Fundamental Mechanics: Class Exam 3

12 April 2018

Name: ______________________________ **Total:** /70

Instructions

- There are 8 questions on 6 pages.
- Show your reasoning and calculations and always explain your answers.

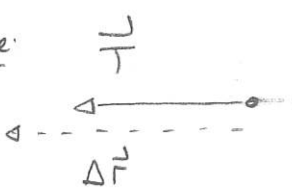
Physical constants and useful formulae

$$g = 9.81\,\mathrm{m/s^2}$$

Question 1

A 2.0 kg box is initially at rest on a horizontal frictionless floor. Subsequently a rope pulls horizontally to the left with a constant force of 27 N and the box moves 3.0 m. Determine the work done by the rope, the work done by gravity and the work done by the normal force. Use these to determine the speed of the box after it has moved 3.0 m from its starting point.

Rope:



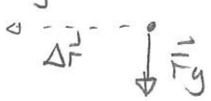
(+1) OR

$W_{rope} = \vec{T}\cdot\Delta\vec{r} = T\Delta r\cos 0^\circ$ (+1)

$= 27N \times 3.0m = 81J$ (+1)

$\boxed{W_{rope} = 81J}$

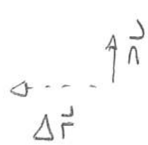
gravity



$W_{grav} = \vec{F}_g\cdot\Delta\vec{r} = mg\,\Delta r\cos 90^\circ = 0$ (+1)

$\boxed{W_{grav} = 0J}$ (+1)

normal



$W_n = \vec{n}\cdot\Delta\vec{r} = n\Delta r\cos 90^\circ = 0$ (+1) /10

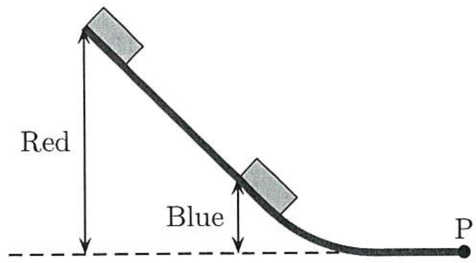
$\boxed{W_n = 0J}$ (+1)

$\Delta K = W_{net}$ (+1) $\Rightarrow \frac{1}{2}mv_f^2 - \cancel{\frac{1}{2}mv_i^2} = 81J$

$\Rightarrow$ (+1) $1.0kg\, v_f^2 = 81J \Rightarrow v_f = 9.0m/s$ (+1)

Question 2

Two identical blocks are released from rest on the same frictionless track. One of the blocks, which is red, is released from a point nine times as high as the other, which is blue. Let v_{red} be the speed of the red block when it reaches point P and v_{blue} be the speed of the blue block at the same point. Determine the ratio of these speeds $\frac{v_{\text{red}}}{v_{\text{blue}}}$. Ignore air resistance.



$W_{nc} = 0 \Rightarrow \Delta E = 0$ initial = release
final = at P

+1 $E_f = E_i \Rightarrow \overbrace{\tfrac{1}{2}mv_f^2}^{+1} + \overbrace{\cancel{mgy_f}}^{+1} = \cancel{\tfrac{1}{2}mv_i^2} + mgy_i$

+3 $v_f^2 = 2gy_i \Rightarrow v_f = \sqrt{2gy_i}$

$V_{red} = \sqrt{2g\,y_{i red}} = \sqrt{2g\,9y_{i blue}} = 3\underbrace{\sqrt{2gy_{i blue}}}_{V_{blue}}$

$\Rightarrow \frac{V_{red}}{V_{blue}} = 3.$ +2

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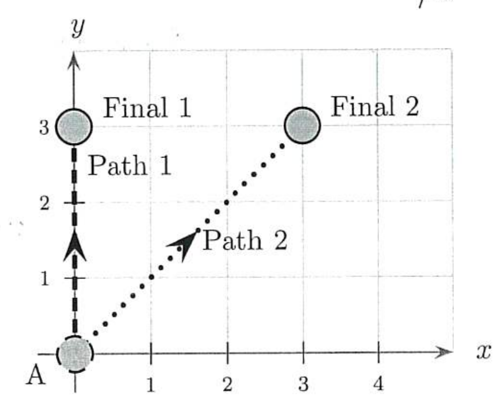
Question 3

Particles move along a horizontal surface. Both particles are initially at location A. Particle 1, with mass 1.0 kg, moves along path 1. Particle 2, with mass 2.0 kg, moves along path 2 where the axes are measured in meters. The same constant force, $\vec{F} = -2\,\text{N}\hat{\mathbf{i}} + 2\,\text{N}\hat{\mathbf{j}}$ acts on both particles. Which of the following (choose one) is true of the work done by the force as each particles moves from point A to its final location?



i) The work done is zero for both particles.
ii) The work done is the same for both particles but is non-zero.
iii) The work done on particle 1 is smaller than that done on particle 2.
iv) The work done on particle 1 is larger than that done on particle 2.

