

Fundamental Mechanics: Class Exam 1

24 February 2023

Name: SOLUTION

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Instructions

- There are 8 questions on 6 pages.
- Show your reasoning and calculations and always explain your answers.

Physical constants and useful formulae

$$g = 9.80 \text{ m/s}^2$$

Question 1

At *one instant* a mouse passes a piece of cheese while running left with speed 8.0 m/s. After moving left for a while, the mouse turns around and runs right. At a *later instant* it passes the same piece of cheese while running right with speed 10.0 m/s. The time elapsed between these instants is 3.0 s. Determine the average acceleration of the mouse between these instants.

earlier \leftarrow \odot $V_0 = -8.0 \text{ m/s}$] +2 \rightarrow time 3.0 s.

later \rightarrow \odot $V = 10.0 \text{ m/s}$

$a_{\text{avg}} = \frac{\Delta V}{\Delta t} = \frac{10.0 \text{ m/s} - (-8.0 \text{ m/s})}{3.0 \text{ s}}$ \rightarrow negative sign missing (-2)

$\boxed{+2}$ $\boxed{+3}$ $= 6.0 \text{ m/s}^2$ 1/6

Question 2

Two carts slide along a horizontal track. At one instant cart A moves right with speed 25 m/s and cart B moves with speed 40 m/s. Geraldine states that, in the 1 s period immediately before, cart B *must* have had a higher acceleration than cart A. Zog disagrees and says it is possible that cart A had a larger acceleration than cart B in this period. Explain who is correct.

Zog is correct
+2

Cart B could have moved with a constant speed of 40 m/s. Then its acceleration was zero

Cart A could have moved with an increasing speed (say 20 m/s \rightarrow 25 m/s) and thus have a non-zero acceleration.

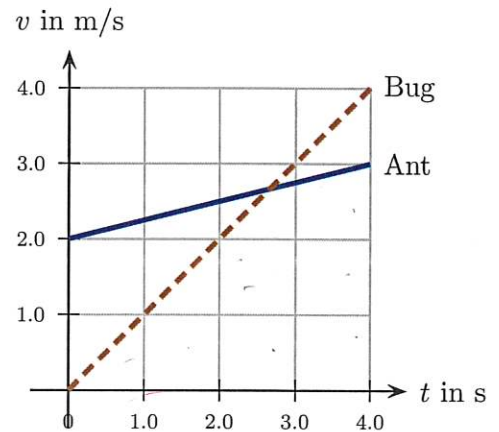
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Question 3

An ant and a bug walk along parallel straight sticks. At 0.0 s they are next to each other. The solid graph illustrates the ant's velocity vs. time. The dashed graph indicates the bug's velocity vs. time. Which of the following (choose one) is true at 4.0 s?

- +3
- i) The ant has traveled the same distance as the bug.
 - ii) The ant is further right than the bug.
 - iii) The bug is further right than the ant.

Briefly explain your choice.



+2 [$\Delta x = \text{area between graph and axis}$. Each block has area $1.0 \text{ m/s} \times 1.0 \text{ s} = 1.0 \text{ m}$.

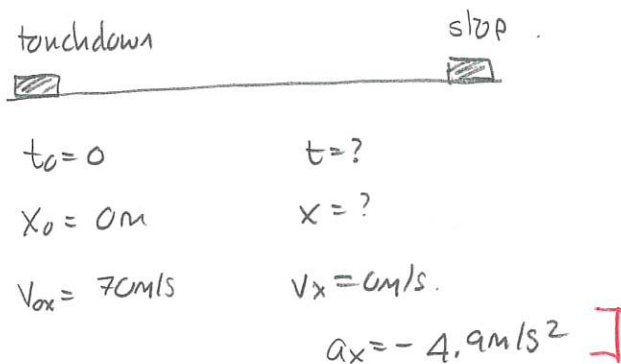
+4 [Bug area = 8 blocks $\Rightarrow \Delta x_{\text{bug}} = 8 \text{ m}$
Ant area = 10 blocks $\Rightarrow \Delta x_{\text{ant}} = 10 \text{ m}$

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Question 4

An airport on a Caribbean island has a straight horizontal runway with length 400 m (very short). An aircraft landing there first touches the runway traveling with speed 70 m/s (typical for a small commercial passenger jet). The pilot would like the aircraft to slow with a constant acceleration less than or equal to 4.9 m/s^2 while it travels in a straight line along the length of the runway. Determine whether this is possible without the aircraft traveling beyond the far end of the runway.

