Phys 321
Spring 2022

Quantum Theory I: Class Exam I

3 March 2022

Name: __________________________ **Total:** **/50**

Instructions

- There are 5 questions on 9 pages.
- Show your reasoning and calculations and always explain your answers.

Physical constants and useful formulae

Charge of an electron	$e = -1.60 \times 10^{-19}\,\mathrm{C}$
Planck's constant	$h = 6.63 \times 10^{-34}\,\mathrm{Js}$ $\quad\hbar = 1.05 \times 10^{-34}\,\mathrm{Js}$
Mass of electron	$m_e = 9.11 \times 10^{-31}\,\mathrm{kg} = 511 \times 10^{3}\,\mathrm{eV/c^2}$
Mass of proton	$m_p = 1.673 \times 10^{-27}\,\mathrm{kg} = 938.3 \times 10^{6}\,\mathrm{eV/c^2}$
Mass of neutron	$m_n = 1.675 \times 10^{-27}\,\mathrm{kg} = 939.6 \times 10^{6}\,\mathrm{eV/c^2}$
Spherical coordinates	$\hat{\mathbf{n}} = \sin\theta\cos\phi\hat{\mathbf{x}} + \sin\theta\sin\phi\hat{\mathbf{y}} + \cos\theta\hat{\mathbf{z}}$
Spin 1/2 state	$\lvert +\hat{\boldsymbol{n}}\rangle = \cos(\theta/2)\lvert +\hat{\boldsymbol{z}}\rangle + e^{i\phi}\sin(\theta/2)\lvert -\hat{\boldsymbol{z}}\rangle$
Spin 1/2 state	$\lvert -\hat{\boldsymbol{n}}\rangle = \sin(\theta/2)\lvert +\hat{\boldsymbol{z}}\rangle - e^{i\phi}\cos(\theta/2)\lvert -\hat{\boldsymbol{z}}\rangle$
Euler relation	$e^{i\alpha} = \cos\alpha + i\sin\alpha$
Spin observables	$\hat{S}_x = \frac{\hbar}{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$ $\quad\hat{S}_y = \frac{\hbar}{2}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}$ $\quad\hat{S}_z = \frac{\hbar}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$

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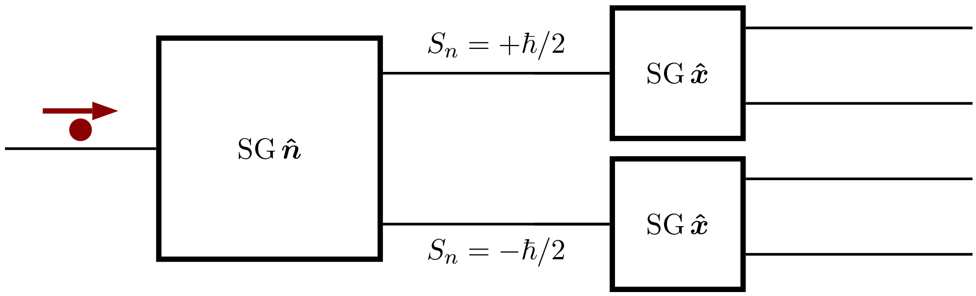
Question 1

A spin-1/2 particle is subjected to an SG $\hat{\boldsymbol{n}}$ apparatus, where $\hat{\mathbf{n}} = -\frac{1}{\sqrt{2}}\,\hat{\mathbf{x}} + \frac{1}{\sqrt{2}}\,\hat{\mathbf{z}}$, and then subsequently an SG $\hat{\boldsymbol{x}}$ apparatus.



a) Suppose that the particle emerges from the first measurement in the beam for which $S_n = +\hbar/2$. Determine an expression for the ket, in terms of $\{|+\hat{z}\rangle, |-\hat{z}\rangle\}$, that represents the state of the particle after it emerges from the SG $\hat{\boldsymbol{n}}$ apparatus.

b) The particle is subsequently subjected to an SG $\hat{\boldsymbol{x}}$ apparatus. List the measurement outcomes and the probabilities with which they occur.

Question 1 continued . . .

c) Suppose that the pair of measurements is regarded as a single measurement, yielding a pair of outcomes: one for S_n and one for S_x. Describe whether this pair of measurements is repeatable, i.e. whether a particle that yields a particular pair of outcomes and is then subjected the same measurement again will yield the same pair of outcomes with certainty.

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Question 2

Determine matrix representations for the measurement operators associated with each of the two outcomes of an S_y measurement.

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Question 3

A large collection of particles are all known to be in the state

$$|\Psi_i\rangle = \frac{4}{5}\,|+\hat{\boldsymbol{z}}\rangle + \frac{3i}{5}\,|-\hat{\boldsymbol{z}}\rangle$$

at an initial instant.

a) A quarter of these particles are subjected to a measurement of S_z. Determine the expected value of the average measurement outcome, $\langle S_z \rangle$.

b) Another quarter of these particles are subjected to a measurement of S_x. Determine the expected value of the average measurement outcome, $\langle S_x \rangle$.

Question 3 continued . . .

c) The remaining half of the particles are subjected to an evolution described by evolution operator

$$\hat{U} = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}.$$

Determine the state of the particles after the evolution.

d) If the remaining particles were subdivided and S_z measured on one half and S_x measured on the other would the resulting expected values of the average measurement outcomes change? Explain your answer.

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Question 4

Consider the two states

$$|\psi_1\rangle = \frac{1}{2}\,|+\hat{\boldsymbol{z}}\rangle + \frac{\sqrt{3}}{2}\,|-\hat{\boldsymbol{z}}\rangle$$
$$|\psi_2\rangle = \frac{1}{2}\,|+\hat{\boldsymbol{z}}\rangle - \frac{\sqrt{3}}{2}\,|-\hat{\boldsymbol{z}}\rangle$$

Can each of these states possibly be associated with each of the outcomes $S_n = +\hbar/2$ and $S_n = -\hbar/2$ for a single spin component measurement? Explain your answer.

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Question 5

An scientist observes the evolution of a spin-1/2 system by observing what happens to particles in the states $\{|+\hat{\boldsymbol{z}}\rangle, |-\hat{\boldsymbol{z}}\rangle\}$. Based on these, the scientist proposes one of the two following evolution operators

$$\hat{U}_1 = |+\hat{\boldsymbol{x}}\rangle\langle+\hat{\boldsymbol{z}}| + |-\hat{\boldsymbol{x}}\rangle\langle-\hat{\boldsymbol{z}}| \qquad \text{or} \qquad \hat{U}_2 = |+\hat{\boldsymbol{x}}\rangle\langle+\hat{\boldsymbol{z}}| + |-\hat{\boldsymbol{y}}\rangle\langle-\hat{\boldsymbol{z}}|$$

Determine the matrices that represent each of these and check which is a possible evolution operator.

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