For 1 $\Delta\vec{r} = 3\hat{j} \Rightarrow W_1 = \vec{F}\cdot\Delta\vec{r} = 6\,J$ /5

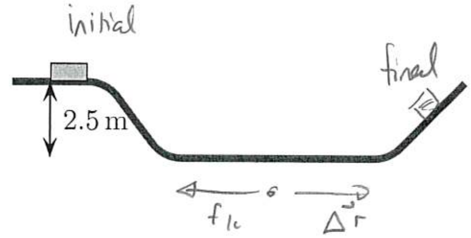
For 2 $\Delta\vec{r} = 3\hat{i} + 3\hat{j} \Rightarrow W_2 = 3(-2) + 3(2) = 0\,J$

$y_i = 2.5\text{m}$ $\qquad$ $y_F = ?$

$V_i = 5.0\text{m/s}$ $\qquad$ $V_f = 0.$

Question 4

A 0.50 kg box is launched at left end of the illustrated track with speed 5.0 m/s. A 10 m long section of the lower horizontal part of the track is rough and the coefficient of kinetic friction is 0.30. The rest of the track is frictionless and air resistance can be ignored. Determine the maximum height that the box reaches on the sloped section of track on the right.



$$\Delta E = W_{nc} \quad \Rightarrow \quad E_f - E_i = W_{nc}$$

$$\Rightarrow \quad E_f = E_i + W_{nc}$$

one of these (+1)

Non-conservative force = friction: $\quad W_{nc} = \overbrace{f_k \Delta r}^{+1} \overbrace{\cos 180°}^{+1}$

We need $f_k = \mu_k n$. But $\sum F_y = 0 \Rightarrow n = mg$. So

$$f_k = \mu_k mg. \quad (+2)$$

or number (+2)

Thus: $\quad W_{nc} = \overbrace{-\mu_k mg \Delta r}.$

So $\quad K_f + U_{gf} = K_i + U_{gi} - \mu_k mg \Delta r$ (+1)

$$\cancel{\tfrac{1}{2} m v_f^2} + mgy_f = \underbrace{\tfrac{1}{2} m v_i^2}_{+1} + \underbrace{mgy_i}_{+1} - \mu_k mg \Delta r$$

(+1) (+1)

$$\Rightarrow \quad gy_f = \tfrac{1}{2} v_i^2 + gy_i - \mu_k g \Delta r$$

(+3)

$$\Rightarrow \quad 9.8\text{m/s}^2\, y_f = \tfrac{1}{2}(5.0\text{m/s})^2 + 9.8\text{m/s}^2 \times 2.5\text{m} - 0.3 \times 9.8\text{m/s}^2 \times 10\text{m}$$

$$= 7.6\text{m}^2/\text{s}^2$$

$$\Rightarrow \quad y_F = \frac{7.6\text{m}^2/\text{s}^2}{9.8\text{m/s}^2} = \boxed{0.78\text{m}}$$

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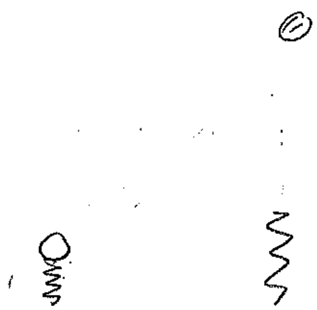
Question 5

A 0.050 kg pebble is launched vertically by a slingshot, which acts as a spring. Prior the launch, the spring is stretched by 0.40 m and the pebble is held at rest against the slingshot material. This is released and the pebble reaches a height of 30 m above its release point. Determine the spring constant of the slingshot that accomplishes this particular launch.