Evaluate distance needed to stop if accel is 4.9 m/s^2 (smaller accel would require larger distance)



$$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$$

$$0 \text{ m}^2/\text{s}^2 = (70 \text{ m/s})^2 + 2 \cdot (-4.9 \text{ m/s}^2) \cdot x \quad \Rightarrow \quad -4900 \text{ m}^2/\text{s}^2 = -2 \cdot 4.9 \text{ m/s}^2 \cdot x$$

$$\Rightarrow x = 500 \text{ m}$$

The aircraft will come to a stop in 500 m. So it is not possible without passing the end of the runway.

- +1 initial final parameters
- +3 negative acceleration
- +1 formula
- +5 calculation.
- +2 conclusion

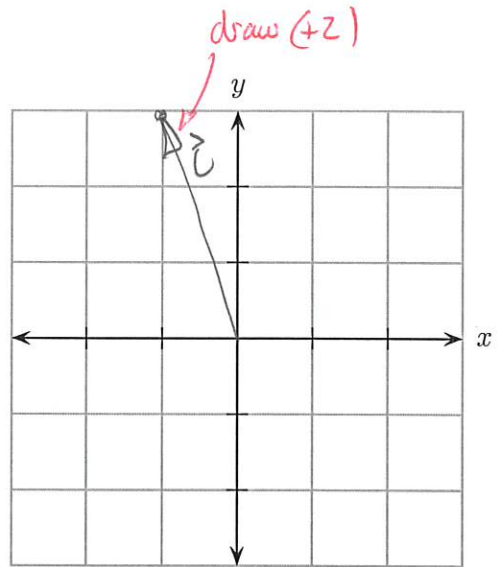
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Question 5

Consider the vectors $\vec{A} = -2\hat{i} + 2\hat{j}$ and $\vec{B} = -\hat{i} - \hat{j}$. Let $\vec{C} = \vec{A} - \vec{B}$. Determine the **components** of \vec{C} , **draw it** as accurately as possible on the coordinate axes that are provided and determine the **magnitude** of \vec{C} .

$$\begin{aligned} \vec{C} &= -2\hat{i} + 2\hat{j} - (-\hat{i} - \hat{j}) \\ &= -2\hat{i} + 2\hat{j} + \hat{i} + \hat{j} \\ &= -\hat{i} + 3\hat{j} \end{aligned}$$

✓ subtract (+2)
 either components or use unit vectors (+2)



$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{1^2 + 3^2} = 3.2\text{m} \quad (+2)$$

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Question 6

A flea jumps from a horizontal surface, leaving the ground with speed 1.8 m/s at an angle of 55° above the horizontal. *The two parts of this question can be answered independently.*

a) Determine how far away the flea lands from its launching spot.



$$\begin{aligned} v_{0y} &= 1.8 \text{ m/s} \sin 55^\circ = 1.47 \text{ m/s} \\ v_{0x} &= 1.8 \text{ m/s} \cos 55^\circ \\ &= 1.03 \text{ m/s} \end{aligned}$$

(+4)

$$\begin{aligned} a_x &= 0 \text{ m/s}^2 \\ a_y &= -9.8 \text{ m/s}^2 \end{aligned}$$

(+1)

$t_0 =$	$t =$
$x_0 = 0$	$x =$
$y_0 = 0\text{m}$	$y = 0\text{m}$
$v_{0x} = 1.03 \text{ m/s}$	$v_x =$
$v_{0y} = 1.47 \text{ m/s}$	$v_y =$

Question 6 continued ...

