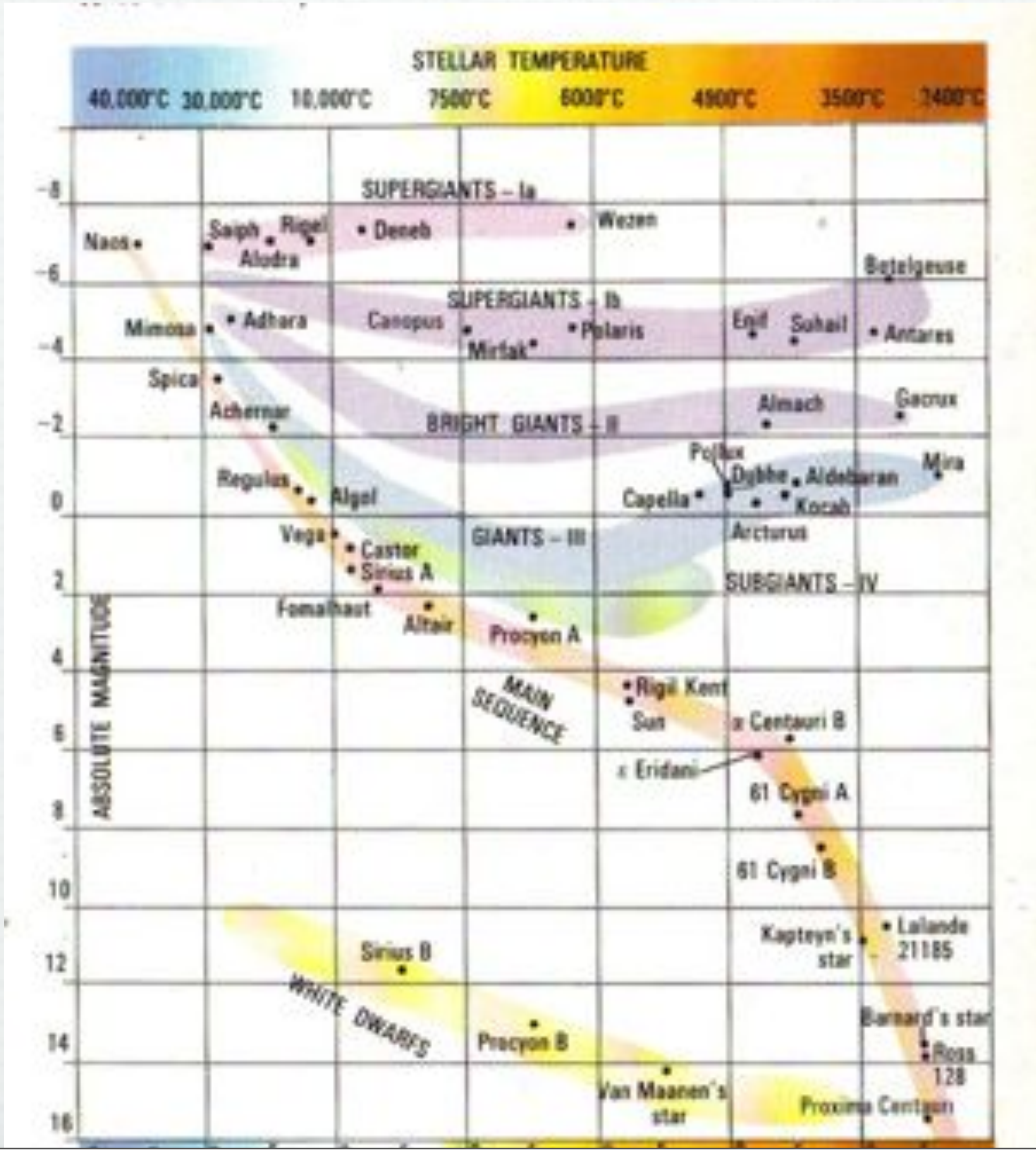
Type 1a SN

* More or less the same light curve
* Sort of
* Must correct for distance but some variability remains
* Not just one mechanism at play
* Maybe not such a standard candle





Evolution of high mass stars



Evolution of high mass stars

* Evolution proceeds as for lower mass stars up to double shell burning
* Star large enough to continue to contract core until carbon burning commences
* Now we have a continual core + shell + shell until carbon burning is finished
* Then triple shell until neon burning is ignited
* Continue until iron builds up in large quantities in core
* Too much iron and it's over

