Phys 131
Spring 2018

Fundamental Mechanics: Final Exam (Version 1)

7 May 2018

Name: Solution **Total:** /150

Instructions

- There are 17 questions on 10 pages.
- Show your reasoning and calculations and always explain your answers.

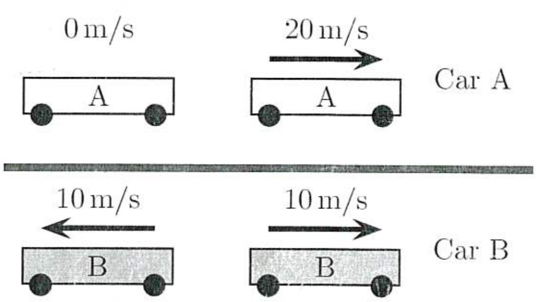
Physical constants and useful formulae

$g = 9.80\,\mathrm{m/s^2}$ $G = 6.67 \times 10^{-11}\,\mathrm{Nm^2/kg^2}$ Disk/solid cylinder: $I = \frac{1}{2}MR^2$

Hoop/hollow cylinder: $I = MR^2$ Hollow sphere: $I = \frac{2}{3}MR^2$ Solid sphere: $I = \frac{2}{5}MR^2$

✓ Question 1

Two cars each travel along a horizontal surface. The speed and direction of travel of each car is indicated at the same initial instant (left of diagram) and at the same final instant (right of diagram). The diagram is NOT to scale. Is the acceleration of car A smaller than, larger than or the same as that of car B? **Explain your answer**



(+1) $a = \frac{\Delta v}{\Delta t}$ $\Delta v = v_f - v_i$

In case A $\Delta v = 20\,\mathrm{m/s} - 0\,\mathrm{m/s} = 20\,\mathrm{m/s}$ (+2)

In case B $\Delta v = 10\,\mathrm{m/s} - (-10\,\mathrm{m/s}) = 20\,\mathrm{m/s}$. (+2)

So Δv is same; Δt is also same

$\Rightarrow$ accelerations are the $\boxed{\text{same}}$ (+1)

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Acceleration is radially inward for uniform circular motion. So at A $\uparrow$

at B $\downarrow$

Question 2

A bug walks at a constant speed around the inside of a loop which is oriented vertically. Point A is at the bottom of the loop and point B is at the top. Which of the following (choose one) is true regarding the directions of the acceleration, $\vec{\mathbf{a}}$ of the bug at the two illustrated points?

i) $\vec{\mathbf{a}}$ is $\uparrow$ at A; $\vec{\mathbf{a}}$ is $\uparrow$ at B.

ii) $\vec{\mathbf{a}}$ is $\uparrow$ at A; $\vec{\mathbf{a}}$ is $\downarrow$ at B.

iii) $\vec{\mathbf{a}}$ is $\downarrow$ at A; $\vec{\mathbf{a}}$ is $\uparrow$ at B.

iv) $\vec{\mathbf{a}}$ is $\downarrow$ at A; $\vec{\mathbf{a}}$ is $\downarrow$ at B.

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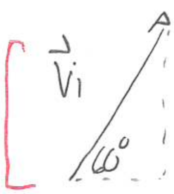
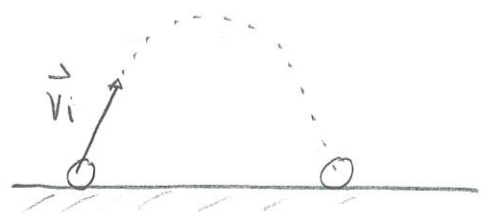
Question 3

A bug jumps from a horizontal table with a speed of 4.0 m/s. The bug leaves the table at an angle of 60° from the table and lands back on the table. Determine the time for which the bug is in the air. Ignore air resistance in this problem.



