Phys 321
Spring 2023

Quantum Theory I: Homework 5

Due: 10 February 2023

1 Photons

Light is incident on a polarizing filter whose transmission axis is oriented at angle θ from the vertical. The light that passes through this polarizing filter is incident on a second polarizing filter whose transmission axis is oriented at angle $\phi > \theta$ from the vertical. Let N_1 be the number of photons that pass through the first filter and N_2 the number that pass through the second filter.

a) Let $|\psi_1\rangle$ represent that state of a photon that passes though the first filter. Express this in the form
$$|\psi_1\rangle = c_1 |\updownarrow\rangle + c_2 |\leftrightarrow\rangle$$
giving c_1 and c_2 in terms of the relevant angles.

b) Use the ket/state formalism for calculating probabilities to show that the probability with which a photon that is incident on the second filter is subsequently transmitted by the second filter is $[\cos(\phi - \theta)]^2$.

c) What would Malus' law predict for the fraction of photons, originally incident on the second filter that are transmitted by the second filter i.e. N_2/N_1? How does this compare to the result determined from the ket formalism?

2 Bra vectors and inner products

Consider the kets

$$\begin{aligned}
|\phi_1\rangle &= \frac{5}{13}|+\hat{\boldsymbol{z}}\rangle - \frac{12}{13}|-\hat{\boldsymbol{z}}\rangle\,, \\
|\phi_2\rangle &= \frac{3i}{5}|+\hat{\boldsymbol{z}}\rangle + \frac{4}{5}|-\hat{\boldsymbol{z}}\rangle\,, \text{and} \\
|\phi_3\rangle &= \frac{1+i}{2}|+\hat{\boldsymbol{z}}\rangle + \frac{1-i}{2}|-\hat{\boldsymbol{z}}\rangle\,.
\end{aligned}$$

a) For each $|\phi_i\rangle$ determine an expression for the associated bra $\langle\phi_i|$ in terms of $\langle+\hat{\boldsymbol{z}}|$ and $\langle-\hat{\boldsymbol{z}}|$.

b) Express each bra $\langle\phi_i|$ as a row vector.

c) Use bra and ket operations to calculate each of $\langle\phi_i|\phi_j\rangle$. *Note: You can use $\langle\phi_j|\phi_i\rangle = (\langle\phi_i|\phi_j\rangle)^*$ to reduce the number of calculations.*