Evolution of high mass stars

* Once iron build up begins the useable fuel in the core decreases
* More iron means the core just contracts and heats until photodisintegration

$$^{56}Fe + \gamma \rightarrow 13\,^4He + 4n$$

$$^4He + \gamma \rightarrow 2p + 2n$$

* When the core mass exceeds the Chandrasekhar limit it begins to collapse and neutronization occurs

$$p + e^- \rightarrow n + \nu_e$$

Core collapse supernovae

* Core collapses until neutron degeneracy sets in
* The outer shell falls inward at nearly a fifth the speed of light
* Core collapses from size of earth to a radius of approximately 50 km in one second
* Core of star reaches 100 billion degrees
* Coe reached density of 8×10^{17} kg m^{-3}
* Core rebounds and neutrino flux acts to detonate the outer star
* About 10^{46} joules of neutrinos are produced with 10^{44} reabsorbed by the star causing it to explode

Evolution of high mass stars

* Energetics
  * 10^{44} joules of kinetic energy
  * 10^{42} joules in photons or 10^{9} solar luminosity
* Remnants
  * Neutron Star
  * Black Hole
* Famous Supernovae
  * Crab Nebula 1054 AD
  * Kepler's SN 1604 (last visible to naked eye)
  * 1987a
  * Core collapse occur a couple to a few time per century per galaxy

Neutron Stars

* At very close range the strong force is repulsive
* The neutrons left behind for a core with a mass less than about 3 solar masses settle down into a system with nuclear densities and a radius

$$R_{ns} \approx 11km(\frac{M_{ns}}{1.4M_{\odot}})^{1/3}$$

* The Escape velocity at the surface is about a third the speed of light
* The acceleration is about 10-100 billion times that of earth
    * An object dropped from 1 meter would hit the surface in .000001 seconds at 2000 kms^{-1}
* The atmosphere has a scale height of less than a centimeter
* The luminosity of newly formed neutron stars can be as high as a fifth a solar luminosity
* They can rotate several times to several hundred times a second

Black Holes

* Or nature's perfect censors
* Throw enough mass in a small enough space and it creates a region of space-time curved so much even light can't get out

$$c^2 d\tau^2 = \left(1 - \frac{r_s}{r}\right) c^2 dt^2 - \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta \, d\varphi^2\right),$$

* Let's play around
* http://casa.colorado.edu/~ajsh/

Black Holes

* Actually emit blackbody radiation
* Particle/Anti Particle pairs
* A one solar mass blackhole emits at one 60 billionth of a kelvin, actually gains more energy from the CMB than it emits
* A moon massed black hole would have a temperature of 2.7 degrees kelvin

$$T = \frac{\hbar c^3}{8\pi G M k_B} \quad \left(\approx \frac{1.227 \times 10^{23}\ \mathrm{kg}}{M}\ \mathrm{K}\right),$$

Black Holes

* Emits at a rate of $P = \frac{\hbar c^6}{15360 \pi G^2 M^2}$
* or $P = \frac{\hbar c^6}{15360 \pi G^2 M_\odot^2} = 9.004 \times 10^{-29}\ \mathrm{W}$
* Truly black, would take 4x10^{27} to power a light bulb
* Evaporates $K_{\mathrm{ev}} = \frac{\hbar c^6}{15360 \pi G^2} = 3.562 \times 10^{32}\ \mathrm{W \cdot kg^2}$

$$P = -\frac{dE}{dt} = \frac{K_{\mathrm{ev}}}{M^2}$$

$$P = -\frac{dE}{dt} = -\left(\frac{d}{dt}\right) M c^2 = -c^2 \frac{dM}{dt}$$

$$-c^2 \frac{dM}{dt} = \frac{K_{\mathrm{ev}}}{M^2}$$

$$\int_{M_0}^{0} M^2 dM = -\frac{K_{\mathrm{ev}}}{c^2} \int_0^{t_{\mathrm{ev}}} dt$$

$$t_{\rm ev} = \frac{c^2 M_0^3}{3K_{\rm ev}} = \left(\frac{c^2 M_0^3}{3}\right)\left(\frac{15360\pi G^2}{\hbar c^6}\right) = \frac{5120\pi G^2 M_0^3}{\hbar c^4} = 8.410\times10^{-17}\left[\frac{M_0}{\rm kg}\right]^3 \text{ s}$$

Let's calculate the evaporation time of a mini black hole created out of 10-1000 TeVs in the large hadron collider

or 10-1000 x 1.6 x 10^{-7} joules

or m = 1.78x$10^{[-23,-21]}$kg

But not likely to occur since this is below the planck mass

Probably need a 1000 light year diameter accelerator to get to the planck mass

Stellar clusters

* Open clusters
  * Hundreds to thousands of stars all formed at the same time
  * Same age
  * May have hydrogen gas left
  * Form from gas clouds with number densities of order 10^2-10^6 cm^{-3}
  * Only 30-40% of gas goes into stars, the rest ejected

Stellar clusters

* Globular clusters
  * Older clusters of up to a million stars
  * Separation distances of order a parsec to a hundred AU
  * Metal poor
  * May contain intermediate mass black holes