$V_{iy} = V_i \sin 60^\circ$
$= 4.0\,m/s \sin 60^\circ = 3.5\,m/s$

+3 $V_{ix} = V_i \cos 60^\circ$
$= 4.0\,m/s \cos 60^\circ = 2.0\,m/s$



$X_i = 0m$	$X_f = ?$
$y_i = 0m$	$y_f = 0m$ +1
$V_{ix} = 2.0\,m/s$	$V_{fx} =$
$V_{iy} = 3.5\,m/s$	$V_{fy} =$
$a_x = 0\,m/s^2$	$a_y = -9.8\,m/s^2$ +1

+2 $y_f = y_i + V_{iy}\Delta t + \frac{1}{2} a_y \Delta t^2$

$0m = 0m + 3.5\,m/s\,\Delta t + \frac{1}{2}(-9.8\,m/s^2)\Delta t^2$

$\Rightarrow 0m = \Delta t\,(3.5\,m/s - 4.9\,m/s^2\,\Delta t)$ + 5

$\Rightarrow \Delta t = 0s$ (describes launch)

or $3.5\,m/s - 4.9\,m/s^2\,\Delta t = 0 \Rightarrow \Delta t = \dfrac{3.5\,m/s}{4.9\,m/s^2} = 0.71s$

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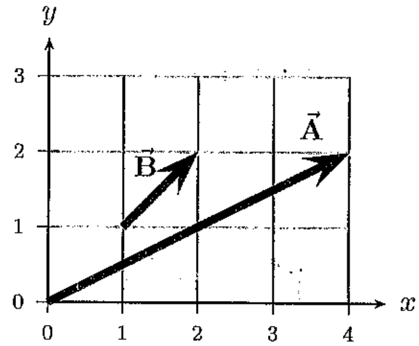
Question 4

Two vectors are as illustrated. Express $\vec{\mathbf{C}} = \vec{\mathbf{A}} - \vec{\mathbf{B}}$ in terms of standard unit vectors ($\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$).



$\vec{A} = 4\hat{i} + 2\hat{j}$

$\vec{B} = \hat{i} + \hat{j}$

So $\vec{C} = 4\hat{i} + 2\hat{j} - (\hat{i} + \hat{j})$

$= 4\hat{i} + 2\hat{j} - \hat{i} - \hat{j} = 3\hat{i} + \hat{j}$

$\vec{C} = 3\hat{i} + \hat{j}$

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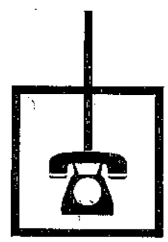
Question 5

A phone, with mass m, is suspended in an elevator as illustrated. The elevator moves up with a decreasing speed. The rope suspending the phone has no slack throughout the motion. Which of the following (choose one) is true regarding the tension in the rope that suspends the phone?



i) $T = 0\,\text{N}$.
ii) $T = mg$.
iii) $T > mg$.
iv) $0 < T < mg$.

Up decreasing speed $\Rightarrow$ accel is $\downarrow$

So $\vec{F}_{net}$ is $\downarrow$

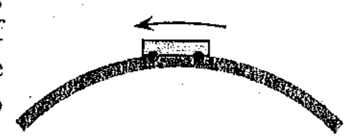
So $F_g > T$

$\Rightarrow mg > T$ No slack $\Rightarrow T \neq 0$.

/5

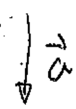
Question 6

A cart of mass m moves over a bump, with a circular cross-section, above Earths surface as illustrated. The cart's speed at the top of the bump is non-zero. Which of the following (choose one) is true regarding the magnitude of the normal force, n, exerted by the loop on the cart?



i) $n < mg$.
ii) $n = mg$.
iii) $n > mg$.