launch max

$y_i = 0m$ $y_f = 30m$

$V_i = 0m/s$ $V_f = 0m/s$

$\Delta x_i = 0.40m$ $\Delta x_f = 0m$

$$E_f = E_i$$

$$\cancel{K_f} + U_{gf} + \cancel{U_{sp\,f}} = \cancel{K_i} + \cancel{U_{gi}} + U_{sp\,i}$$

$$mgy_f = \frac{1}{2}k(\Delta x_i)^2$$

$$\Rightarrow \frac{2mgy_f}{(\Delta x_i)^2} = k$$

$$\Rightarrow k = \frac{2 \times 0.050kg \times 9.8m/s^2 \times 30m}{(0.40m)^2}$$

$$k = 184 N/m$$

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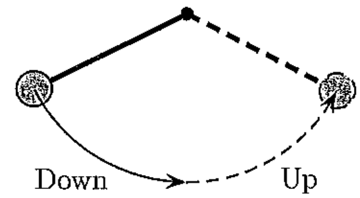
Question 6

A ball at the end of a string swings in a vertical plane in a circular arc. The ball swings down from the left and then up to the right. The highest points at the two ends of the arc are illustrated. Let $W_{\text{string down}}$ be the work done by the string as the ball swings down from the left high point to the lowest point. Let $W_{\text{string up}}$ be the work done by the string as the ball swings up from the lowest point to the right high point. Which of the following (choose one) is true?



i) $W_{\text{string down}} > 0$ and $W_{\text{string up}} > 0$.

ii) $W_{\text{string down}} > 0$ and $W_{\text{string up}} < 0$.

iii) $W_{\text{string down}} < 0$ and $W_{\text{string up}} > 0$.

iv) $W_{\text{string down}} < 0$ and $W_{\text{string up}} < 0$.

v) $W_{\text{string down}} = 0$ and $W_{\text{string up}} = 0$.

Tension is always perpendicular to direction of motion

$W_{string} = \vec{T} \cdot \Delta\vec{r} = T\Delta r \cos 90^\circ$
$= 0$

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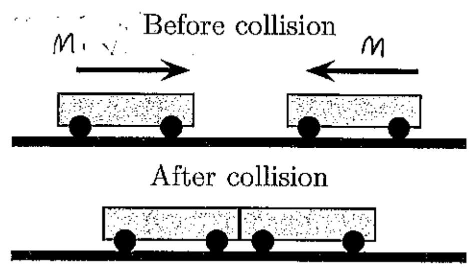
Question 7

Two identical carts travel on a frictionless surface toward each other with identical speeds and collide and stick together as illustrated. Which of the following (choose one) is true after the collision?



i) The carts are at rest and total kinetic energy is the same as before the collision.

ii) The carts are at rest and total kinetic energy is **not** the same as before the collision.

iii) The carts are **not** at rest and the total kinetic energy is the same as before the collision.

iv) The carts are **not** at rest and the total kinetic energy is **not** the same as before the collision.

Briefly explain your choice.

$p_{tot\,f} = p_{tot\,i} = \underbrace{MV}_{\text{left cart}} + \underbrace{M(-v)}_{\text{right cart}} = 0$

$(M+M)V_f = 0 \Rightarrow V_f = 0$ Carts are at rest.

$KE = \frac{1}{2}Mv^2$ non-zero before
$KE = \frac{1}{2}Mv^2 = 0$ after
} KE not same.

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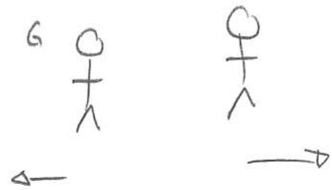
Question 8

Two ice skaters, Zog and Geraldine, hold each other while skating right at a constant velocity of 10 m/s. Zog has mass 100 kg and Geraldine has mass 60 kg. At one moment Zog pushes Geraldine and after that she moves left with a constant velocity of 5.0 m/s. Determine the velocity (including direction) of Zog after they separate.

Before :

$V_{Gi} = V_{Zi} = 10 m/s$

After



$V_{Gf} = -5.0 m/s$ $\quad V_{Zf} = ?$

$\vec{P}_{tot\,f} = \vec{P}_{tot\,i}$ (+2)

$\Rightarrow \underbrace{M_G V_{Gi} + M_Z V_{Zi}}_{+1} = \underbrace{M_G V_{Gf} + M_Z V_{Zf}}_{+1}$

$\Rightarrow 60kg \times 10 m/s + 100kg \times 10 m/s = 60kg \underbrace{(-5.0 m/s)}_{+2} + 100kg\ V_{Zf}$

(+5)

$\Rightarrow 1600 kg\,m/s = -300 kg\,m/s + 100kg\ V_{Zf}$

$\Rightarrow 1900 kg\,m/s = 100 kg\ V_{Zf}$

$\Rightarrow V_{Zf} = 19 m/s$ (right)

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