$$X = X_0 + v_{0x}t + \frac{1}{2}a_x t^2 \quad (+1)$$

$$X = 1.03 \text{ m/s}t$$

$$X = 1.03 \text{ m/s} \times 0.301 \text{ s}$$

$$= 0.31 \text{ m}$$

We need time:

$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 \quad (+1)$$

$$\Rightarrow 0 = v_{0y}t - \frac{1}{2}gt^2 \Rightarrow 0 = t(v_{0y} - \frac{1}{2}gt)$$

$$\Rightarrow v_{0y} - \frac{1}{2}gt = 0 \Rightarrow v_{0y} = \frac{1}{2}gt$$

$$\Rightarrow t = \frac{2v_{0y}}{g} = \frac{2 \times 1.47 \text{ m/s}}{9.8 \text{ m/s}^2}$$

$$= 0.301 \text{ s} \quad (+5)$$

- b) Someone plans to place a tiny hoop so that the flea passes through the hoop at the midpoint of its jump. How high must the hoop be above the ground?



$$t_0 = 0 \quad t =$$

$$x_0 = 0 \quad x =$$

$$(+1) \left[\begin{array}{l} y_0 = 0 \\ y = \end{array} \right.$$

$$v_{0x} = 1.03 \text{ m/s}$$

$$v_{0y} = 1.47 \text{ m/s} \quad v_y = 0 \text{ m/s}$$

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0) \quad (+1)$$

$$0 \text{ m}^2/\text{s}^2 = (1.47 \text{ m/s})^2 + 2(-9.8 \text{ m/s}^2)y$$

$$\Rightarrow -2.16 \text{ m}^2/\text{s}^2 = -19.6 \text{ m/s}^2 y$$

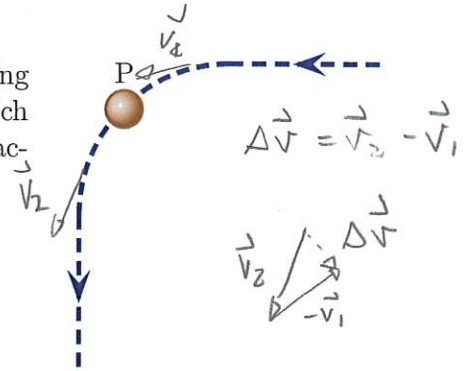
$$\Rightarrow y = \frac{2.16 \text{ m}^2/\text{s}^2}{19.6 \text{ m/s}^2} = 0.11 \text{ m} \quad (+3)$$

$$y = 0.11 \text{ m}$$

Question 7

An asteroid moves with constant speed, constantly curving around the circular corner along the illustrated trajectory. Which of the following (choose one) is true regarding the asteroid's acceleration at the point labeled P?

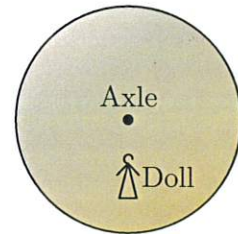
- i) $\vec{a} = 0$
- ii) $\vec{a} \neq 0$, pointing \downarrow
- iii) $\vec{a} \neq 0$, pointing \searrow
- iv) $\vec{a} \neq 0$, pointing \swarrow



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Question 8

A doll is glued to a disk with radius 0.600m that rotates about its center. The doll is halfway from the center to the edge. The acceleration of the doll is 2.50 m/s^2 . Determine the speed with which the doll moves and the time taken to complete one rotation.



+1 $\left[a = \frac{v^2}{r} \Rightarrow v^2 = ar \right]$

Here $r = 0.600\text{m}/2 = 0.300\text{m}$ +1

$\Rightarrow v^2 = 2.50\text{m/s}^2 \times 0.300\text{m}$

$= 0.75\text{m}^2/\text{s}^2$

$\Rightarrow v = \sqrt{0.75\text{m}^2/\text{s}^2} = 0.866\text{m/s}$ +3

+1 $\left[v = \frac{2\pi r}{T} \Rightarrow T = \frac{2\pi r}{v} = \frac{2\pi \times 0.300\text{m}}{0.866\text{m/s}} = 2.2\text{s} \right]$ +2

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