Acceleration is radially inward



So $\vec{F}_{net}$ is $\downarrow$

So $F_g > n \Rightarrow n < mg$

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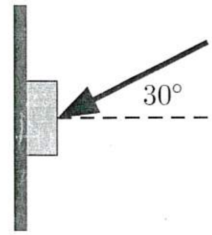
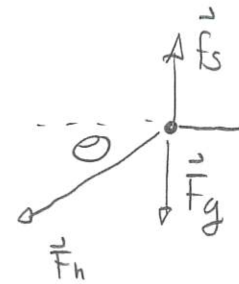
Question 7

A 1.5 kg book is pushed against a rough vertical wall by a hand which exerts a force at an angle of 30° from the horizontal. The coefficient of static friction between the book and wall is 0.80. Determine the minimum force that must be exerted by the hand to keep the book at rest.



+1
$$\sum F_x = ma_x = 0$$
$$\sum F_y = ma_y = 0$$
because at rest



+2

Need magnitudes / components of forces

$F_g = mg$

$F_{hx} = -F_h\cos 30^\circ \qquad F_{hy} = -F_h\sin 30^\circ$ ~~+4~~ +4

$f_s \le \mu_s n$ +1

	x	y
+1 $\vec{F}_g$	0	$-mg$
+1 $\vec{n}$	n	0
F_h	$-F_h\cos 30^\circ$	$-F_h\sin 30^\circ$
+1 f_s	0	f_s

Then

+4
$$\sum F_x = 0 \Rightarrow n - F_h\cos 30^\circ = 0$$
$$\Rightarrow n = F_h\cos 30^\circ$$

+1
$$\sum F_y = 0 \Rightarrow -mg - F_h\sin 30^\circ + f_s = 0$$
$$\Rightarrow f_s = mg + F_h\sin 30^\circ$$

Now $f_s \le \mu_s n$

+2 ~~+4~~
$$\Rightarrow mg + F_h\sin 30^\circ \le \mu_s F_h\cos 30^\circ$$
$$\Rightarrow mg \le F_h(\mu_s\cos 30^\circ - \sin 30^\circ)$$
$$\Rightarrow 1.5\,\text{kg} \times 9.8\,\text{m/s}^2 \le F_h(0.80\cos 30^\circ - \sin 30^\circ)$$
$$\Rightarrow 14.7\,\text{N} \le F_h\, 0.19 \Rightarrow \boxed{F_h \ge \frac{14.7\,\text{N}}{0.19} = 77\,\text{N}}$$

+1

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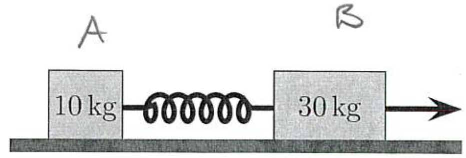
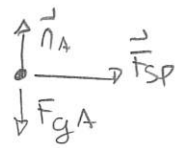
Question 8

Two boxes on a frictionless surface are separated by a spring with spring constant 200 N/m. A rope pulls the block on the right with a constant tension of 80 N. The blocks move together while a constant separation is maintained between them.



a) Determine the acceleration of the 30 kg box and the force exerted by the spring on this box.

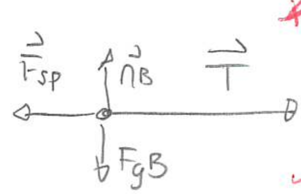
Block A



(+2) $\sum F_x = m_A a$ (+1)

$\Rightarrow \boxed{F_{sp} = m_A a}$ (+1)

Block B



(+2) $\sum F_x = m_B a$

$\Rightarrow \boxed{T - F_{sp} = m_B a}$ (+1)

Combining $\quad T - m_A a = m_B a \quad \Rightarrow \quad T = (m_A + m_B)\, a$

$\Rightarrow \quad 80N = 40kg\; a$

$\Rightarrow \quad \boxed{a = 2.0 m/s^2}$ (+3)

Then: $\quad F_{sp} = m_A a \quad \Rightarrow \quad F_{sp} = 10kg \times 2.0 m/s^2$

$\Rightarrow \quad \boxed{F_{sp} = 20N}$ (+3)

b) Determine the force exerted by the spring on the 10 kg box.

same as force exerted by spring on 10kg box

$\boxed{F_{sp} = 20N}$ (+3)

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Momentum is conserved

$$p_{tot\,f} = p_{tot\,i} = 0$$

$$M_p V_{pf} + M_b V_{bf} = 0 \qquad V_{bf} = -\frac{M_p}{M_b} V_{pf}$$

positive

negative

Question 9

A person stands at rest in a boat on a perfectly still lake; the boat is at rest (compared to the water). The person starts to walk east in the boat. Which of the following (choose one) is true while the person is walking east? Ignore air resistance and any friction between the boat and water and assume that there are no currents in the water.

Moves west

i) The boat remains at rest.
ii) The boat moves east.
iii) The boat moves west.
iv) The boat moves west if the person moves fast enough. Otherwise it remains at rest.

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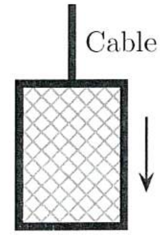
Question 10

An elevator, which is suspended by a cable, moves down.



a) The elevator slows down as it descends. Is the work done by the cable positive, negative or zero? Explain your answer.

$W = \vec{F}\cdot\Delta\vec{r}$ where $\vec{F}$ is force exerted by cable

+4

From the diagram

$$\vec{F}\cdot\Delta\vec{r} = F\Delta r\cos 180^\circ = -F\Delta r$$

so $\boxed{W < 0}$

b) The elevator speeds up as it descends. The cable does not become slack. Is the work done by the cable positive, negative or zero? Explain your answer.

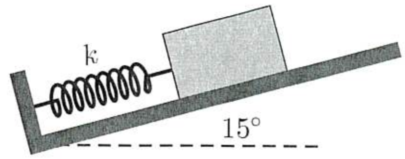
The arrangement of force and displacement is the same as for part a $\Rightarrow$ $\boxed{W < 0}$

+4

/8

Question 11

A 2.0 kg box can move along a ramp tilted at 15° above the horizontal. It is held at rest against a spring with unknown spring constant k. The spring is compressed by 0.50 m. The box is released and it leaves the spring with speed 5.0 m/s as the spring reaches its equilibrium point. Ignoring friction and air resistance, determine k.



Work done by non-conservative forces is zero. So $\Delta E = 0$

$y_i = 0$ $\quad$ $y_f = ??$

$V_i = 0$ $\quad$ $V_f = 5.0\,m/s$

$\Delta s_i = 0.50\,m$ $\quad$ $\Delta s_f = 0\,m$

$y_f = \Delta s_i \sin 15°$
$= 0.50\,m \sin 15° = 0.13\,m$ +3

+1 $E_f = E_i$

+1 $K_f + U_{gf} + U_{spf} = K_i + U_{gi} + U_{spi}$

$\underbrace{\tfrac{1}{2} m v_f^2}_{+1} + \underbrace{m g y_f}_{+1} + \underbrace{\tfrac{1}{2} k (\cancel{\Delta s_f})^2}_{+1} = \tfrac{1}{2} m \cancel{v_i^2} + m g \cancel{y_i} + \tfrac{1}{2} k (\Delta s_i)^2$

somewhat

+8

$\underbrace{\tfrac{1}{2}\, 2.0\,kg \times (5.0\,m/s)^2}_{+2} + \underbrace{2.0\,kg \times 9.8\,m/s^2 \times 0.13\,m}_{+2} = \underbrace{\tfrac{1}{2} k\, (0.50\,m)^2}_{+2}$

$\Rightarrow 25\,J + 2.5\,J = \tfrac{1}{2}\, 0.25\,m^2\, k$

$\Rightarrow 27.5\,J = 0.125\,m^2\, k$

$\Rightarrow k = \dfrac{27.5\,J}{0.125\,m^2} \Rightarrow \boxed{k = 220\,N/m}$ +2

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Question 12

Consider the vectors,

$$\vec{\mathbf{A}} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$$
$$\vec{\mathbf{B}} = 40\hat{\mathbf{i}} + 30\hat{\mathbf{j}}$$
$$\vec{\mathbf{C}} = -40\hat{\mathbf{i}} + 30\hat{\mathbf{j}}$$

$\vec{A} \cdot \vec{B} = 3 \times 40 + 4 \times 30$
$= 120 + 120 = 240$
$\vec{A} \cdot \vec{C} = 3 \times (-40) + 4 \times 30$
$= 0$

Which of the following (choose one) is true?

i) $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = \vec{\mathbf{A}} \cdot \vec{\mathbf{C}}$
ii) $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} > \vec{\mathbf{A}} \cdot \vec{\mathbf{C}}$
iii) $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} < \vec{\mathbf{A}} \cdot \vec{\mathbf{C}}$

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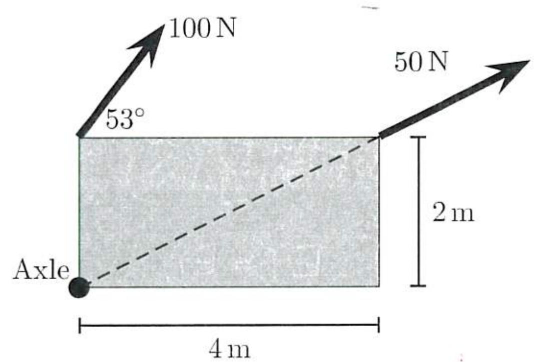
Question 13

A rectangular plate can pivot about an axle at the lower left corner and perpendicular to the page. Only two forces act on the plate, as illustrated.



a) Determine the net torque about the axle.

$\tau_{net} = \tau_{100N} + \tau_{50N}$

+1 The 50N force produces no torque because $\vec{r} \times \vec{F} = 0$

The torque produced by the 100N force is

$\tau = rF\sin\phi$ +1

$= 2m \times 100N \sin 323^\circ = -120\ Nm \Rightarrow \tau_{net} = -120Nm$

+1 +1 +1

$\vec{F}$ $\phi = 270^\circ + 53^\circ = 323^\circ$

b) Suppose that the plate is initially rotating clockwise about the axle and the two forces then act for a while. The forces maintain their magnitudes and angles (relative to the plate). Which of the following (choose one) is true *while the forces act?*

+3

i) The plate continues to rotate clockwise at a constant rate.
ii) The plate immediately speeds up and then rotates at a constant rate.
iii) The plate rotates at a rate which constantly increases.

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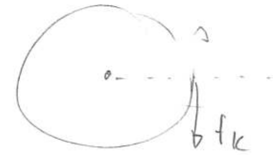
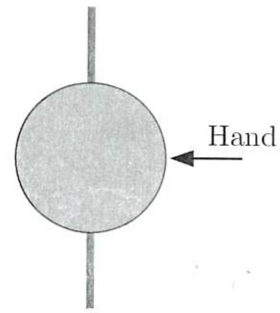
$\tau_{net} = I\alpha \Rightarrow \alpha$ negative. Then initial angular velocity is negative $\Rightarrow$ angular speed constantly increases



Question 14

A 2.0 kg solid sphere with radius 0.10 m rotates about an axle with negligible mass. The sphere rotates at 1800 rpm. A hand pushes with a constant force on the sphere at its midpoint and this brings the sphere to a stop in 6.0 s. Determine the magnitude of the frictional force exerted by the hand.



+1 $\tau_{net} = I\alpha$

Get α from kinematics:

$\omega_i = \frac{1800 \text{ rev}}{\text{min}} = \frac{1800 \text{ rev}}{\text{min}} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} = 60\pi \text{ rad/s}$

+1 $\omega_F = \omega_i + \alpha \Delta t$

+3 $\Rightarrow 0 \text{ rad/s} = 60\pi \text{ rad/s} + \alpha\, 6.0 \text{ s} \Rightarrow \alpha = -10\pi \text{ rad/s}^2$

Then $\underbrace{I = \frac{2}{5} MR^2}_{+1} = \frac{2}{5} \times 2.0 \text{ kg} \times (0.10 \text{ m})^2 = 0.0080 \text{ kg m}^2$ +1

So $\tau_{net} = 0.0080 \text{ kg m}^2 (-10\pi \text{ rad/s}^2) = -0.25 \text{ Nm}$ +2

Then $\underbrace{\tau_{net} = fR \sin\phi}_{+1} = fR \sin 270° = f\,(0.10 \text{ m})(-1)$

$\Rightarrow -0.10 \text{ m}\, f = -0.25 \text{ Nm}$

$\Rightarrow \boxed{f = 2.5 \text{ N}}$ +2

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Question 15

A satellite with mass m_s is a distance above the earth's surface exactly equal to the earth's radius. If the earth's radius is denoted R_E and the mass of the earth M_E, which of the following (choose one) represents the force exerted by the earth on the satellite?

i) $F = G\frac{M_E m_s}{2R_E}$

ii) $F = G\frac{M_E m_s}{R_E^2}$

iii) $F = G\frac{M_E m_s}{2R_E^2}$

iv) $F = G\frac{M_E m_s}{4R_E^2}$

$F = G\frac{m_s M_E}{r^2}$ $r = 2R_E$

$= G\frac{m_s M_E}{(2R_E)^2}$

$= G\frac{m_s M_E}{4R_E^2}$

/5

Question 16

A 2.0 kg solid sphere with radius 0.10 m rolls along a horizontal surface without slipping with a constant speed of 10 m/s. It approaches a ramp and rolls up the ramp without slipping. Determine the maximum vertical height reached by the sphere along the ramp.

$y_i = 0\,m$ $\quad y_f = ?$

$V_i = 10\,m/s$ $\quad V_f = 0\,m/s$

$\omega_i =$ $\quad V_i = 0\,rad/s$

+1 $\quad E_f = E_i$

+3 $\quad K_f + U_{gf} = K_i + U_{gi}$

$\cancel{K_{rot f}} + \cancel{K_{trans f}} + U_{gf} = K_{rot i} + K_{trans i} + \cancel{U_{gi}}$

$$\cancel{m} g y_f = \underbrace{\tfrac{1}{2} I \omega_i^2}_{+1} + \underbrace{\tfrac{1}{2} m v_i^2}_{+1}$$

$$= \tfrac{1}{2}\tfrac{2}{5}\cancel{m}R^2\omega_i^2 + \tfrac{1}{2}\cancel{m}v_i^2$$

$$\underbrace{I = \tfrac{2}{5} m R^2}_{+2}$$

$$\underbrace{V_i = R\omega_i}_{+1} \Rightarrow g y_f = \tfrac{1}{5} V_i^2 + \tfrac{1}{2} V_i^2$$

+2

$$\Rightarrow 9.8\,m/s^2\, y_f = \tfrac{1}{5}(10\,m/s)^2 + \tfrac{1}{2}(10\,m/s)^2 = 70\,m^2/s^2$$

$$\boxed{y_f = 7.1\,m}$$

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Question 17

Titan is a moon of the planet Saturn. Titan has mass 1.35×10^{23} kg and radius 2.57×10^{6} m. Determine the acceleration due to Titan's gravity at the surface of Titan. **Note: to receive full credit for this problem, your solution must with Newton's second law and use this to derive the answer.**

$$\vec{F}_{net} = m\vec{a}$$

$$F_g = ma$$

$$G\frac{\cancel{m} M_T}{r^2} = \cancel{m} a$$

$$a = G\frac{M_T}{r^2} = 6.67\times10^{-11}\,Nm^2/kg^2 \;\frac{1.35\times10^{23}\,kg}{(2.57\times10^{6}\,m)^2}$$

$$a = 1.36\,m/s^2$$

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